Control in Haptics: Towards Optimal Haptic Feedback

WHC-Workshop: Haptic Methods and Technologies for Virtual Assembly Simulations

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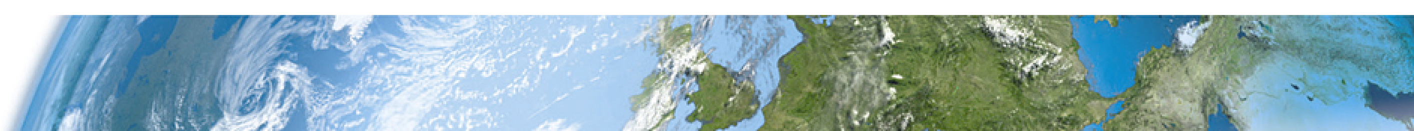
Institute of Robotics and Mechatronics
German Aerospace Center (DLR)

06.06.2017, IEEE World Haptics Conference

Motivation

“*Haptic rendering is the process of computing and generating forces in response to user interactions with virtual objects*”

[Salisbury et al., 1995]



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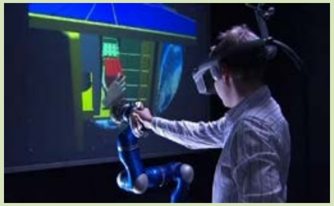
"*Haptic rendering is the process of computing and generating forces in response to user interactions with virtual objects*"

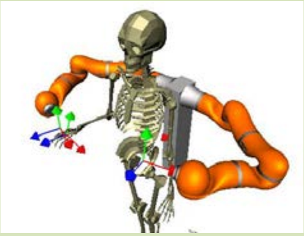
[Salisbury et al., 1995]

Haptic Rendering Applications

assembly simulation



training of mechanics



rehabilitation

augmentation in telerobotics

ergonomics analysis, computer games, education, …

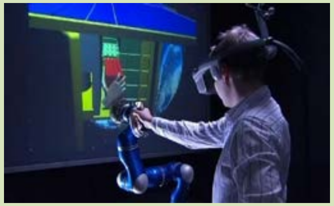
Motivatio

“Haptic r ... *forces in*
respons

[Salisbu

HUG – DLR’s bimanual haptic device
[Hulin et al., ENACTIVE2008], [Hulin et al., ICRA2011]

assembly simulation



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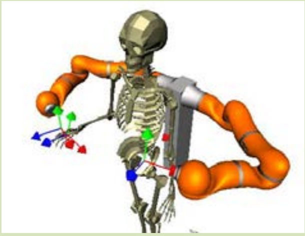

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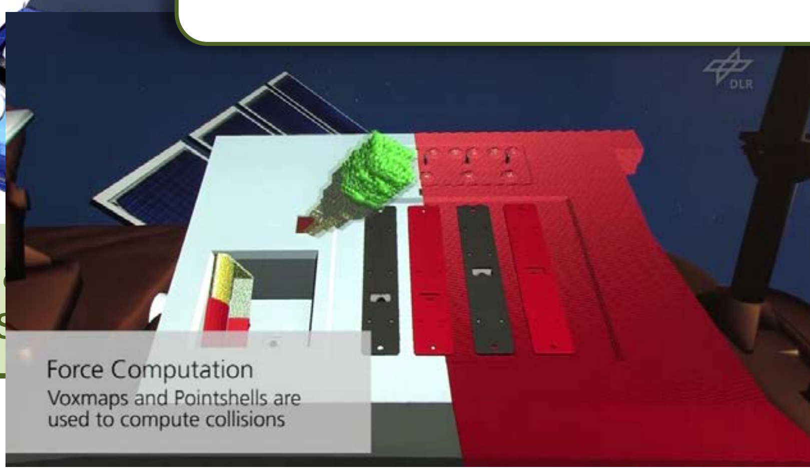
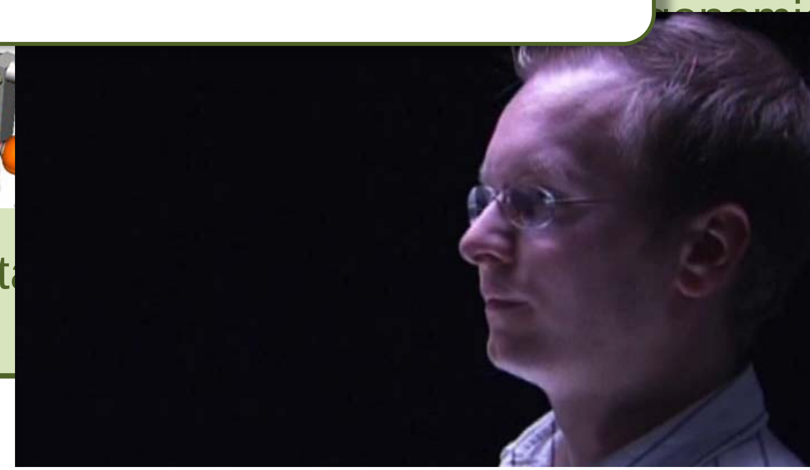
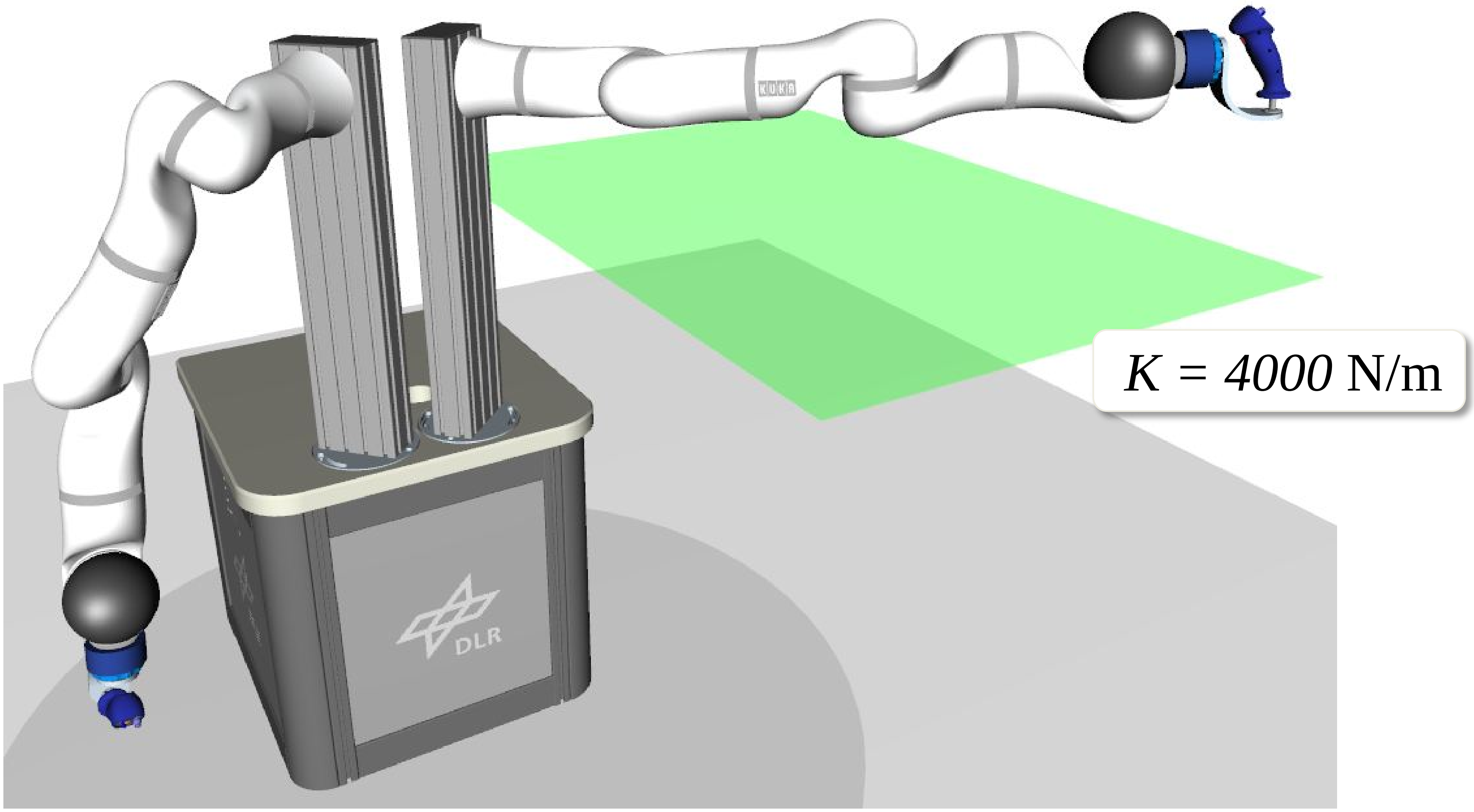
Motivation

Fundamental Requirement: Stability!





Motivation
Virtual springs are active!



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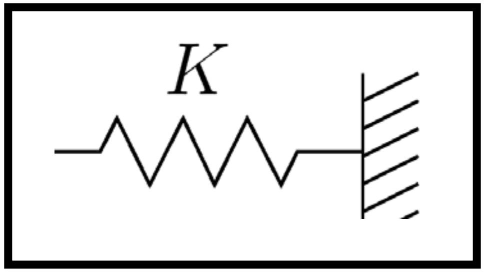

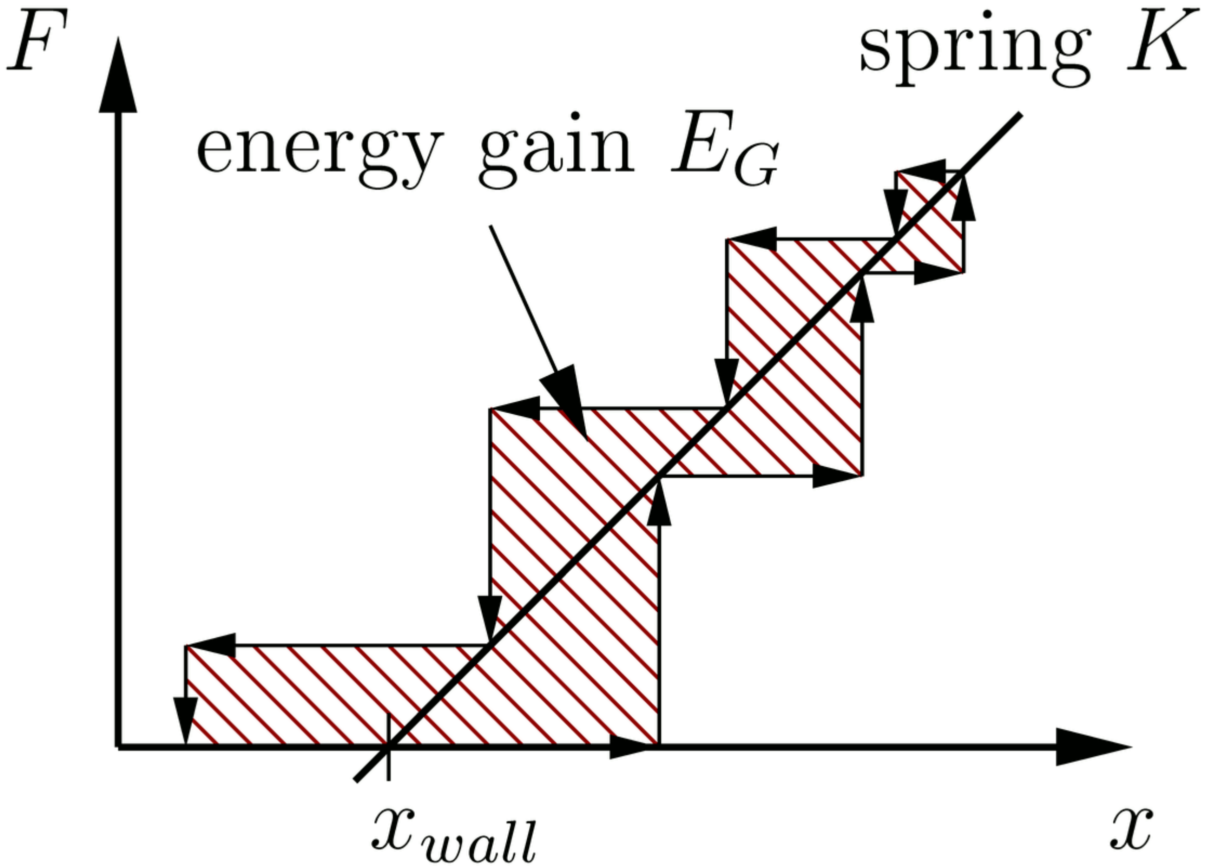

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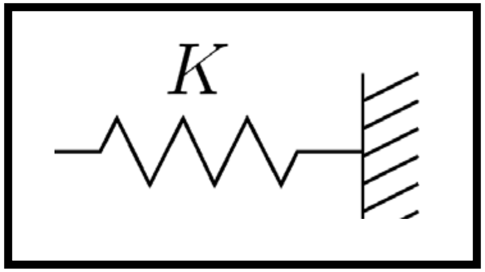

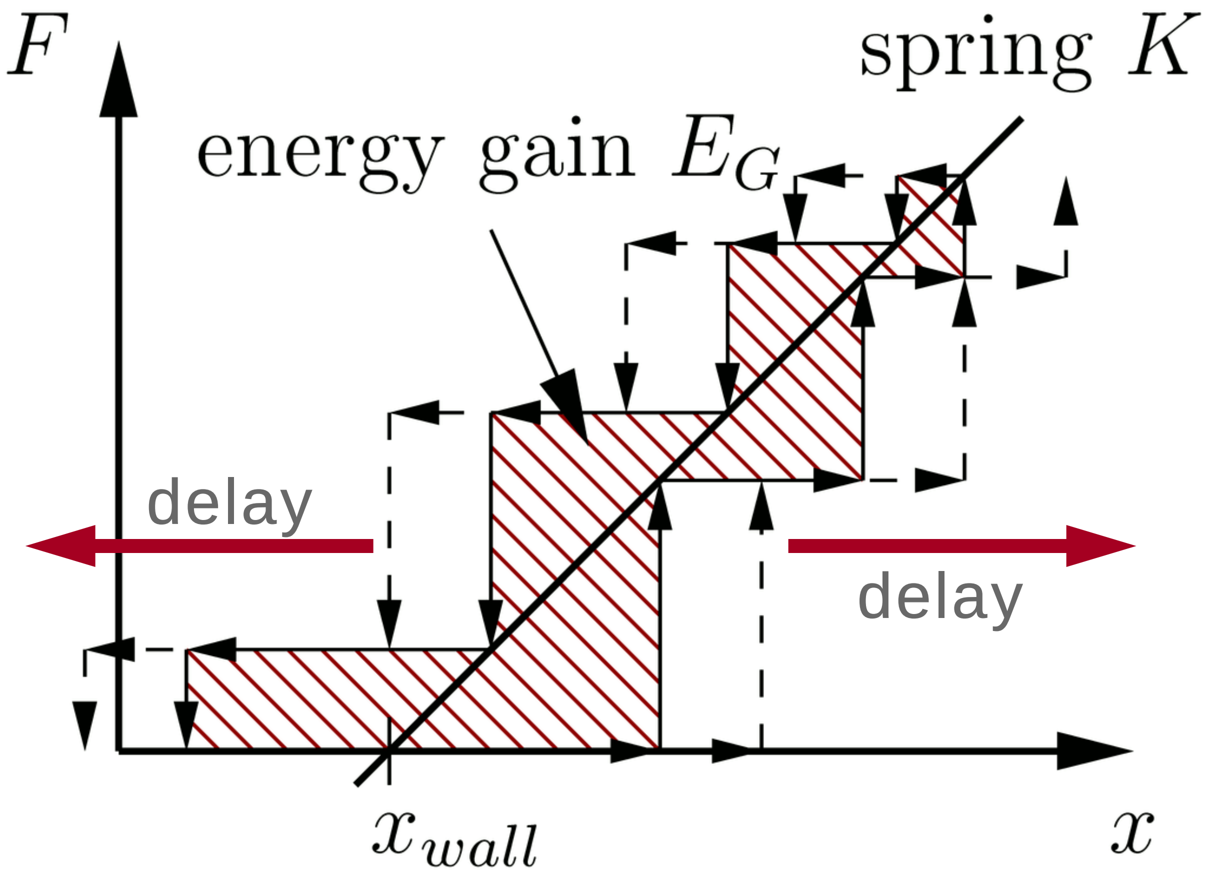

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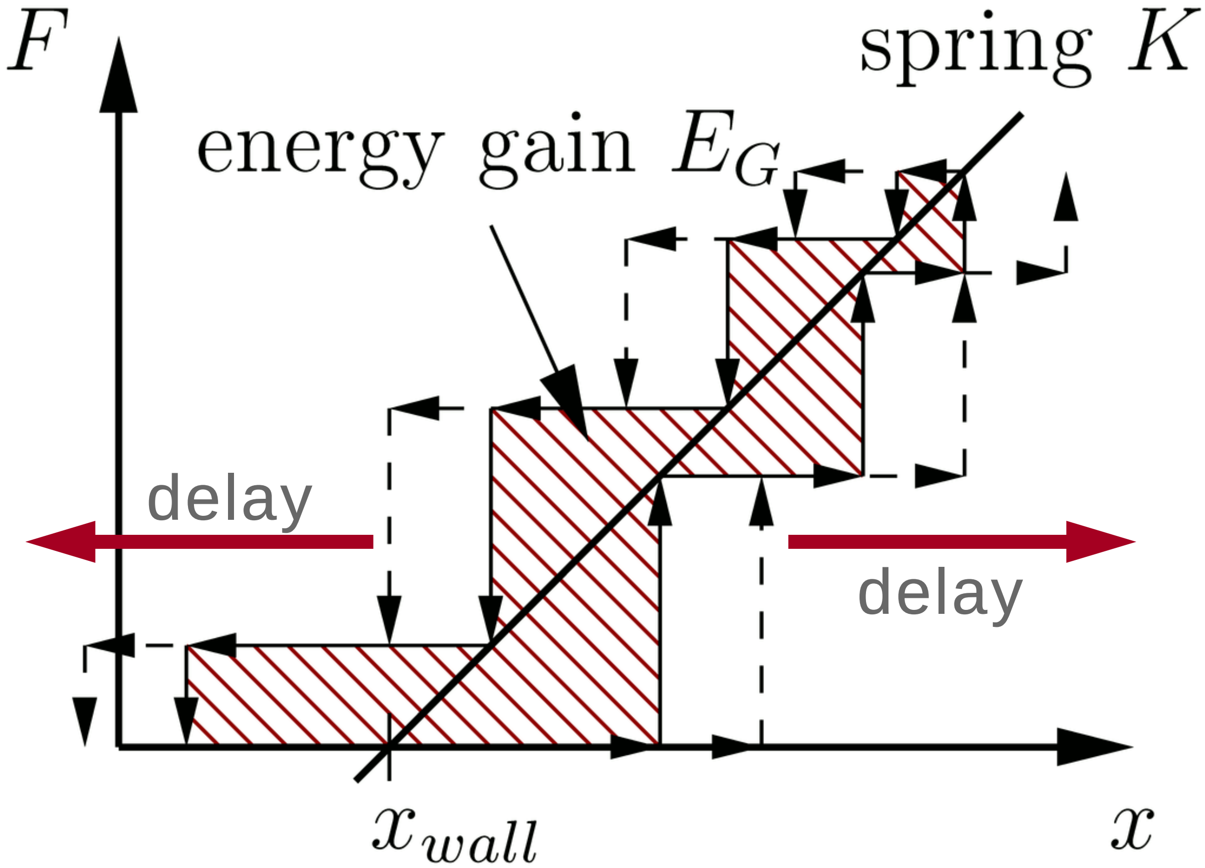
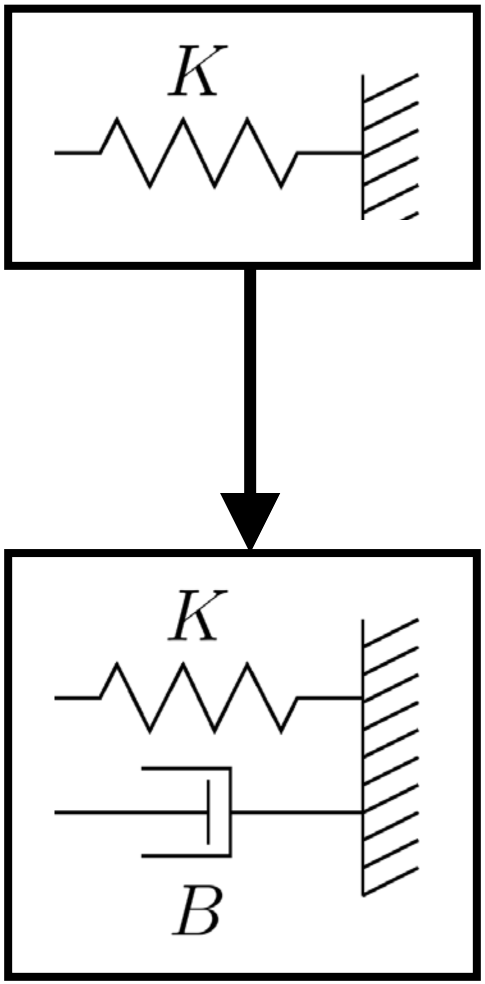
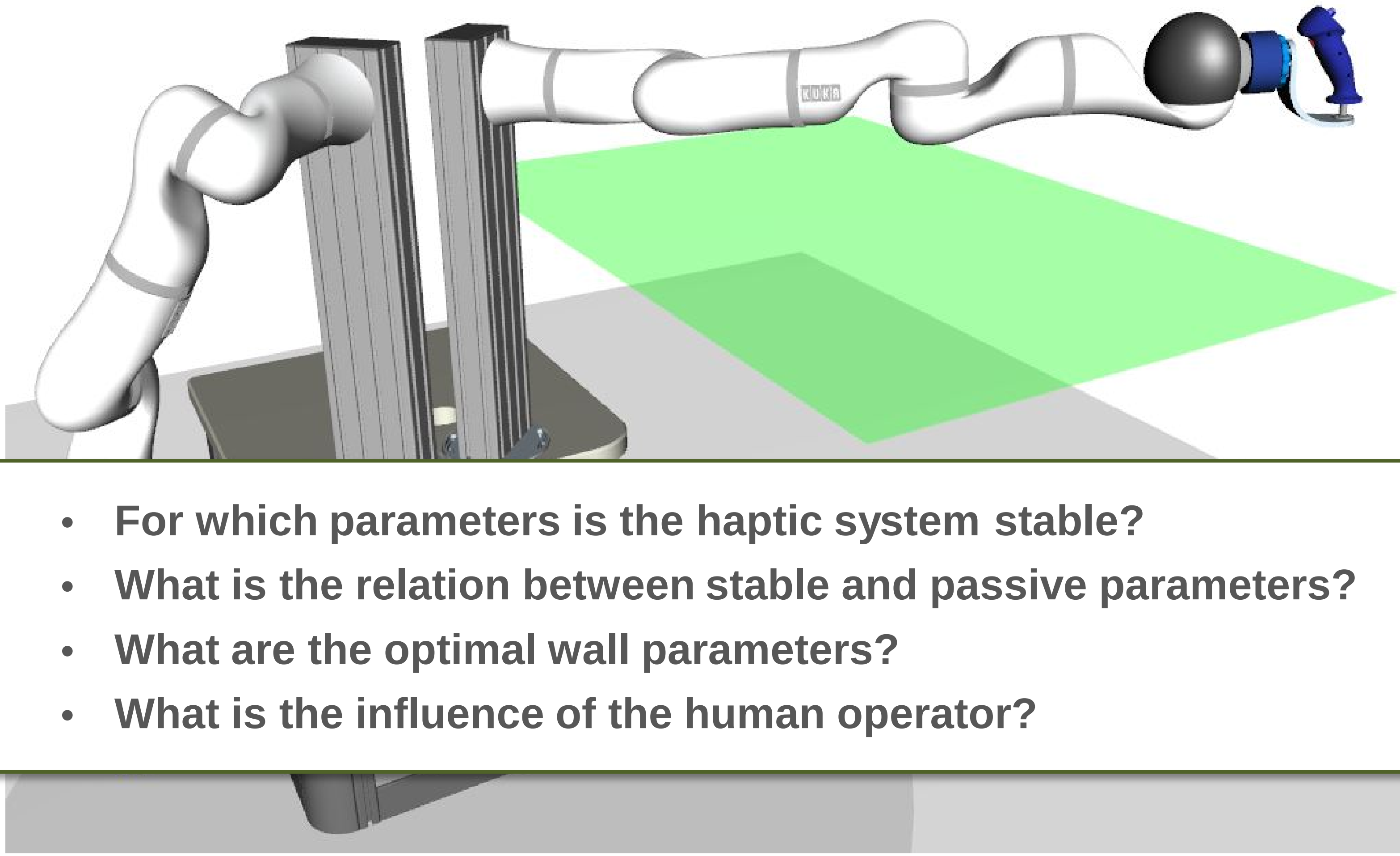




Motivation
Virtual springs are active!

K = 4000 N/m
B = 60 Ns/m



- For which parameters is the haptic system stable?
- What is the relation between stable and passive parameters?
- What are the optimal wall parameters?
- What is the influence of the human operator?

State of the Research

[Hulin Dissertation]

State of the Research

Table 2.1: Key approaches in control of haptic rendering.

		1990	1994	1996	2000	2002	2003	2005	2006	2007	2008	2015	2014
timeline		1990	1994	1996	2000	2002	2003	2005	2006	2007	2008	2015	2014
first author		Minsky	Colgate	Salcudean	Miller	Hannaford	Gil	Abbott	Diolaiti	Iskakov	Diaz	Paine	Hulin
reference		[81]	[23]	[99]	[80]	[42]	[35]	[4]	[33]	[60]	[30]	[90]	[48, 50]
analysis													
	stability	+	−	+	−	−	+	−	+	+	+	+	+
	passivity	−	+	−	+	+	−	+	+	−	−	+	+
	optimal control	−	−	−	−	−	−	−	−	−	−	+	+
linear properties													
m	mass	+	+	+	+	−	+	+	+	+	+	+	+
K	discrete-time stiffness	$\pm^a$	+	+	+	$\pm^b$	+	+	+	+	+	$\pm^a$	+
B	discrete-time damping	$\pm^a$	+	+	+	+	+	−	−	+	−	$\pm^a$	+
b	viscous friction	+	+	−	+	−	+	+	+	+	+	+	+
	human operator	+	$\pm^c$	−	$\pm^c$	$\pm^c$	+	$\pm^c$	−	−	−	−	+
d	delay	+	−	$\pm^d$	$\pm^d$	$\pm^d$	−	−	+	−	−	+	+
	velocity filtering	−	+	−	−	−	−	−	−	−	−	+	−
	structural elasticities	−	−	−	−	−	−	−	−	−	+	−	−
nonlinear properties													
	unilateral wall	−	+	−	+	−	−	+	−	−	−	−	$\pm^e$
c	static friction	−	−	+	−	−	−	+	+	+	−	−	−
Δ	pos. sensor quantization	−	−	−	−	−	−	+	+	+	−	−	−

[a] The effect of time-discretization was only considered within numerical simulations.

[b] The time-domain passivity approach does not require any assumption on the virtual environment but it takes into account the effect of time-discretization, as it assumes a constant force between two sampling instants.

[c] The passivity analysis is independent of a human operator grasping the haptic device. Thus, although not investigated directly, this work takes into account humans operators.

[d] A constant delay of up to one sampling period was considered.

[e] The unilateral wall is taken into account for passivity and not for stability.

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	human operator	+	$\pm^c$	−	$\pm^c$	$\pm^c$	+	$\pm^c$	−	−	−	−	+
d	delay	+	−	$\pm^d$	$\pm^d$	$\pm^d$	−	−	+	−	−	+	+
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[a]The effect of time-d… simulations

[b]The time-domain p… on the virt… s into account the effect of time-discretization, as it… nstants.

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$$b > \frac{KT}{2} + |B|$$

$$c \geq \frac{K\Delta}{2}$$

State of the Research

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d	delay	+	−	$\pm^d$	$\pm^d$	$\pm^d$	−	−	+	−	−	+	+
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Outline

1. Stability (9 Minutes)

2. Optimal Control (7 Minutes)

3. Experiments (5 Minutes)

Part 1: Stability

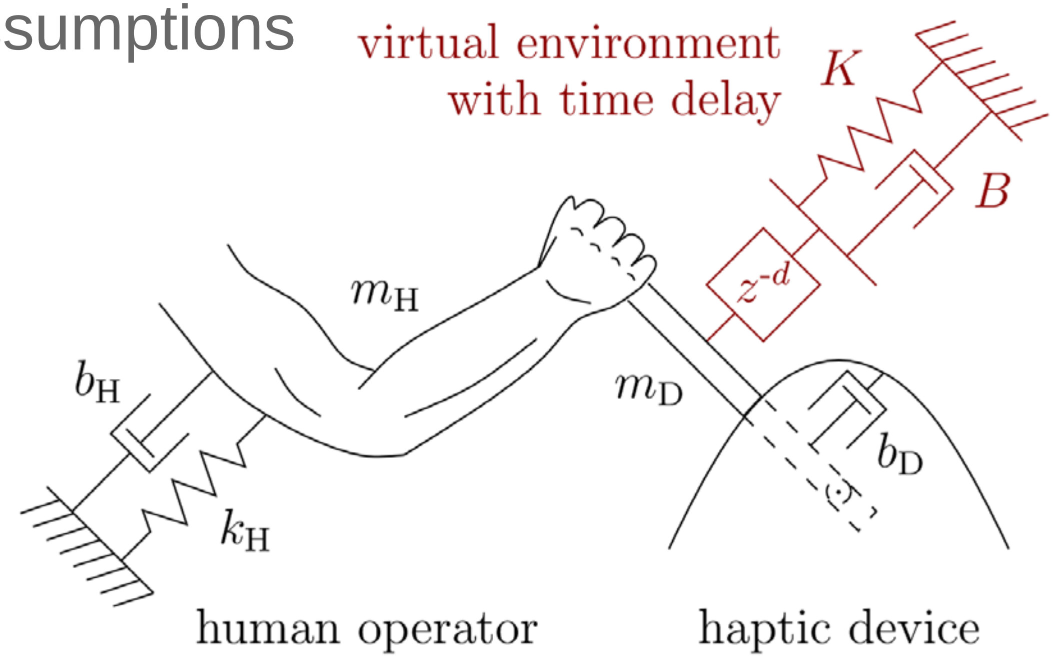
Stability Analysis of Haptic Systems

[Hulin et al., SYROCO2006],
[Hulin et al., IROS2006],
[Hulin et al., IROS2008], …

DLR

System Description

Assumptions



- 1 DoF
- Linear model of human
- Delay is permitted
- Other nonlinear effects are neglected
- Direct coupling between m_{D} and m_{H}

Virtual Wall

K : virtual stiffness

B : virtual damping

T : sampling period

t_d : time delay ($t_d = d \cdot T$)

Haptic Device

b_{D} : physical damping

m_{D} : mass

Human

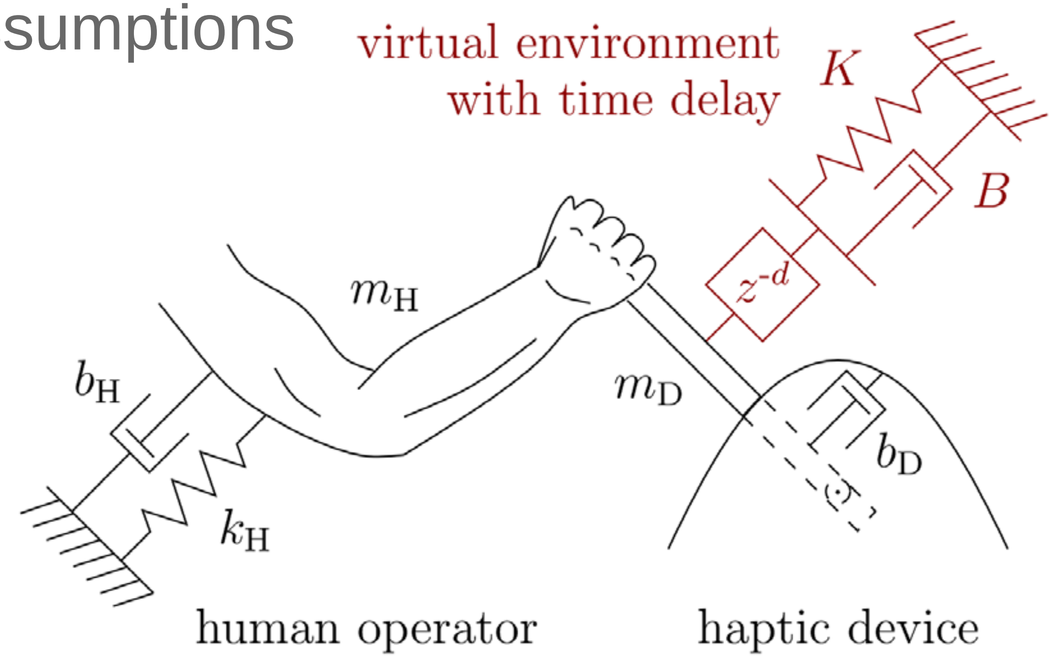
k_{H} : physical stiffness

b_{H} : physical damping

m_{H} : mass

System Description

Assumptions



$$
\begin{aligned}
m &= m_{\mathrm{D}} + m_{\mathrm{H}} \\
b &= b_{\mathrm{D}} + b_{\mathrm{H}} \\
k &= k_{\mathrm{H}}
\end{aligned}
$$

Virtual Wall

K : virtual stiffness

B : virtual damping

T : sampling period

t_d : time delay ($t_d = d \cdot T$)

Haptic Device

b_{D} : physical damping

m_{D} : mass

Human

k_{H} : physical stiffness

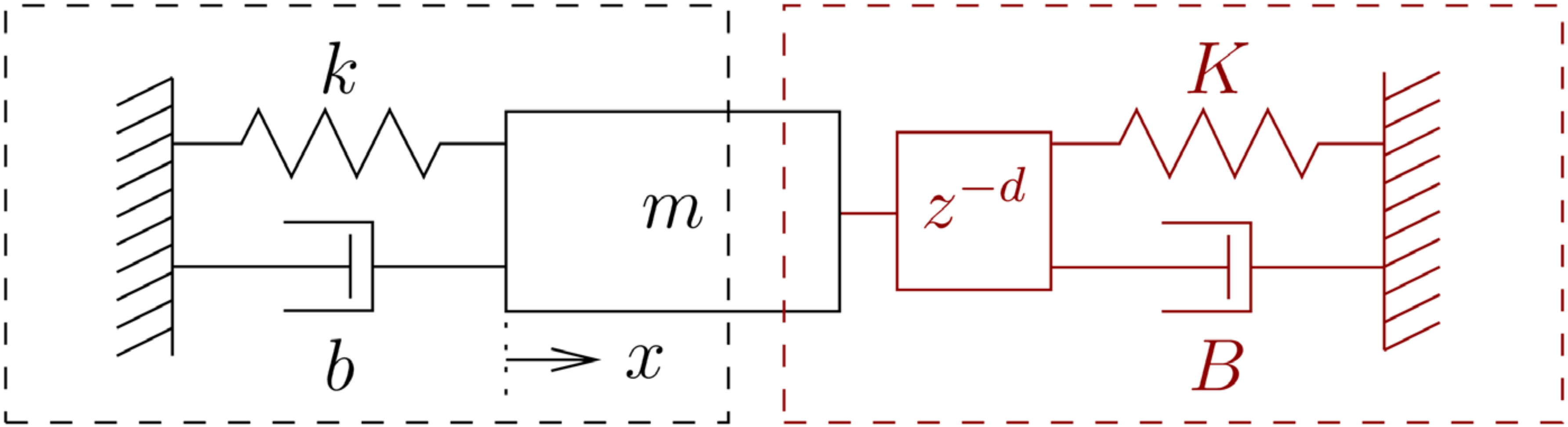
b_{H} : physical damping

m_{H} : mass

System Description
Assumptions

K : virtual stiffness
B : virtual damping
T : sampling period
t_d : time delay ($t_d = d \cdot T$)
k : physical stiffness
b : physical damping
m : mass

real world | virtual world



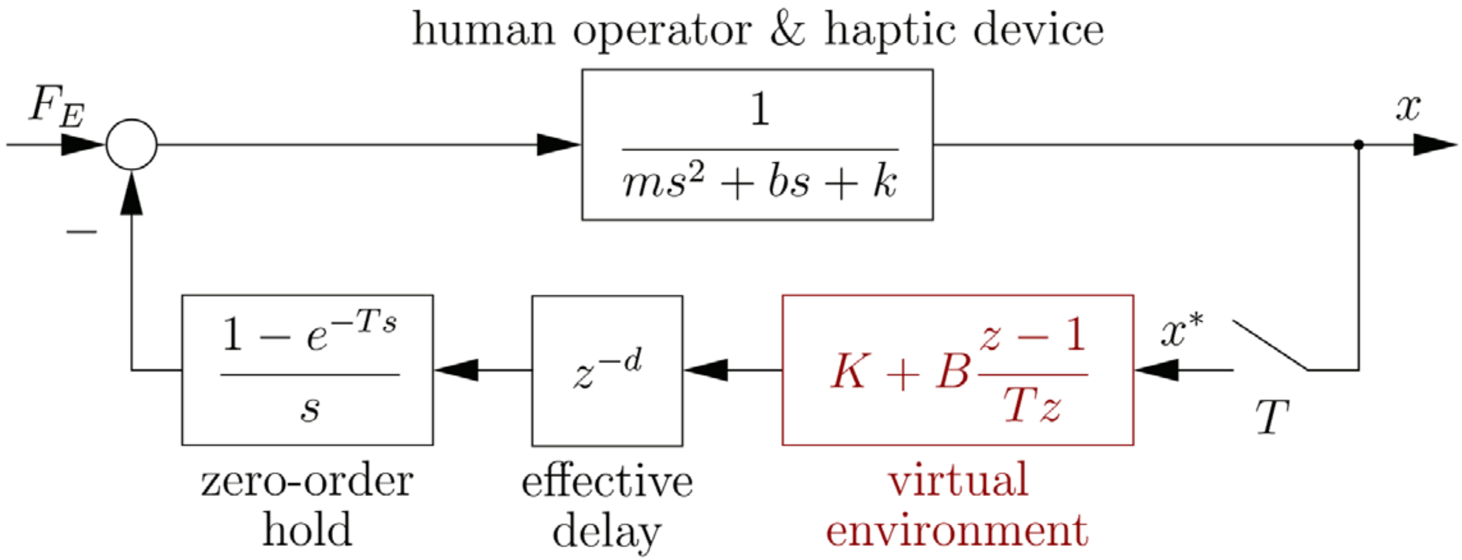
System with 7 parameters

System Description

Control Loop

Time Delay

$$t_d = d \cdot T$$



Consists of continuous- and discrete-time blocks

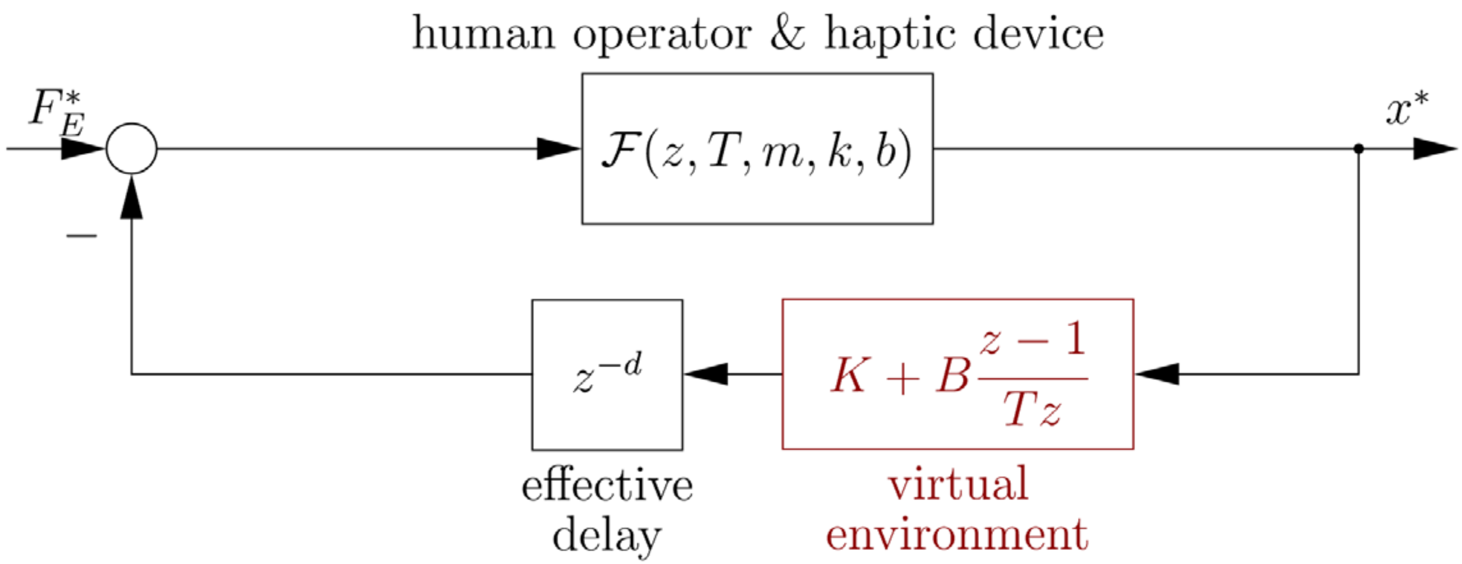
► Use ZOH-Equivalent of continuous-time block
(= Exact description!)

System Description

Control Loop

Time Delay

$$t_d = d \cdot T$$



$$\mathcal{Z}\left\{H_x(s)H_0(s)\right\} = \frac{2(z + e^{-bT/m}) - (z+1)(c_2 + c_3) + bT(z-1)(c_2 - c_3)/(mc_1)}{2k\left(z^2 - (c_2+c_3)z + e^{-bT/m}\right)}$$

with

$$\begin{aligned} c_1 &= \sqrt{(bT/m)^2 - 4kT^2/m} \\ c_2 &= e^{-(bT/m + c_1)/2} \\ c_3 &= e^{-(bT/m - c_1)/2} \end{aligned}$$

System Description
Normalization

Time Delay

$$t_d = d \cdot T$$

$$\begin{aligned}
\alpha &= K \cdot T^2/m && : \text{normalized virtual stiffness} \\
\beta &= B \cdot T/m && : \text{normalized virtual damping} \\
\gamma &= k \cdot T^2/m && : \text{normalized physical stiffness} \\
\delta &= b \cdot T/m && : \text{normalized physical damping}
\end{aligned}$$

Characteristic Equation

$$\begin{aligned}
p(z) = &\big((c_3 + c_2 - 2)c_1 + (c_3 - c_2)\delta\big)(\alpha + \beta)\ z^2 \\
&+ \Big(\big((c_3 + c_2 - 2e^{-\delta})c_1 + (c_2 - c_3)\delta\big)\alpha \\
&\quad + 2\big((1 - e^{-\delta})c_1 + (c_2 - c_3)\delta\big)\beta\Big)\ z \\
&- 2\big(z^2 - z(c_3 + c_2) + e^{-\delta}\big)c_1\gamma\ z^{1+d} \\
&+ \big((2e^{-\delta} - c_3 - c_2)c_1 + (c_3 - c_2)\delta\big)\beta
\end{aligned}$$

with

$$\begin{aligned}
c_1 &= \sqrt{\delta^2 - 4\gamma} \\
c_2 &= e^{-(\delta + c_1)/2} \\
c_3 &= e^{-(\delta - c_1)/2}
\end{aligned}$$

- Mass m and Sampling Period T dropped out!

System Description
Normalization

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+ 2\big((1 - e^{-\delta})c_1 + (c_2 - c_3)\delta\big)\beta\Big)\ z \\
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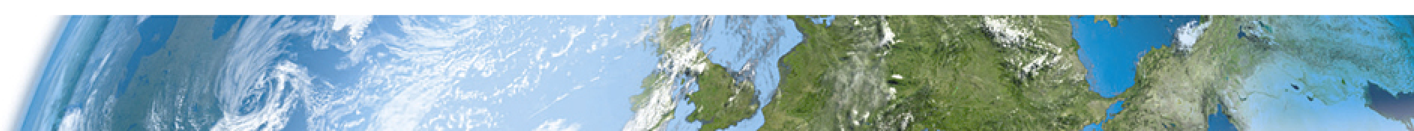
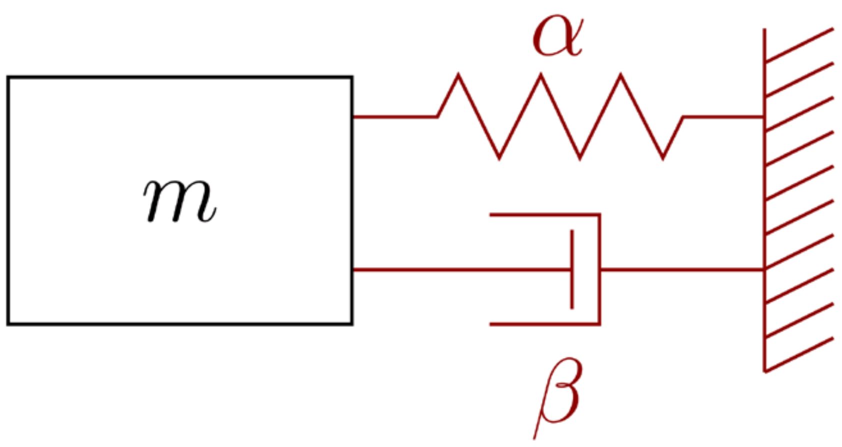
DLR



$$d = t_d \,/\, T$$
$$\alpha = KT^2 \,/\, m$$
$$\beta = BT \,/\, m$$
$$\gamma = kT^2 \,/\, m$$
$$\delta = bT \,/\, m$$

Normalized Stability Boundaries

Simple case: $\gamma = \delta = 0$



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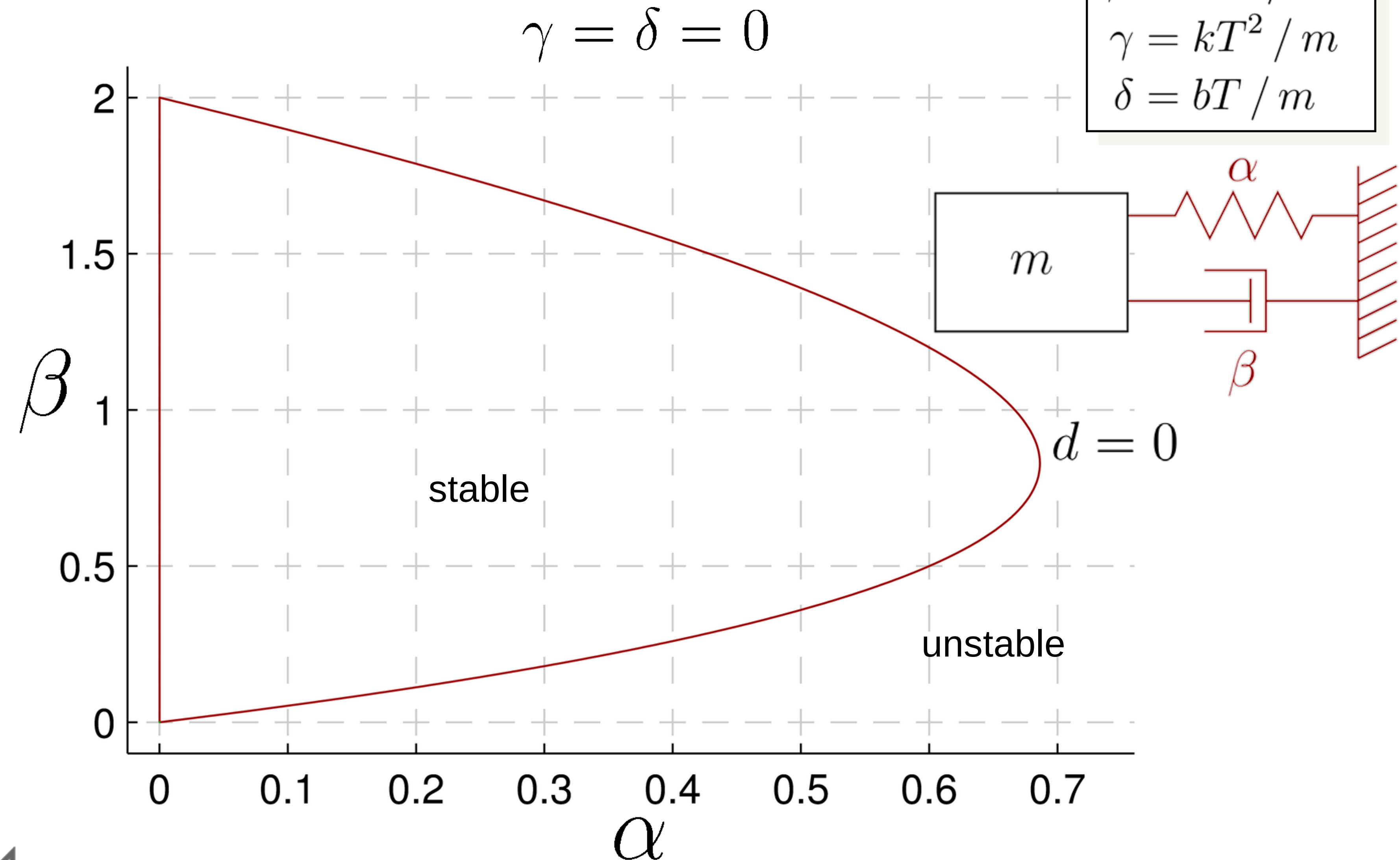

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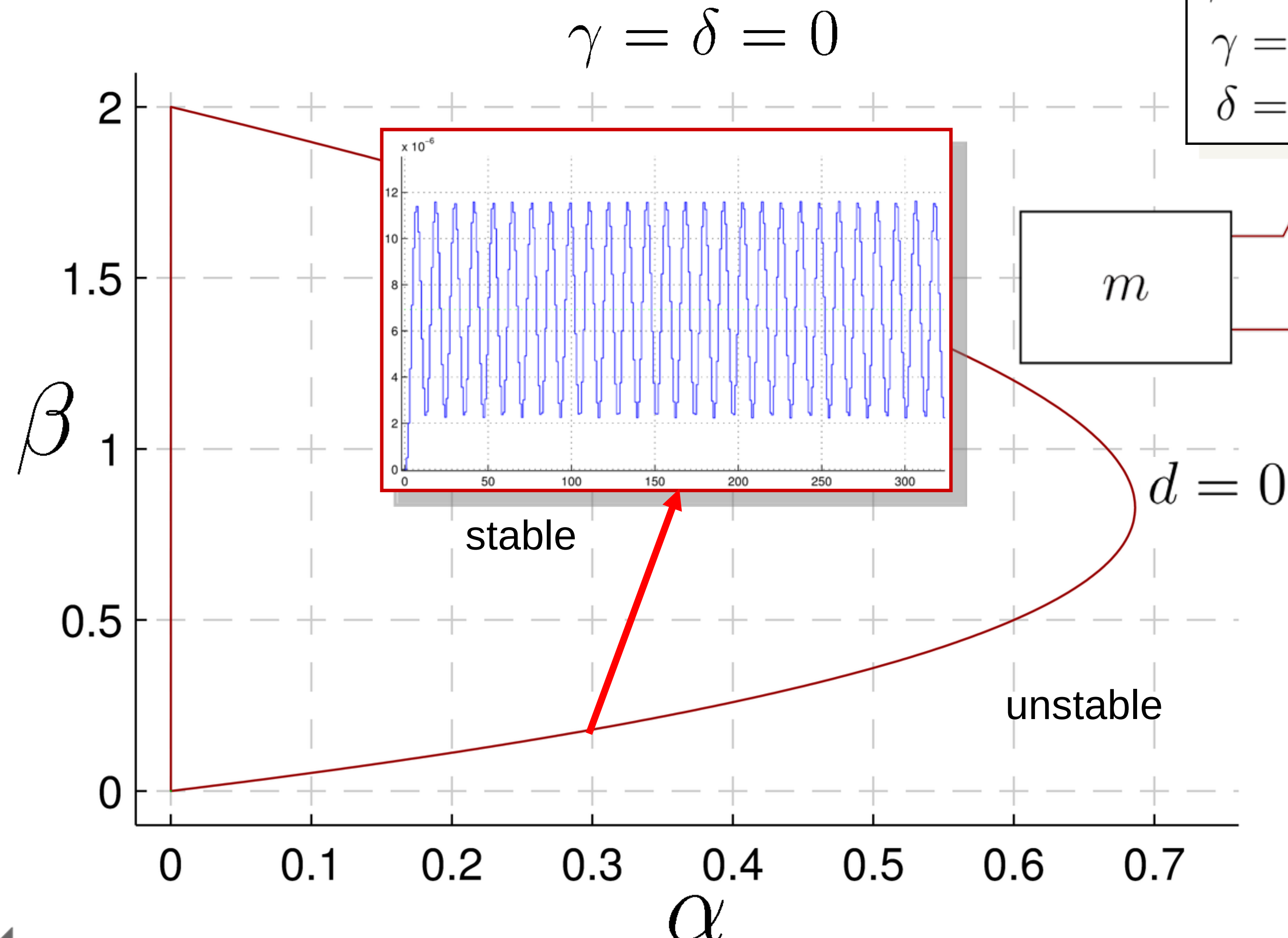

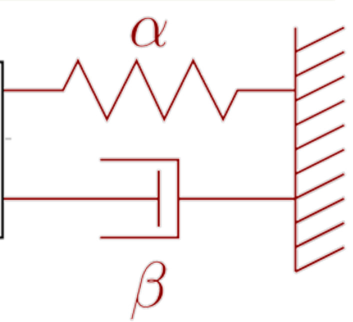

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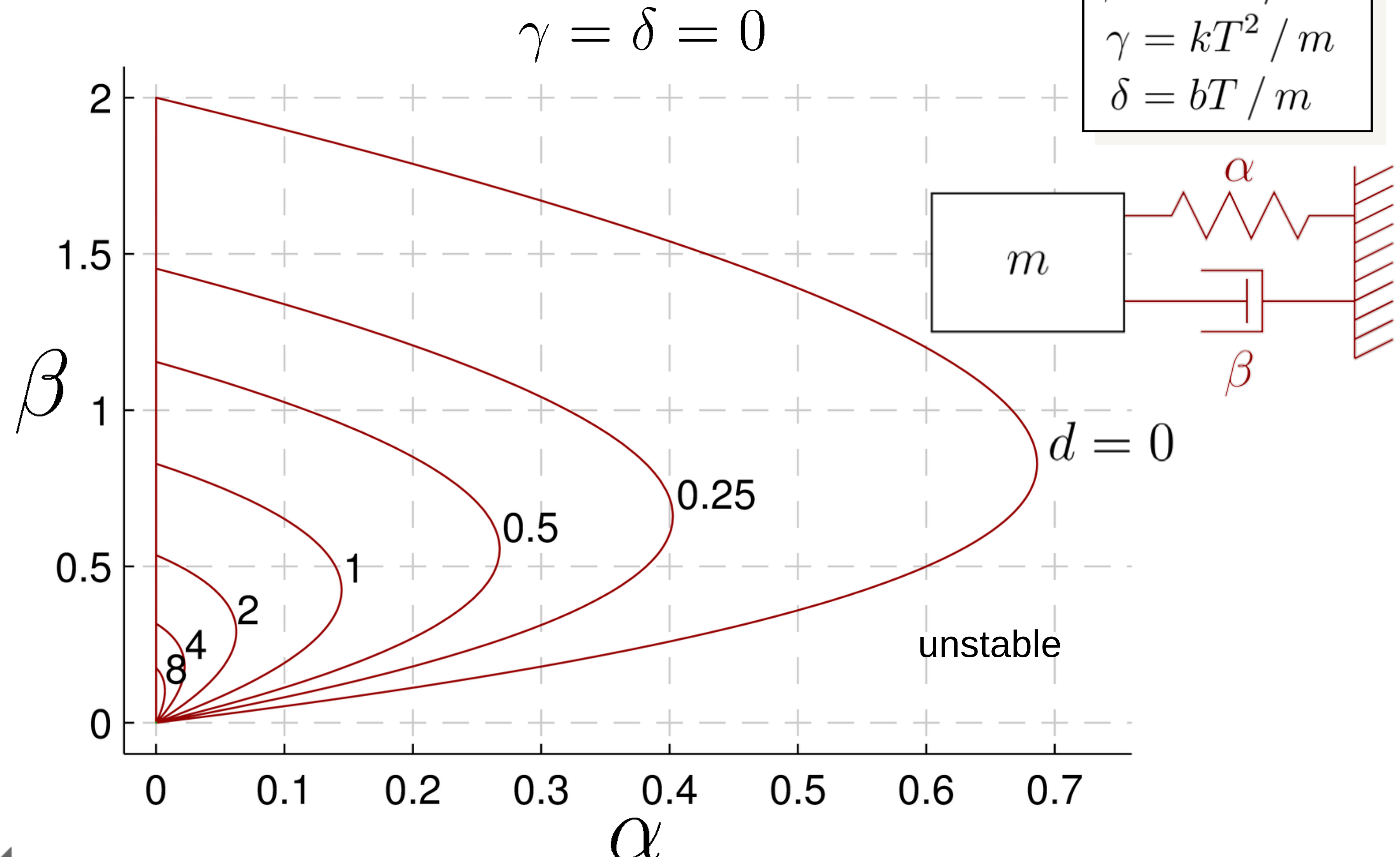


Realistic Parameter Range

for haptic devices holds

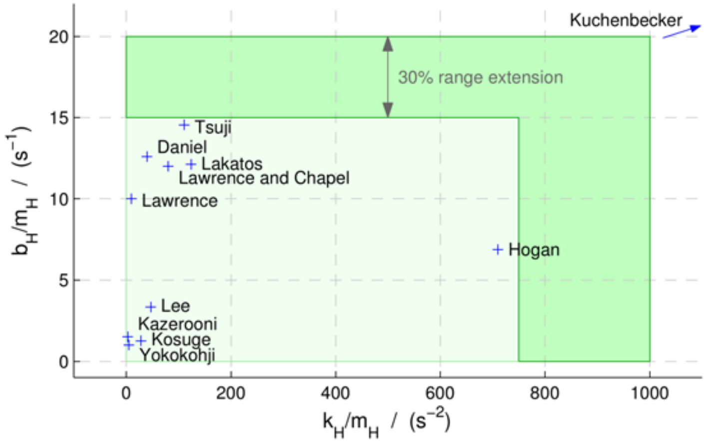
$$b_L/m_L < 0.625s^{-1}$$
$$T \leq 0.001s$$

Device	*m* *[Kg]*	*b* *[Ns/m]*	*T* *[ms]*
Delta	0.250	0.01	0.33
Freedom 6	0.250	0.01	1
Impulse Engine	0.032	0.02	0.2
MIT Toolhandle	0.119	0.001	1
Omega	0.220	0.01	0.33
Phantom 1.0	0.072	0.005	1

[Diolaiti 2006]

for human arms holds

$$k_H/m_H \leq 710\,\mathrm{s}^{-2}$$
$$b_H/m_H \leq 14.6\,\mathrm{s}^{-1}$$



[Gil 2004, Hulin 2014]

+ 30%

$$0 \leq \gamma = k \cdot T^2/m \leq 1 \cdot 10^{-3}$$
$$0 \leq \delta = b \cdot T/m \leq 20 \cdot 10^{-3}$$

DLR

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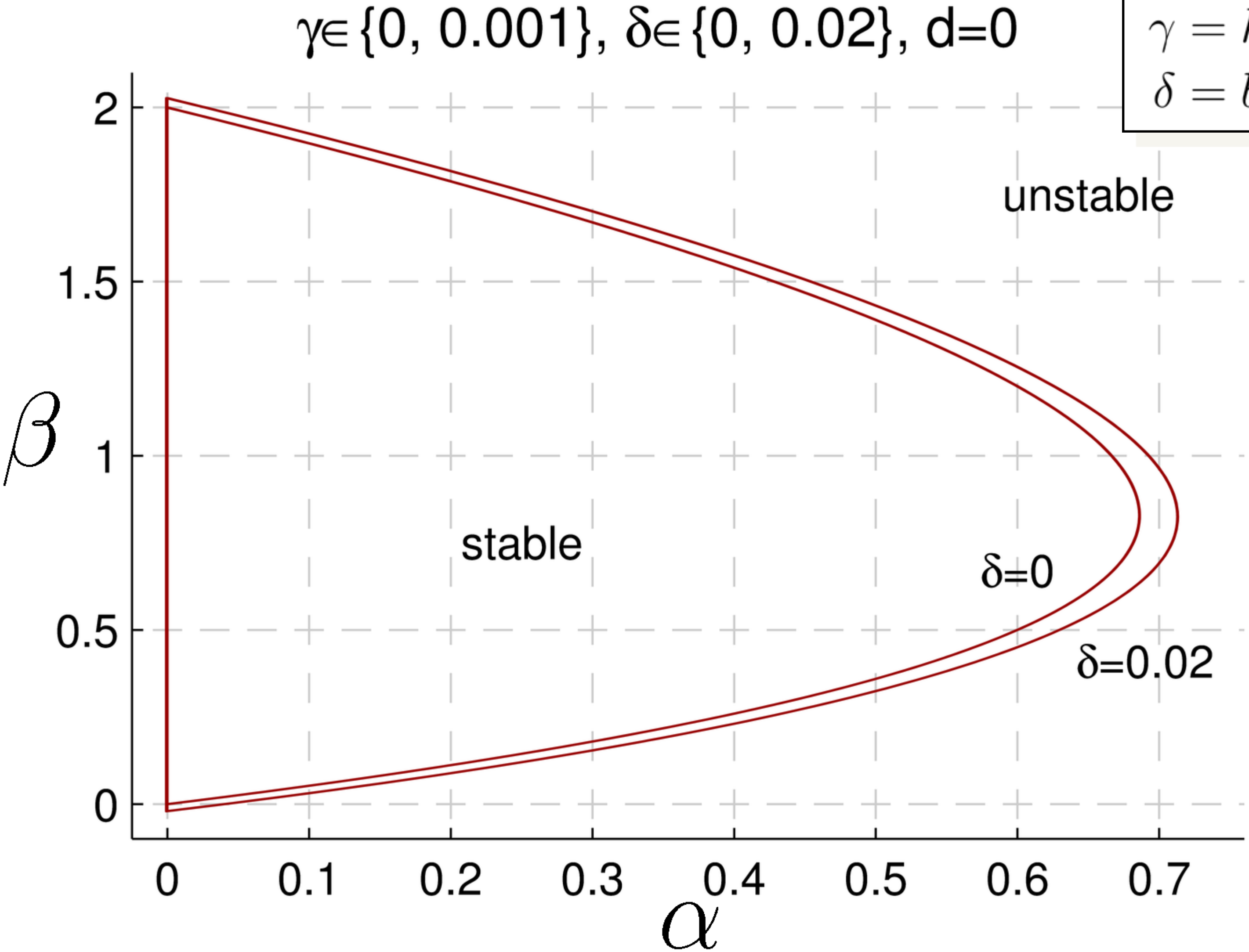

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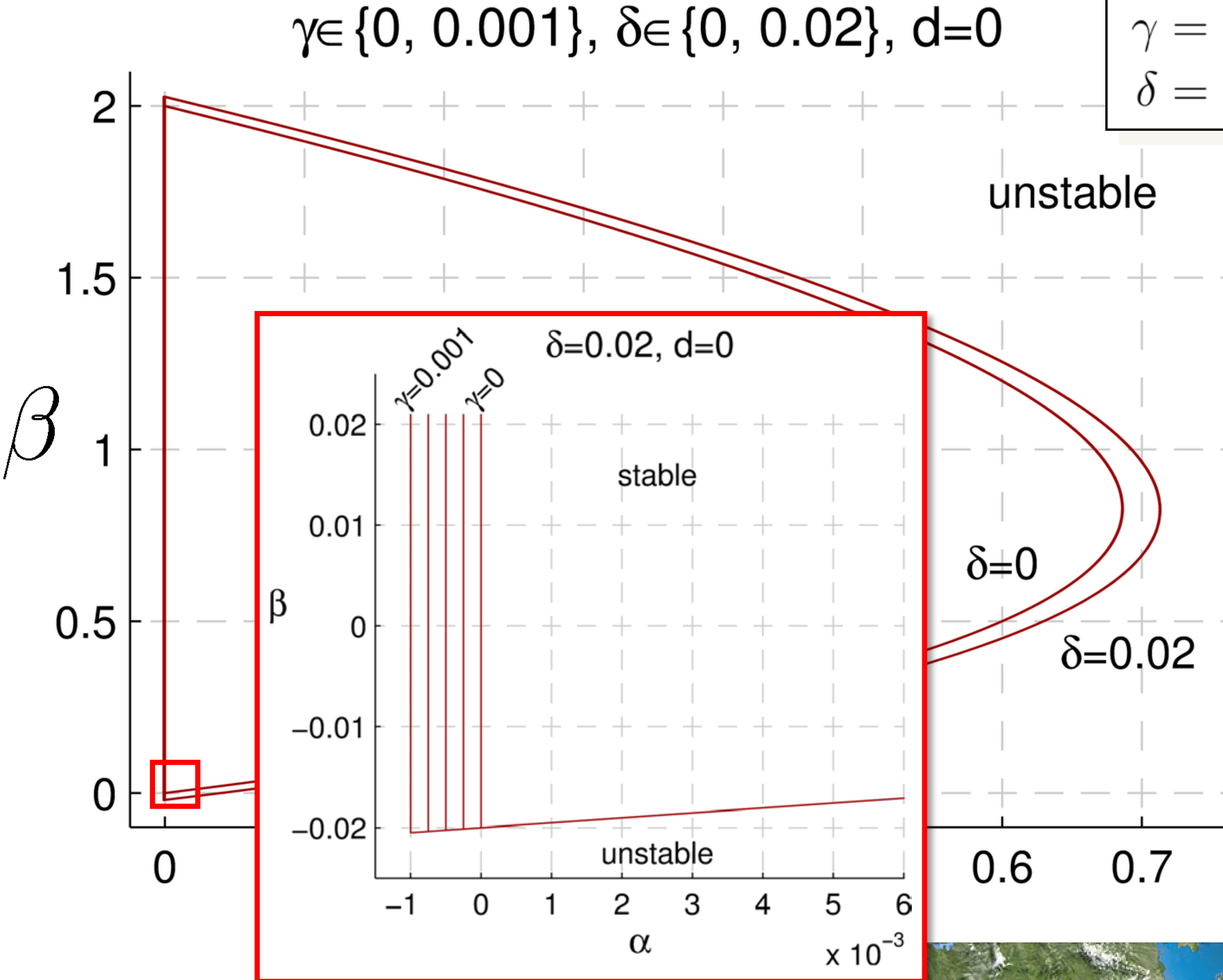
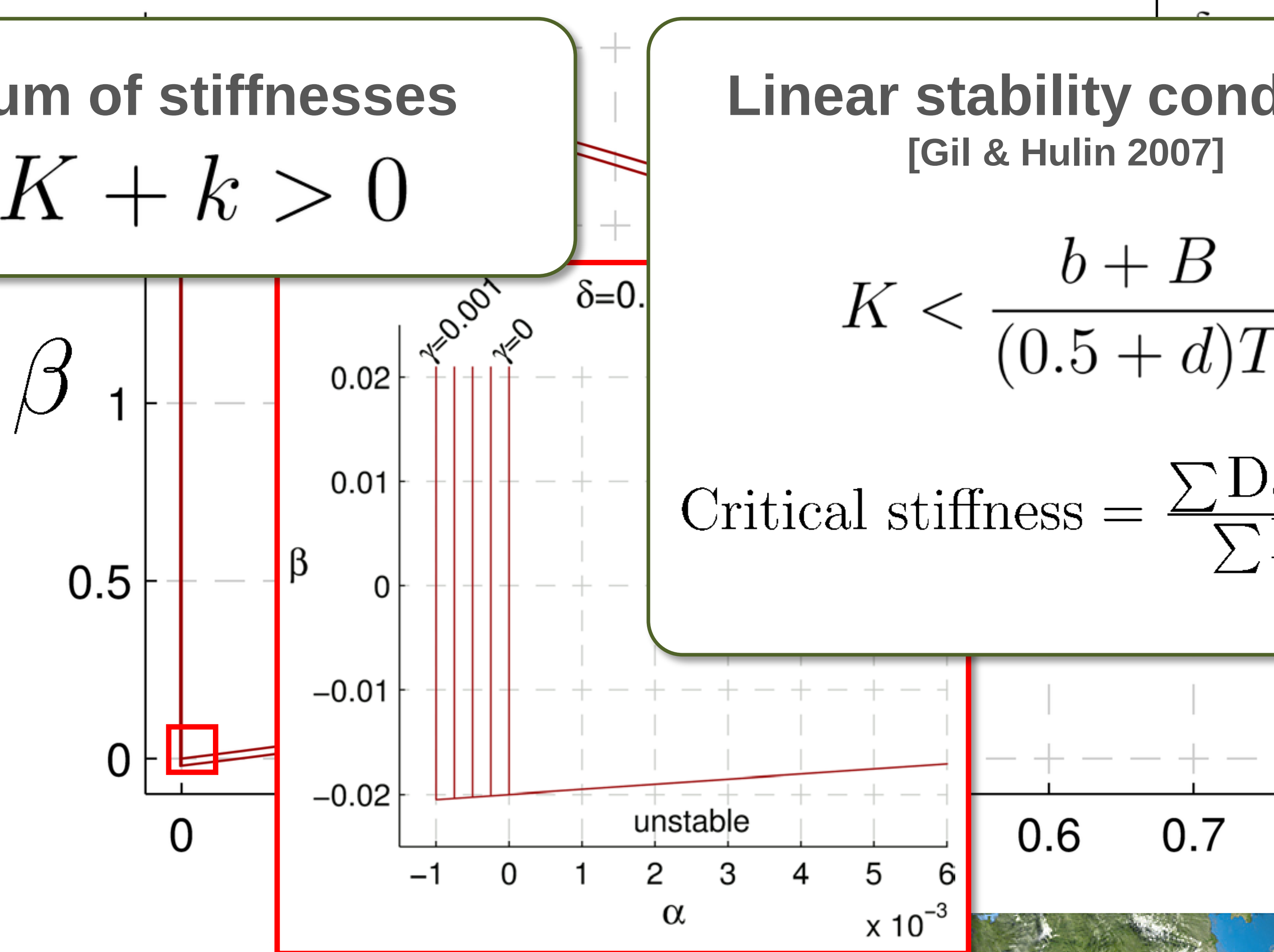


Normalized Stability Boundaries

$$d = t_d \,/\, T$$
$$\alpha = KT^2 \,/\, m$$
$$\beta = BT \,/\, m$$
$$\gamma = kT^2 \,/\, m$$

γ∈{0, 0.001}, δ∈{0, 0.02}, d=0



Sum of stiffnesses

$$K + k > 0$$

Linear stability condition

[Gil & Hulin 2007]

$$K < \frac{b + B}{(0.5 + d)T}$$

$$\text{Critical stiffness} = \frac{\sum \text{Damping}}{\sum \text{Delay}}$$

Normalized Stability Boundaries

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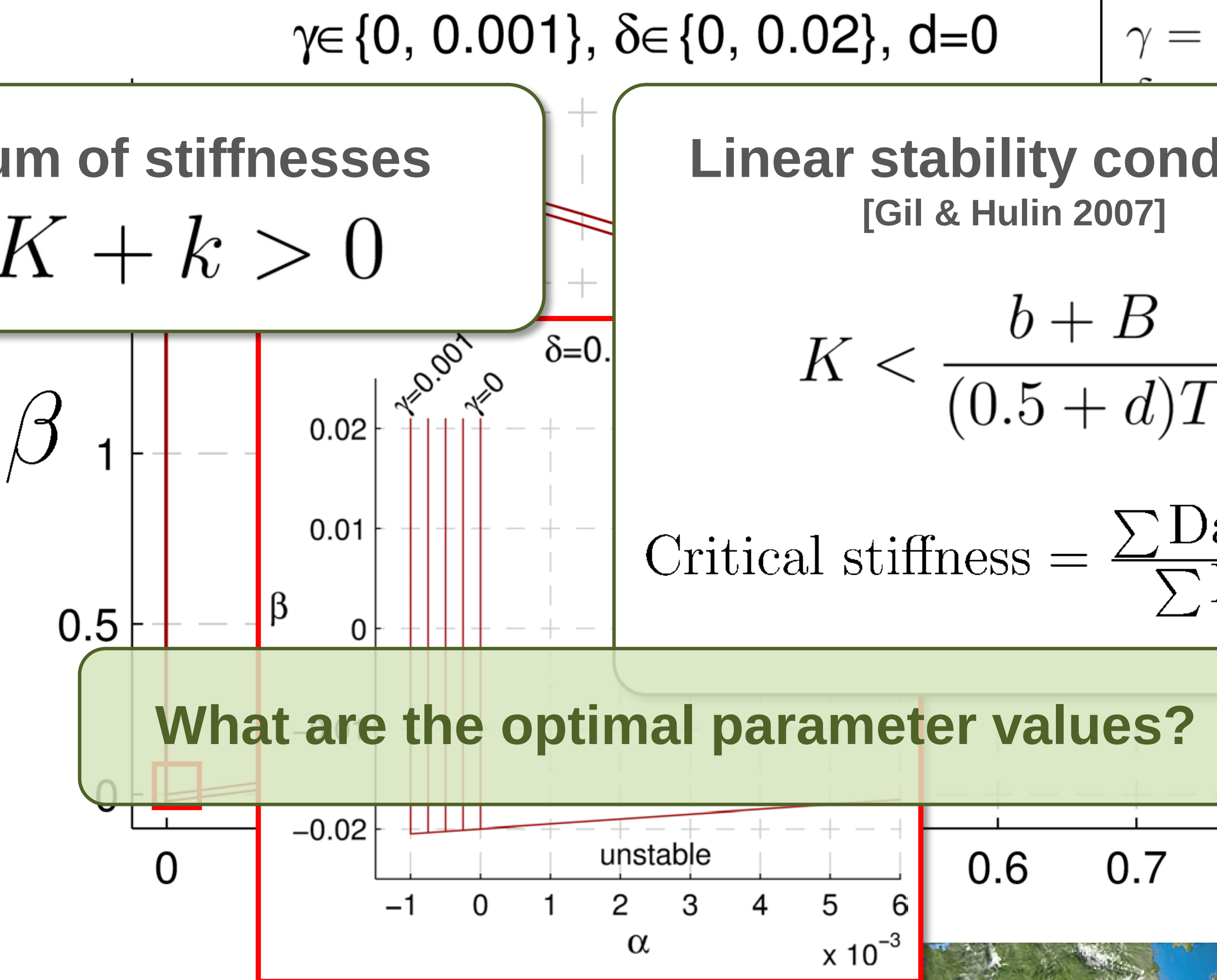
$$\gamma = kT^2 / m$$



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Linear stability condition

[Gil & Hulin 2007]

$$K < \frac{b + B}{(0.5 + d)T}$$

$$\text{Critical stiffness} = \frac{\sum \text{Damping}}{\sum \text{Delay}}$$

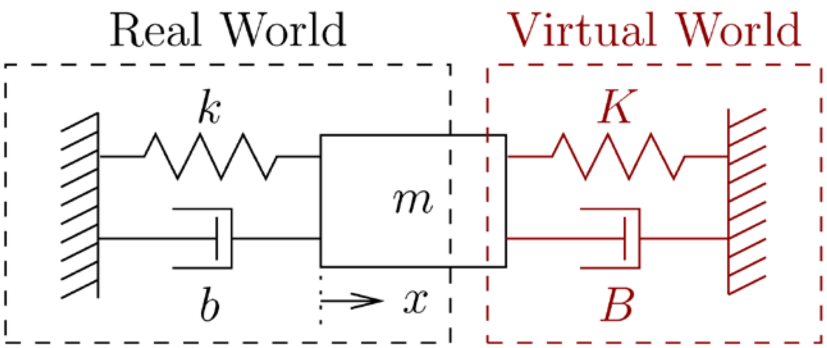
What are the optimal parameter values?

Part 2: Control Design

Optimal Control for Haptic Interaction

[Hulin et al., SYROCO2006],
[Hulin et al., IROS2013],
[Hulin, RA-L/ICRA 2017]

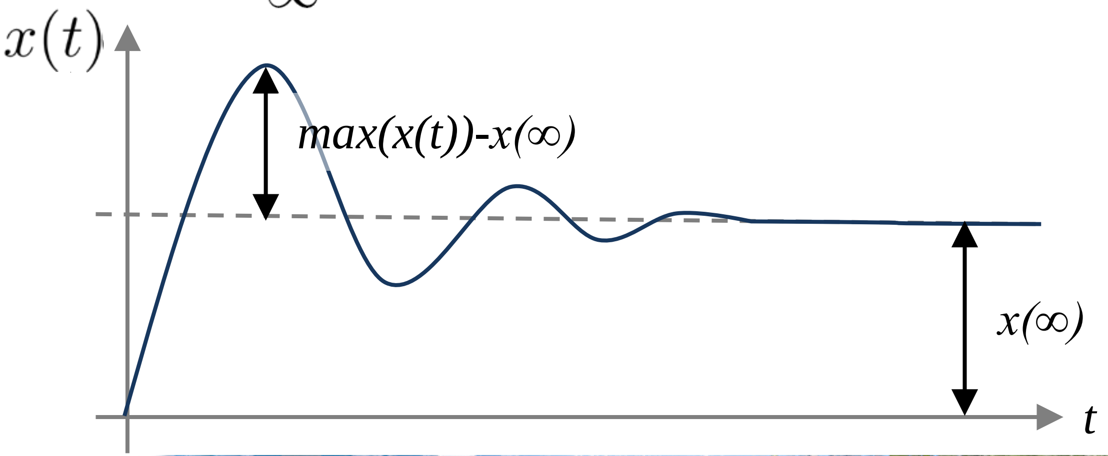
Response-based Control Design
Minimum Overshoot



Motivation: Optimal controller for the mass spring damper system: minimum overshoot

Cost function:

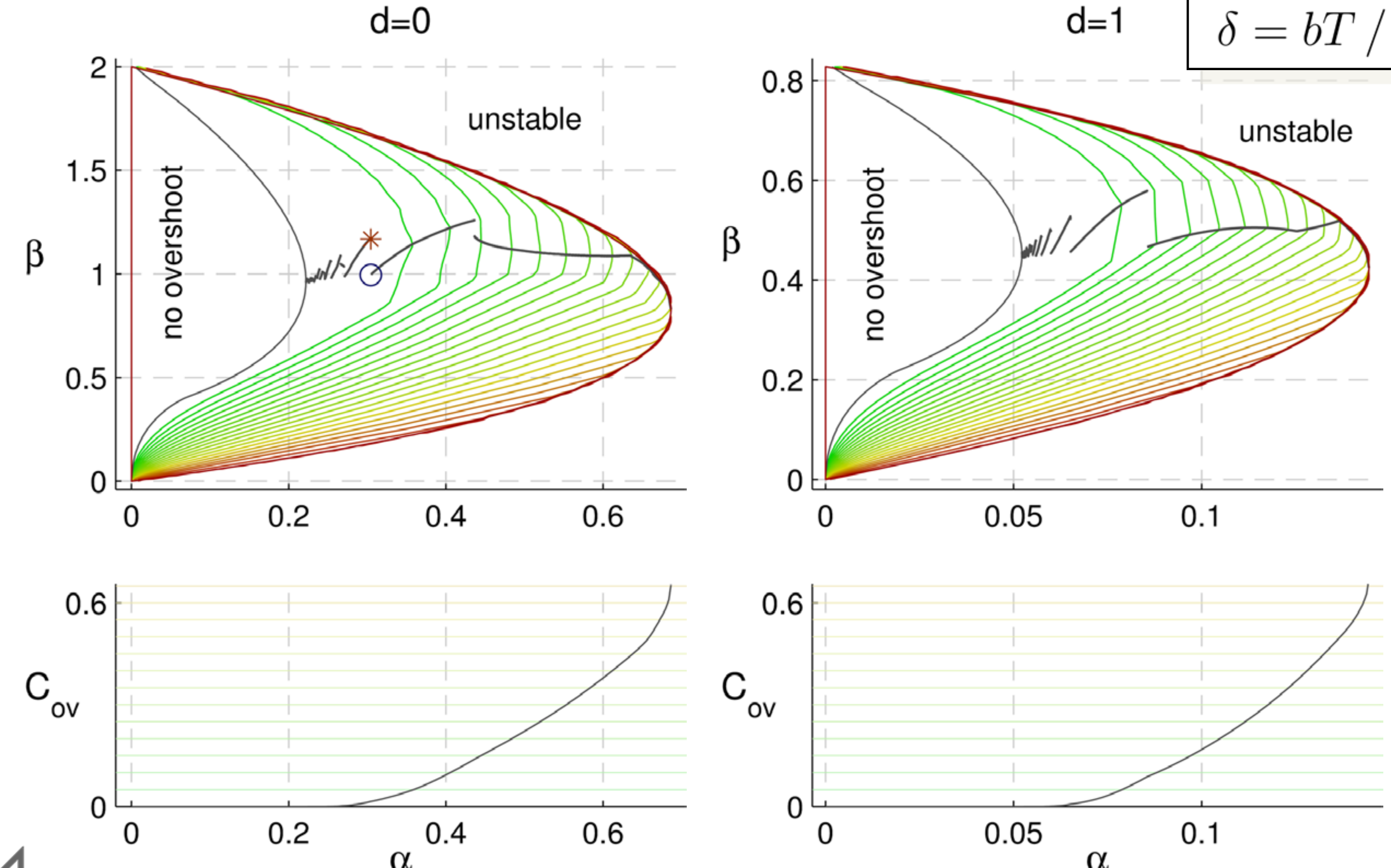
$$C_{\mathrm{ov}} := \frac{\max(x(t)) - x_\infty}{x_\infty}$$



$$d = t_d \,/\, T$$
$$\alpha = KT^2 \,/\, m$$
$$\beta = BT \,/\, m$$
$$\gamma = kT^2 \,/\, m$$
$$\delta = bT \,/\, m$$

Response-based Control Design

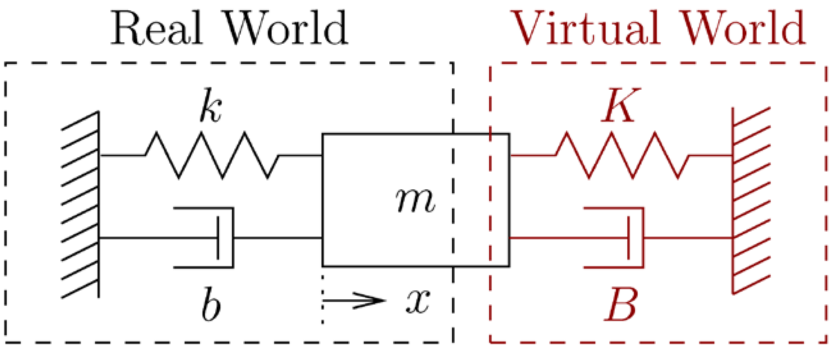
Minimum Overshoot



Response-based Control Design

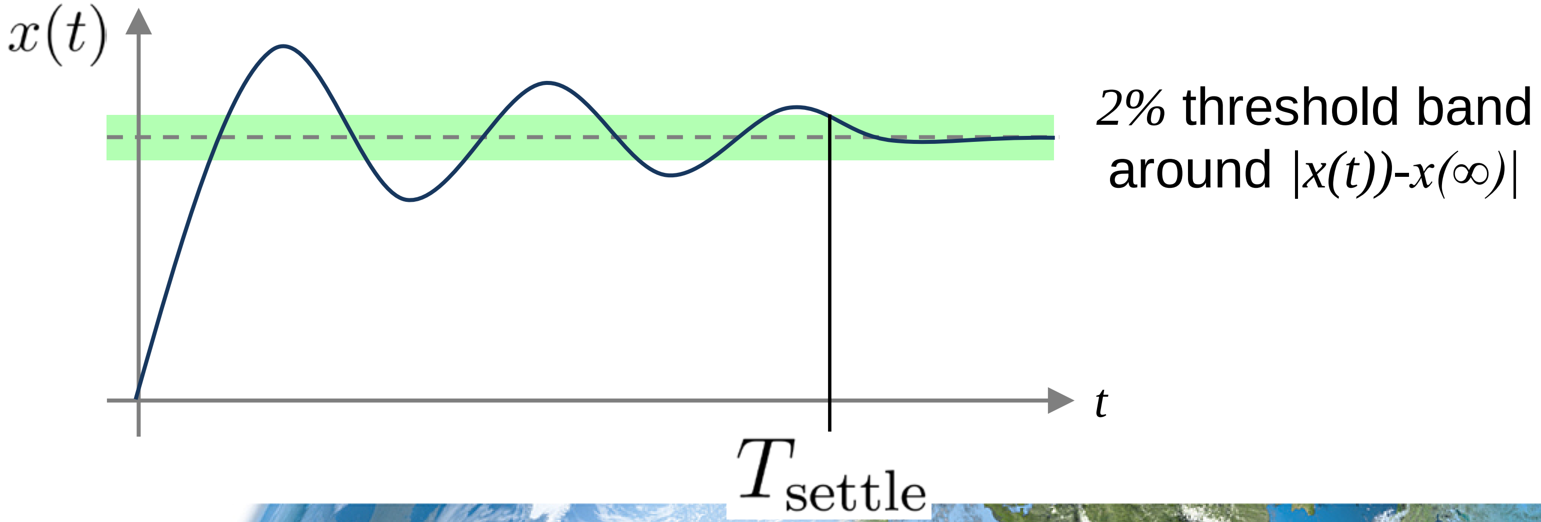
Minimum Settling Time

Motivation: Optimal controller for the mass spring damper system: minimum settling time



Cost function:

$$C_{\text{settle}} := T_{\text{settle}}/T$$



$$d = t_d \,/\, T$$
$$\alpha = KT^2 \,/\, m$$
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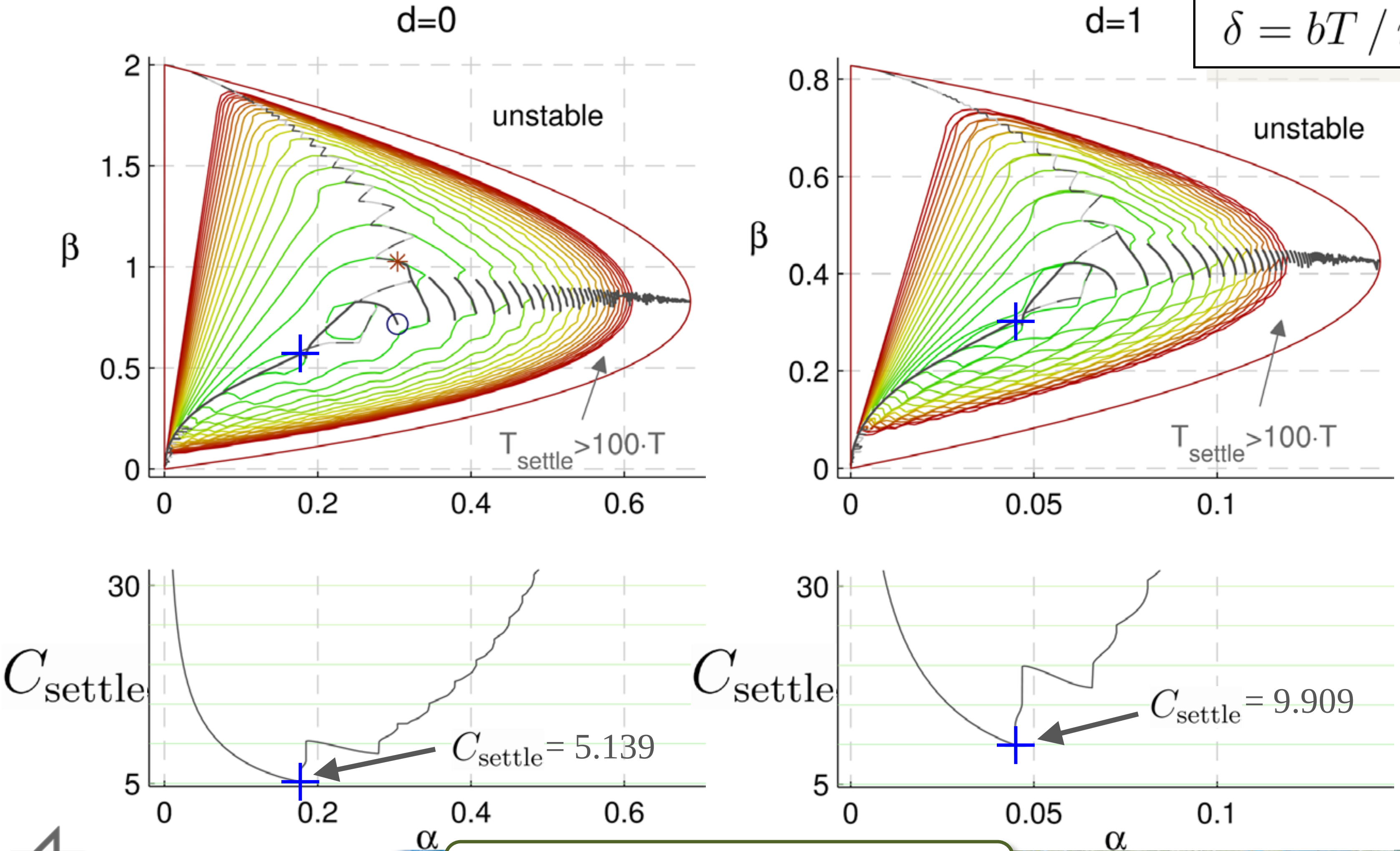
$$\delta = bT \,/\, m$$

Response-based Control Design

Minimum Settling Time



$$C_{\text{settle}} := T_{\text{settle}} / T$$

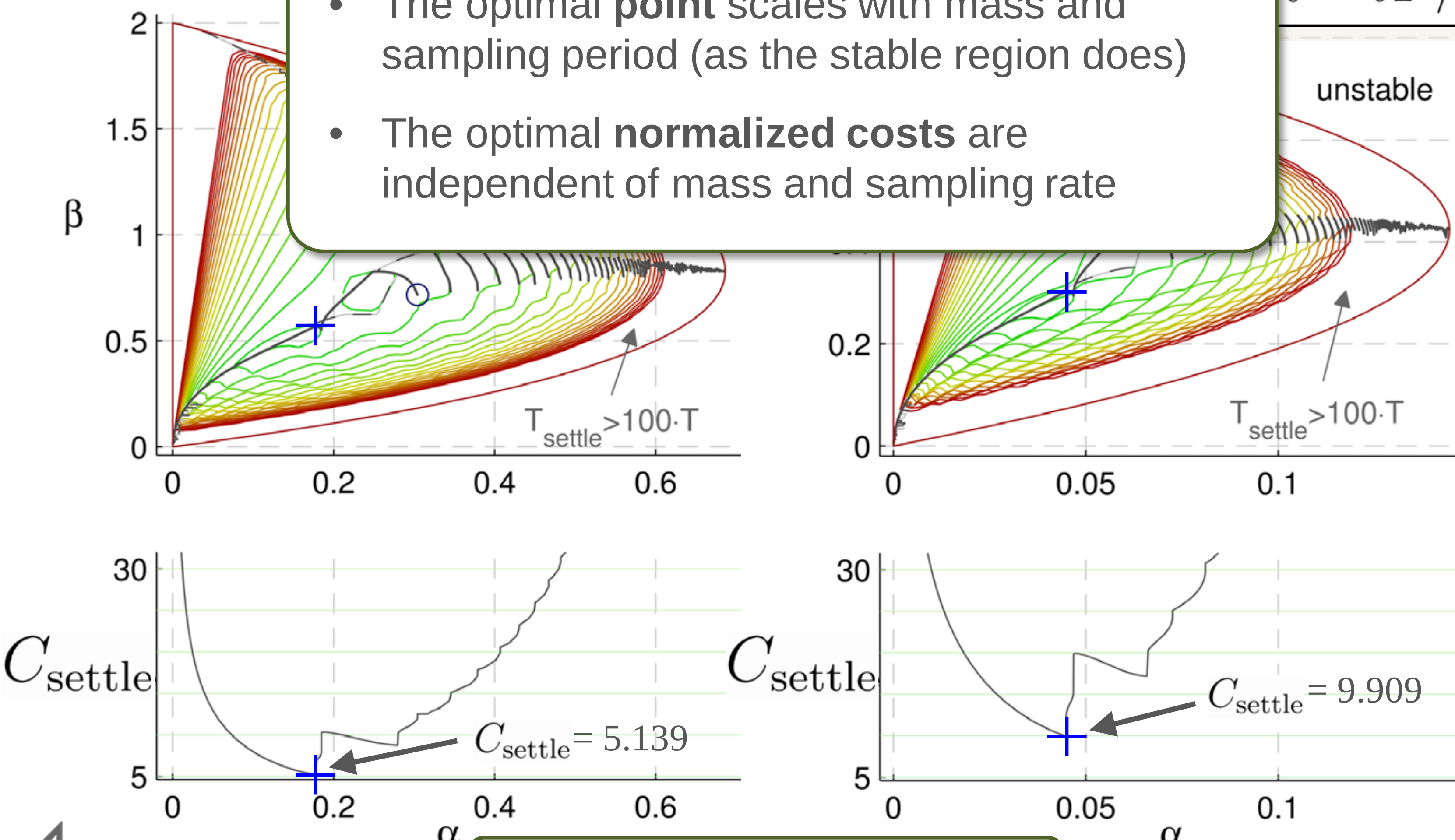
DLR

$$d = t_d \,/\, T$$
$$\alpha = KT^2 \,/\, m$$
$$\beta = BT \,/\, m$$
$$\gamma = kT^2 \,/\, m$$
$$\delta = bT \,/\, m$$

Response-based Control Design

Minimum Settling Time

- The optimal **point** scales with mass and sampling period (as the stable region does)
- The optimal **normalized costs** are independent of mass and sampling rate



$$C_{\text{settle}} := T_{\text{settle}} / T$$

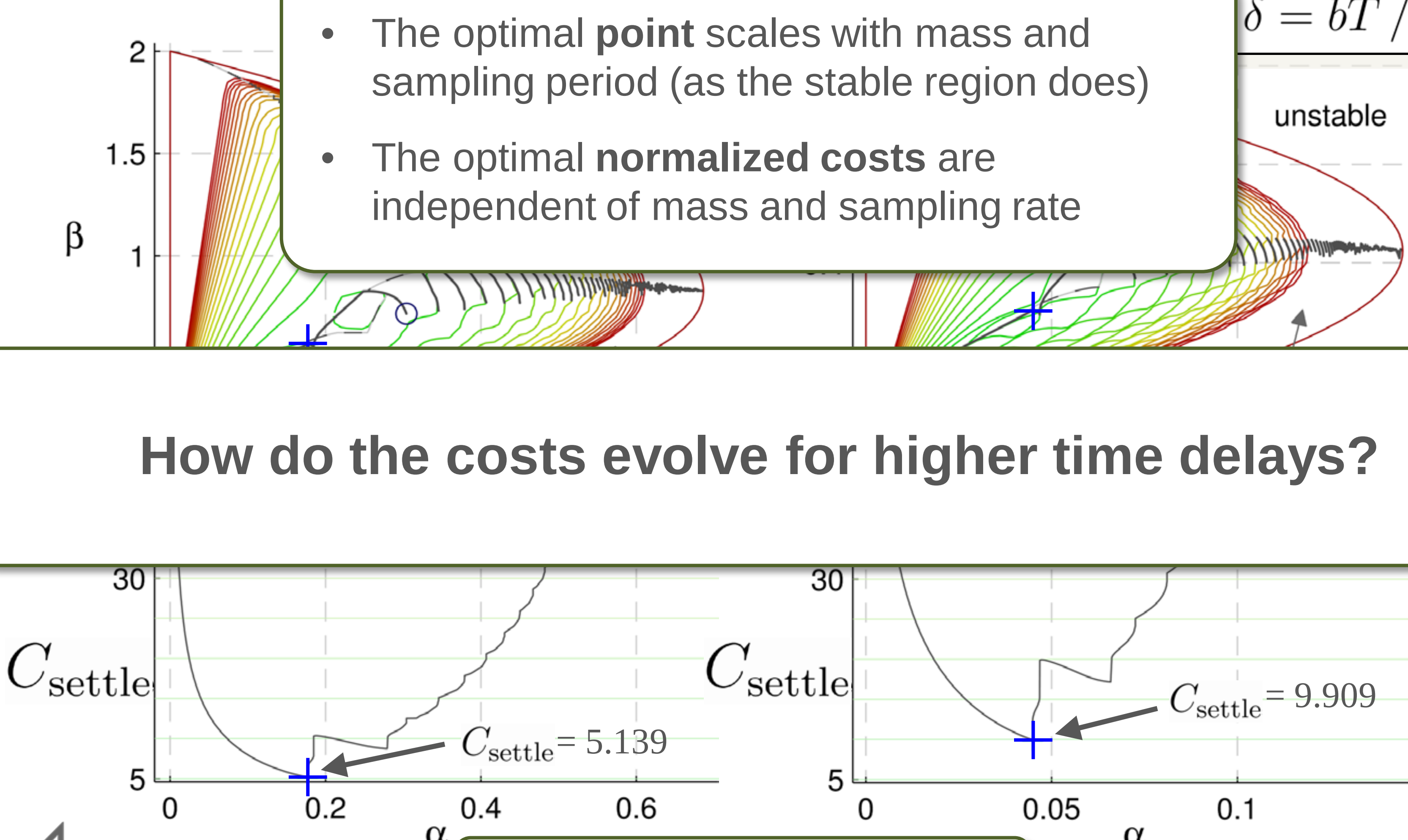
DLR

Response-based Control Design

Minimum Settling Time

$$
\begin{aligned}
d &= t_d \,/\, T \\
\alpha &= K T^2 \,/\, m \\
\beta &= B T \,/\, m \\
\gamma &= k T^2 \,/\, m \\
\delta &= b T \,/\, m
\end{aligned}
$$

- The optimal **point** scales with mass and sampling period (as the stable region does)
- The optimal **normalized costs** are independent of mass and sampling rate



How do the costs evolve for higher time delays?

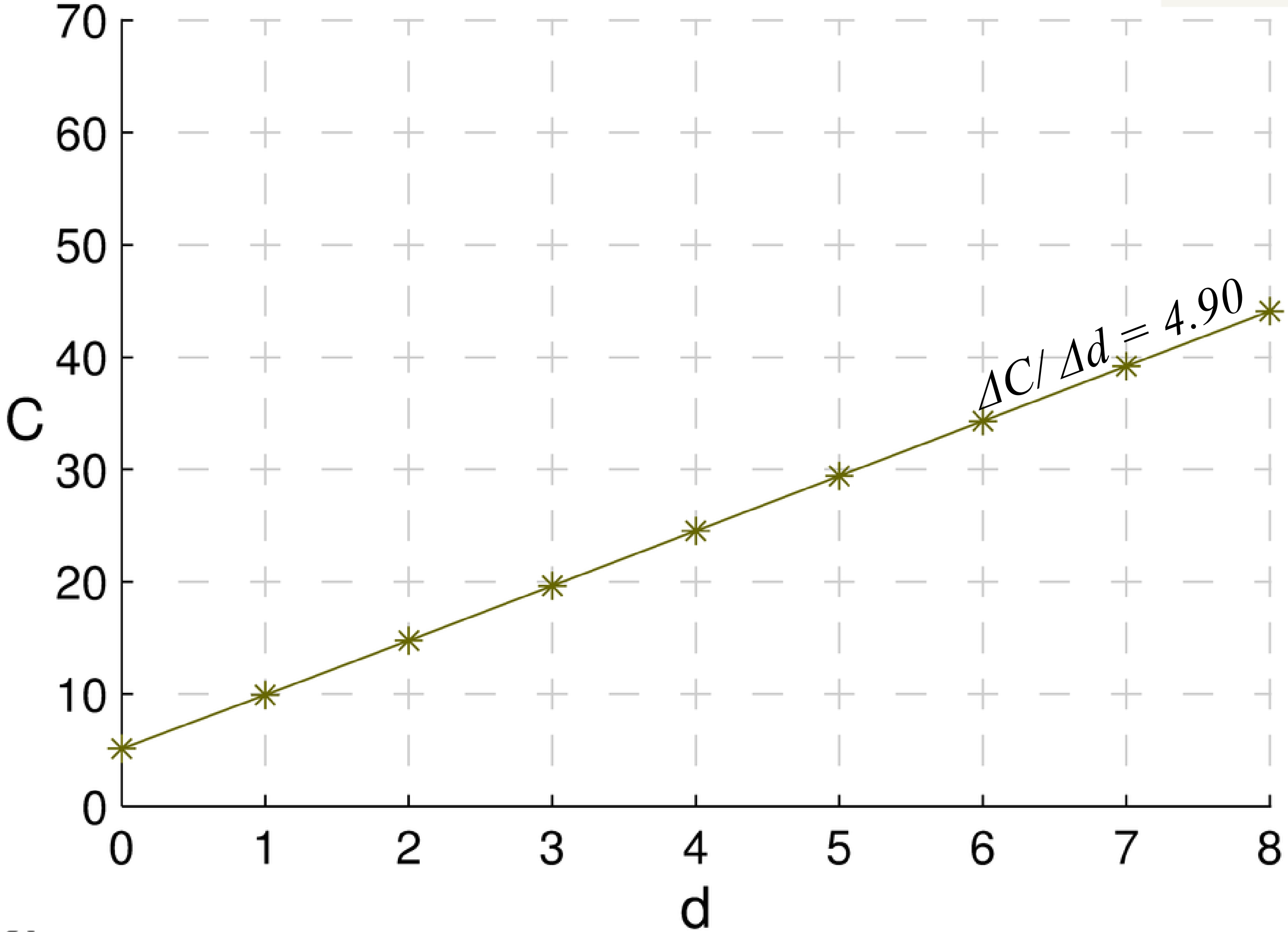
$$C_{\text{settle}} := T_{\text{settle}} / T$$

Control Design

Influence of Delay

Time Delay

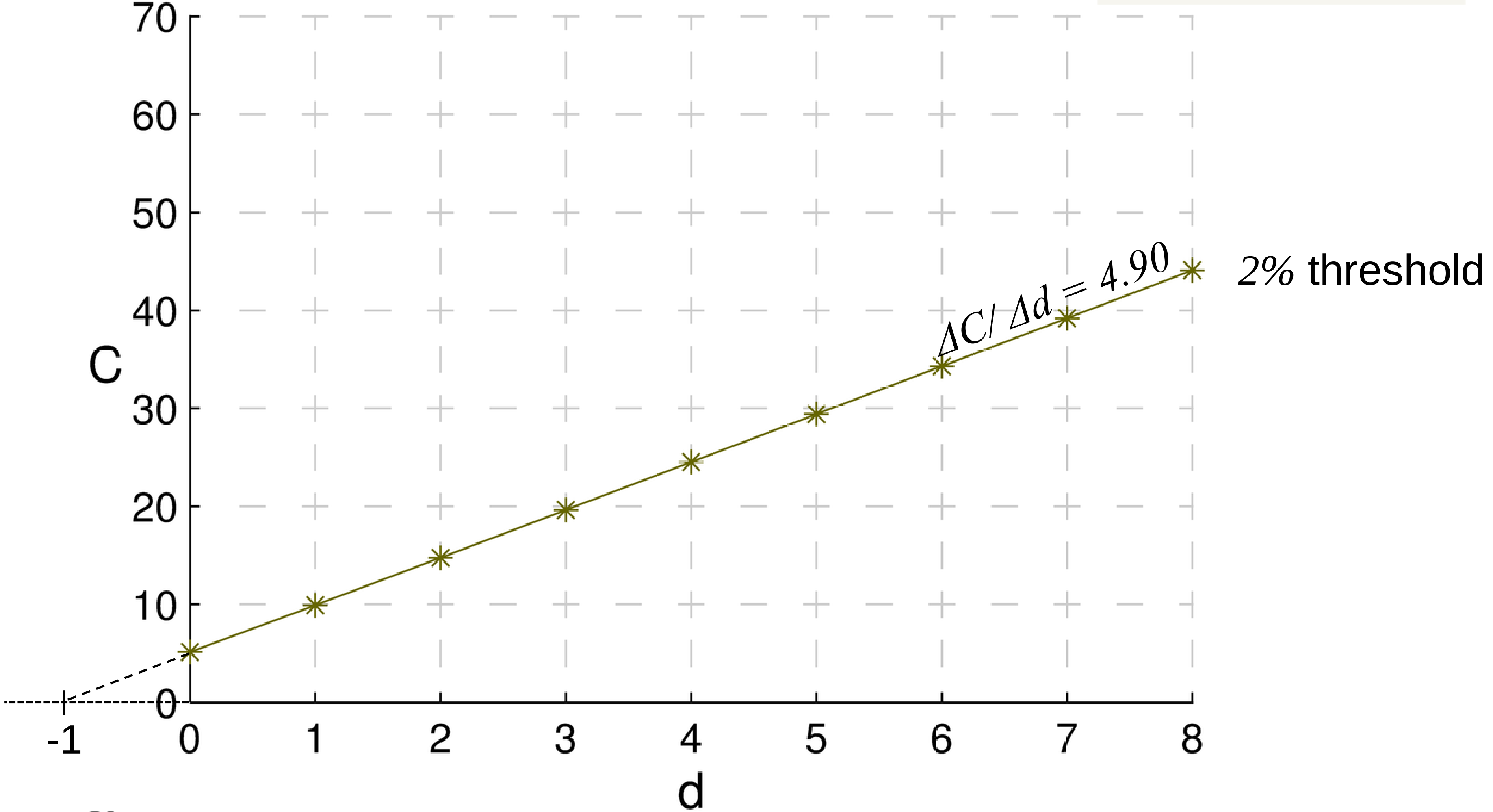
$$t_d = d \cdot T$$



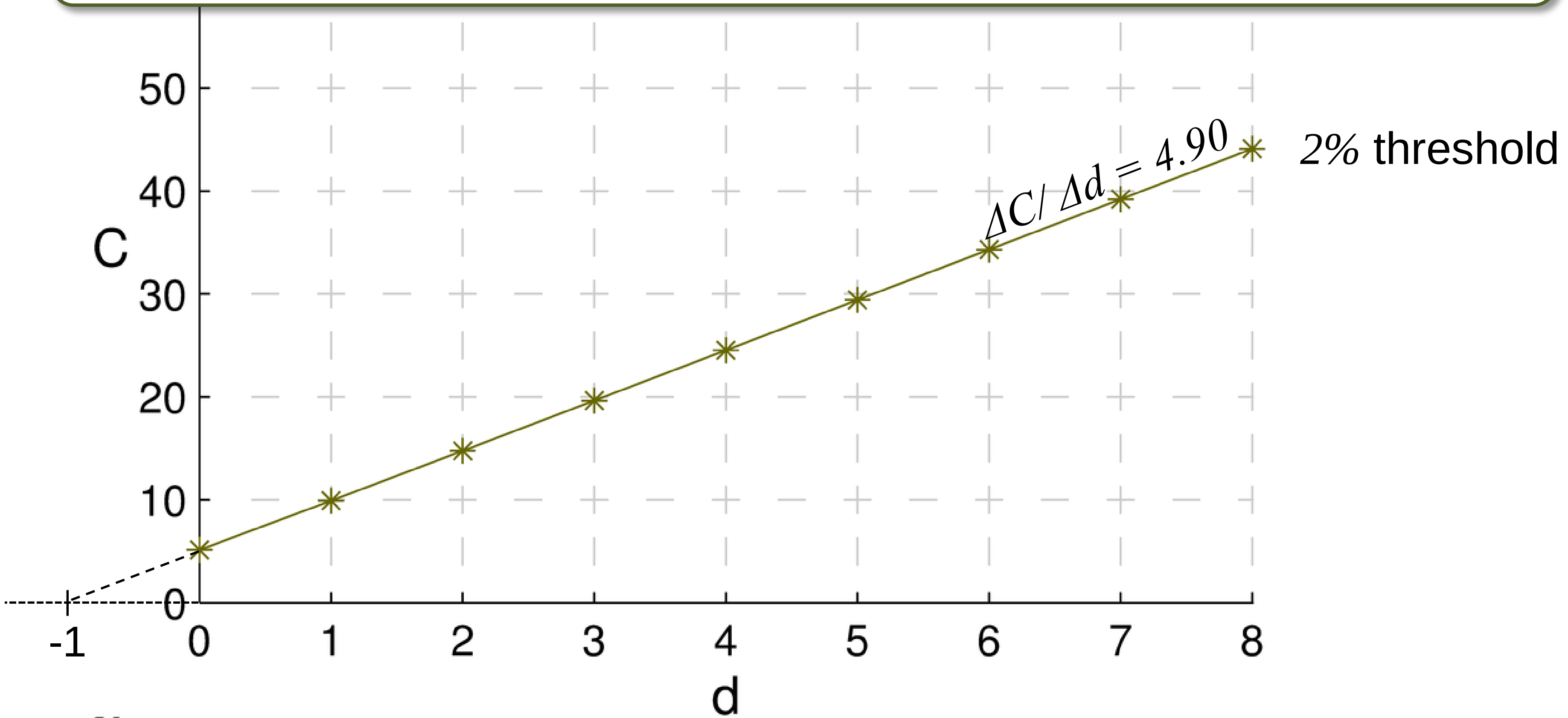
2% threshold

Control Design
Influence of Delay

Time Delay
$$t_d = d \cdot T$$



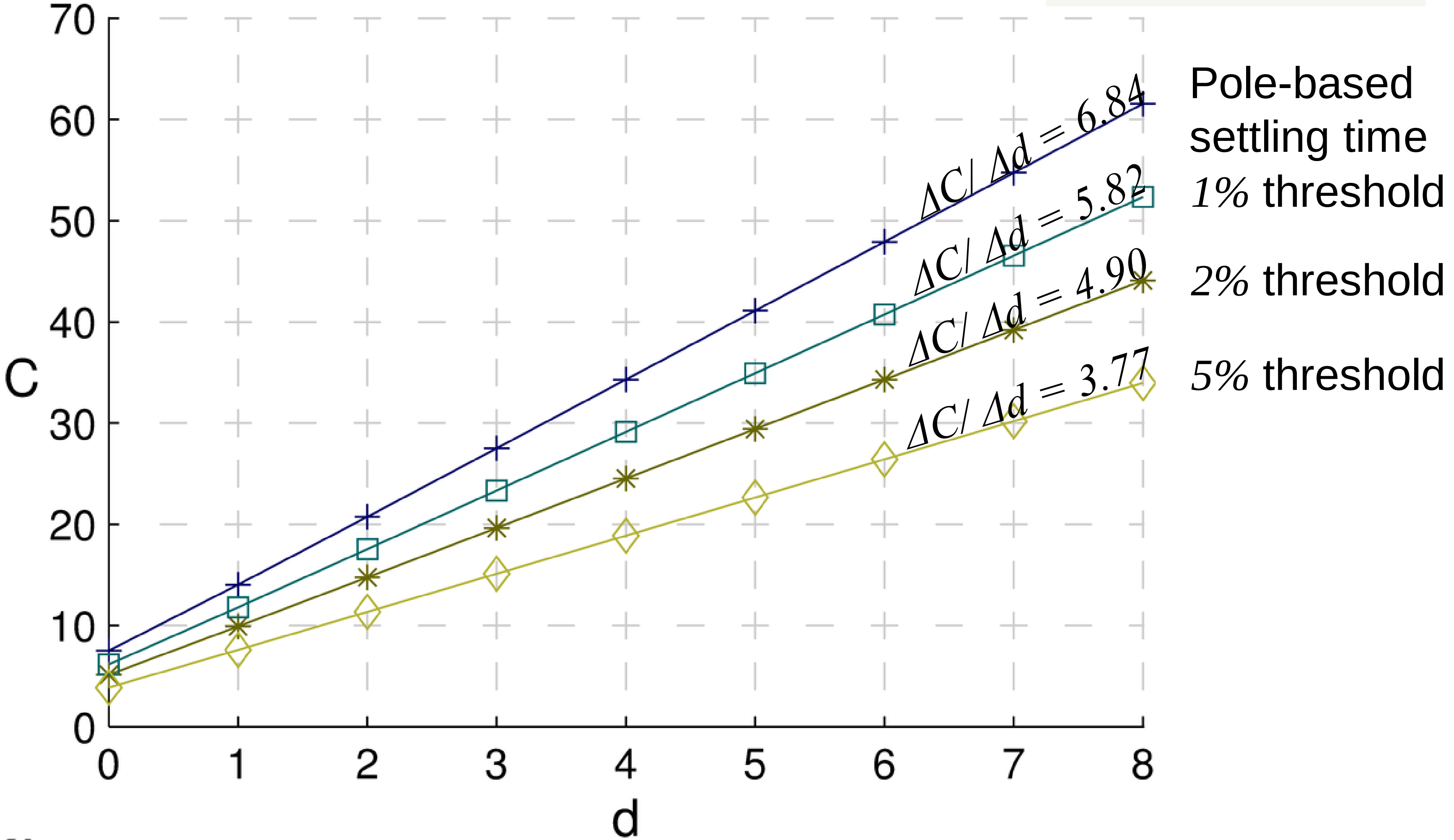
1. **Each sampling period of additional time delay causes the optimal settling time to increase by approximately five sampling periods.**
2. **The effect of discrete-time sampling corresponds to a delay of one whole sampling period in terms of cost.**



Control Design
Influence of Delay

Time Delay

$$t_d = d \cdot T$$



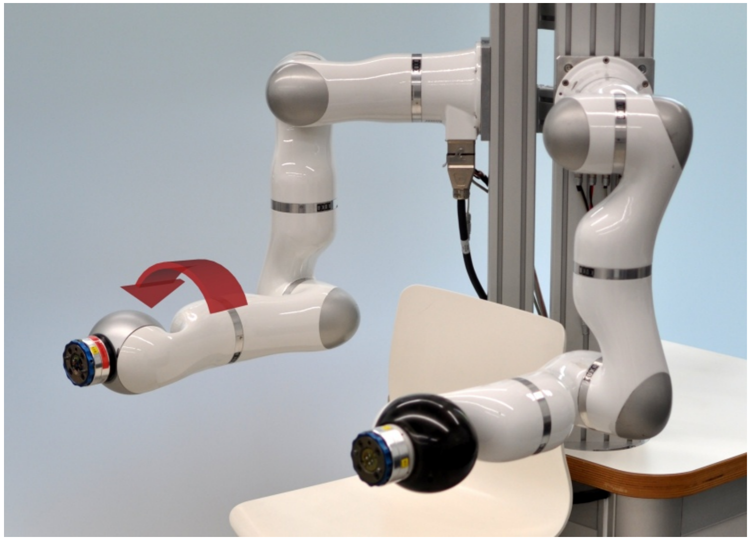
Part 3: Experiments

Verification of the theoretical approach on real systems

[Hulin et al., IROS2013],
[Hulin, RA-L/ICRA 2017]

Experiments

- Two different devices with widely differing properties:
    - Novint Falcon
    - DLR/KUKA Light-Weight Robot (the right arm of HUG)
- Influence of human operator (Falcon)
- Influence of time delay (LWR)
- Influence of modifications in the HW (Falcon)



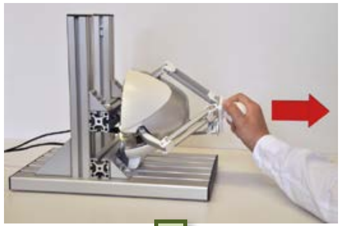
Experiments on the Novint Falcon
Experimental Procedure

A set of seven experiments:

- Influence of human operator
- Influence of modifications in the HW

human grip	Falcon A	Falcon B	Falcon B stiff
no operator	1	2	3
comfortable grip	4	–	5
firm grip	6	–	7

Experiments on the Novint Falcon
Experimental Procedure



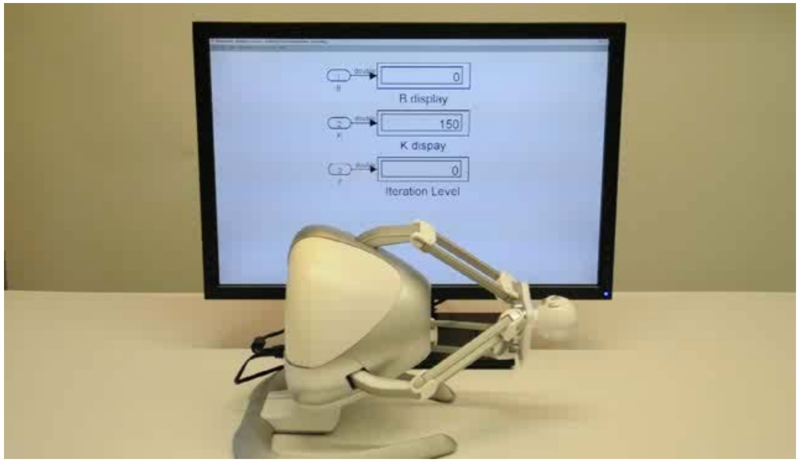
A set of seven experiments:

- Influence of human operator
- Influence of modifications in the HW

human grip	Falcon A	Falcon B	Falcon B stiff
no operator	1	2	3
comfortable grip	4	–	5
firm grip	6	–	7

For each experiment a grid was defined:

no human	**with human**
ΔK=200N/m, ΔB=4Ns/m	ΔK=400N/m, ΔB=10Ns/m
~2200 grid points	~650 grid points



DLR

Experiments on the Novint Falcon
Parameter Estimation

Determine physical parameters to draw theoretical curves:

- Mass: conservation of linear momentum

$$m = \int_{t_1}^{t_2} F(\tau)\mathrm{d}\tau \Big/ \Big(\dot{x}(t_2) - \dot{x}(t_1)\Big)$$

- Viscous damping: negative virtual damping

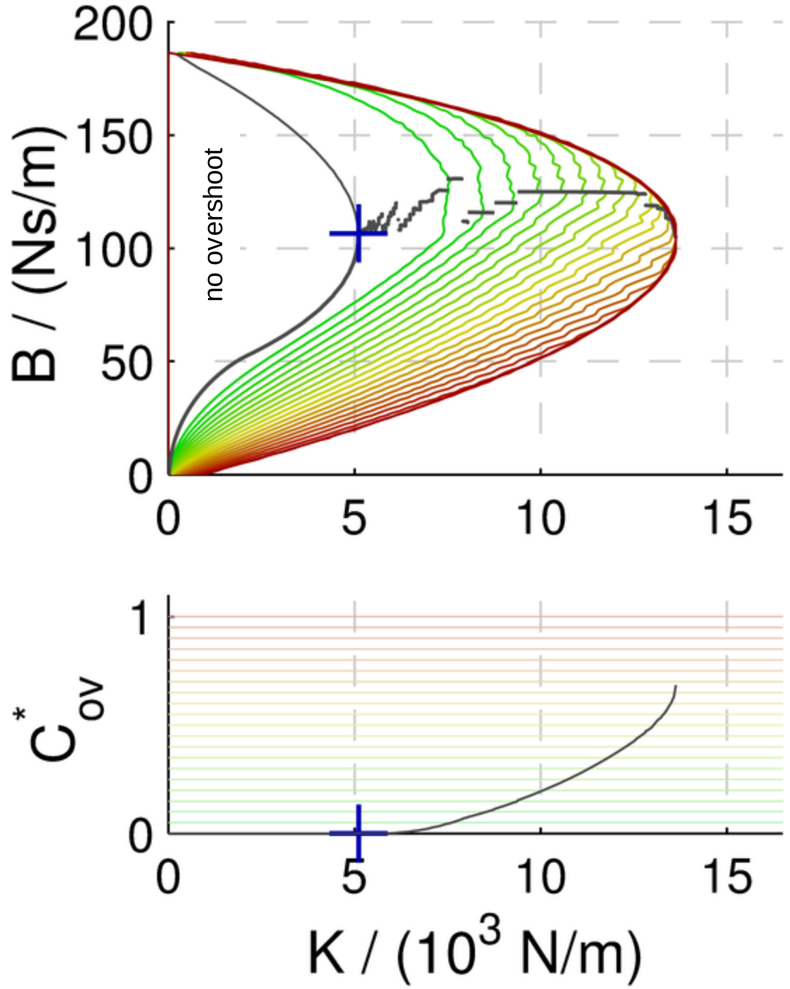
human grip	dynamic mass m	viscous damping b	
	Falcon A&B[a]	Falcon A	Falcon B
no operator	0.58 kg	4 Ns/m	2 Ns/m
comfortable grip	0.65 kg	9 Ns/m	7 Ns/m
firm grip	1.00 kg	34 Ns/m	32 Ns/m

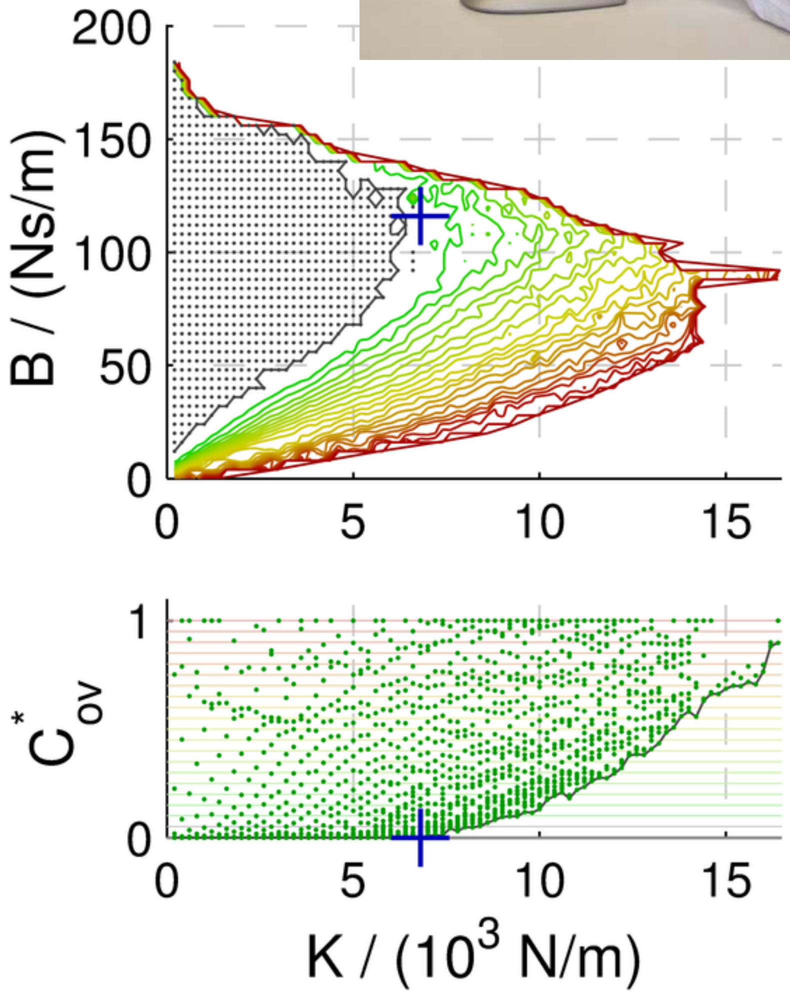
- Delay: first change in step response

$3\,\mathrm{ms} \leq t_d < 6\,\mathrm{ms}$ (average delay: $t_d = 5\,\mathrm{ms}$)

Experiments on the Novint Falcon

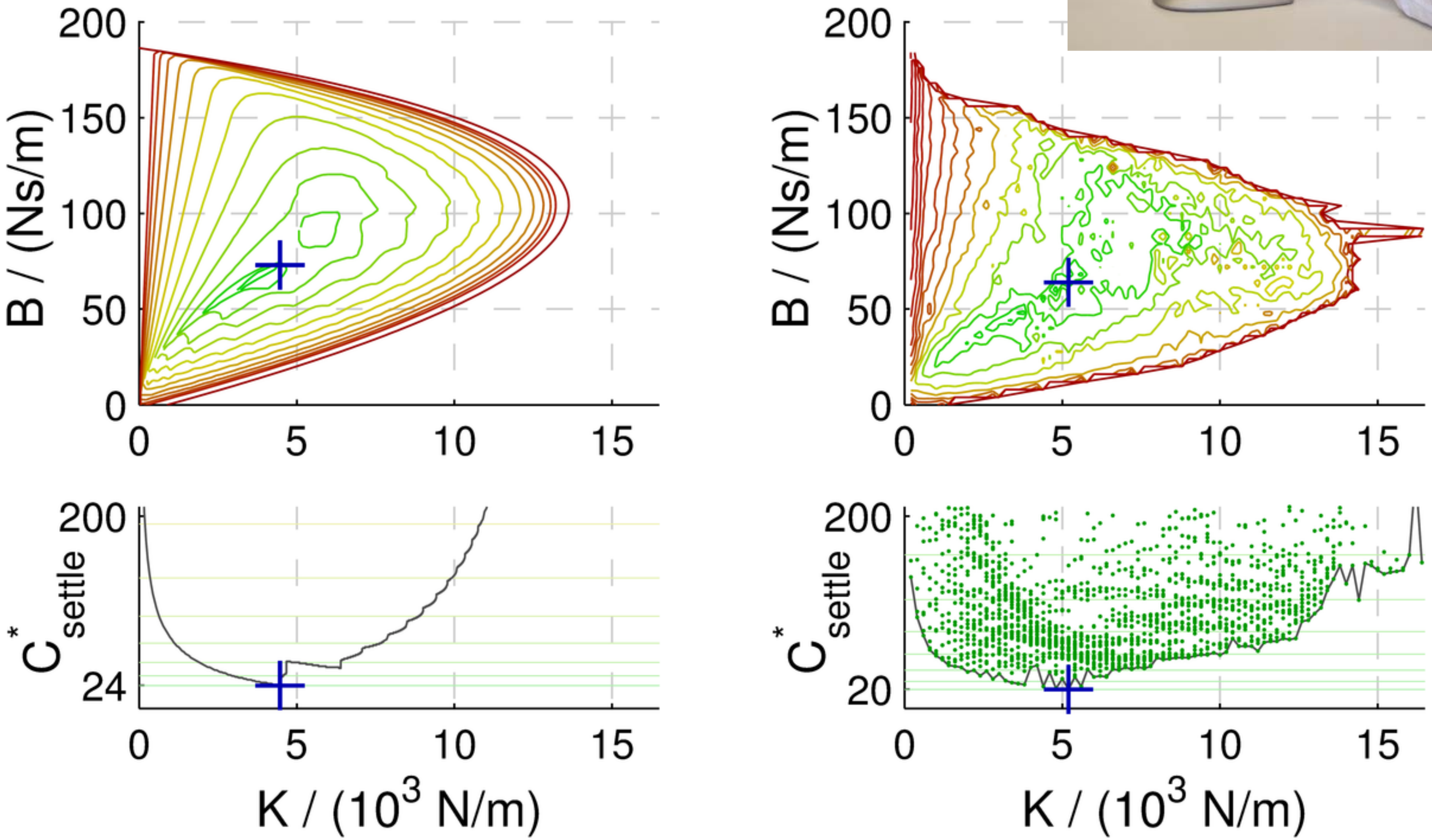
Results: Overshoot





Experiments on the Novint Falcon

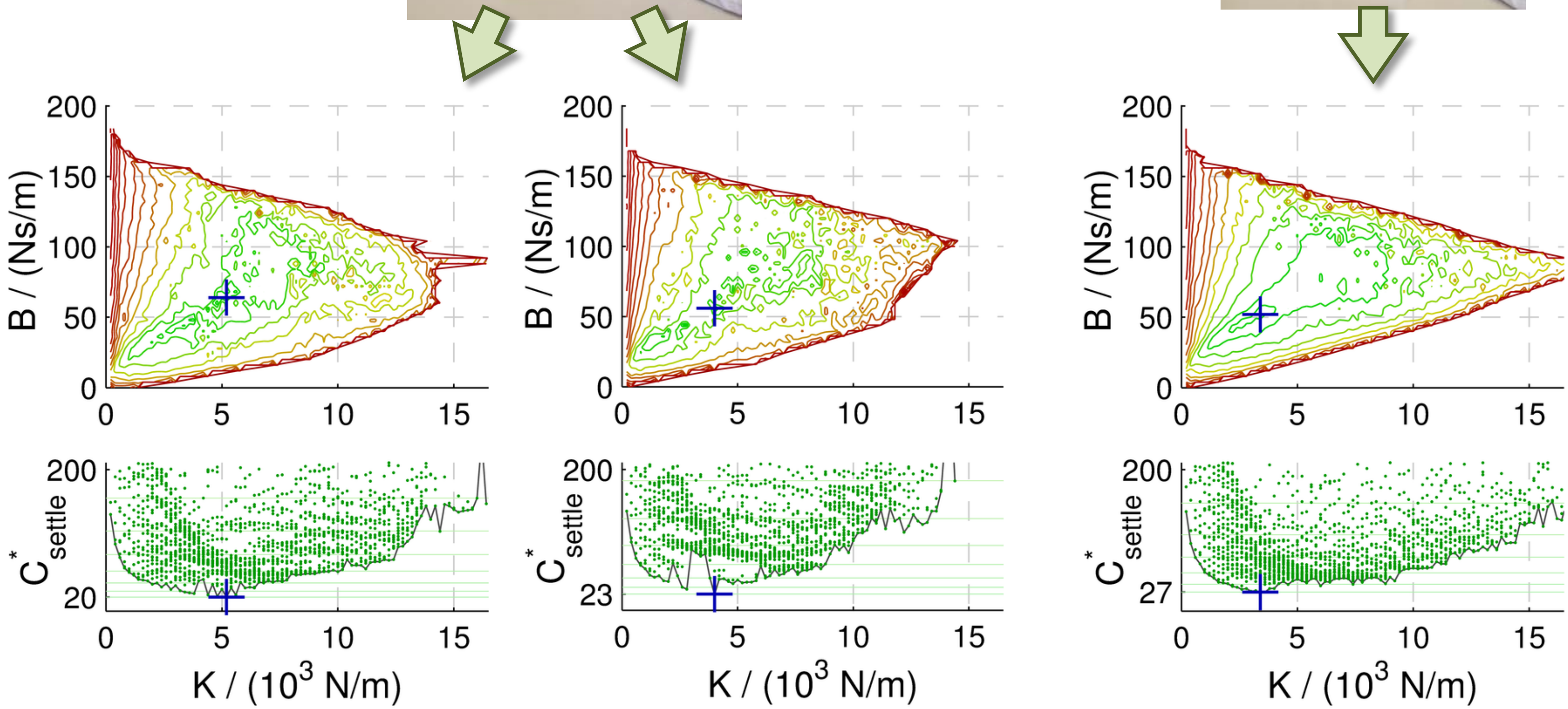
Results: 2% Settling Time



Experiments on the Novint Falcon

Results: 2% Settling Time

Possible Explanation [Ciáurriz 2013]:
Compliance resp. elasticities in the mechanical structure



Experiments on the Novint Falcon

Results: 2% Settling Time

human grip	dynamic mass m	viscous damping b	
	Falcon A&B[a]	Falcon A	Falcon B
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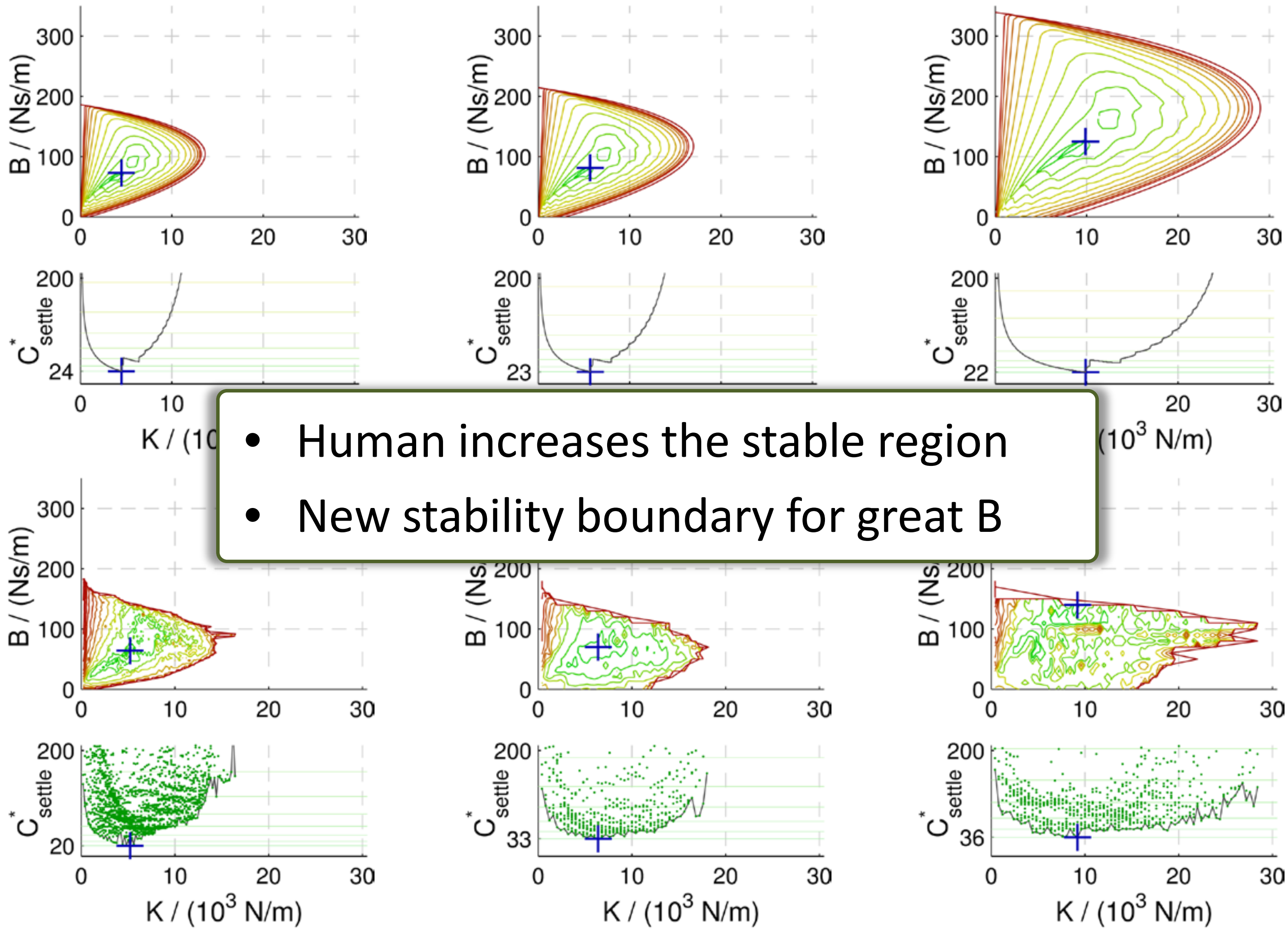
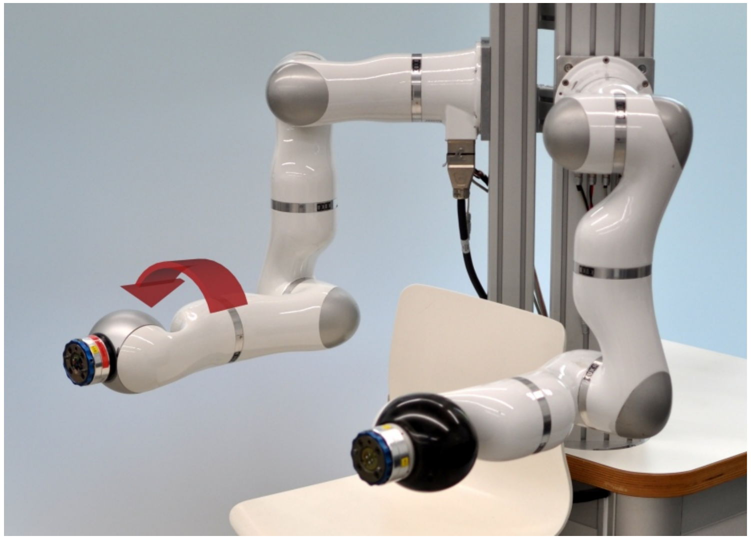


- Human increases the stable region
- New stability boundary for great B

Experiments on the LWR
Experimental Procedure



A set of seven experiments:

- Influence of time delay

For each experiment a grid was defined:

$\Delta K_{\mathrm{rot}} = 200$ Nm/rad
$\Delta B_{\mathrm{rot}} = 1$ Ns/m
→ up to 1025 grid points

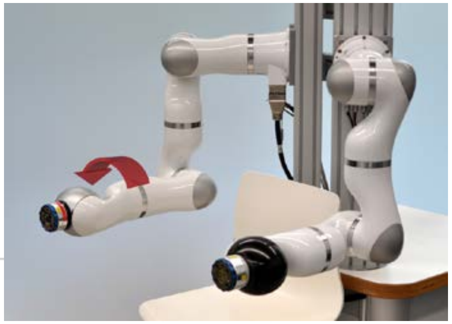
Parameter estimation results:

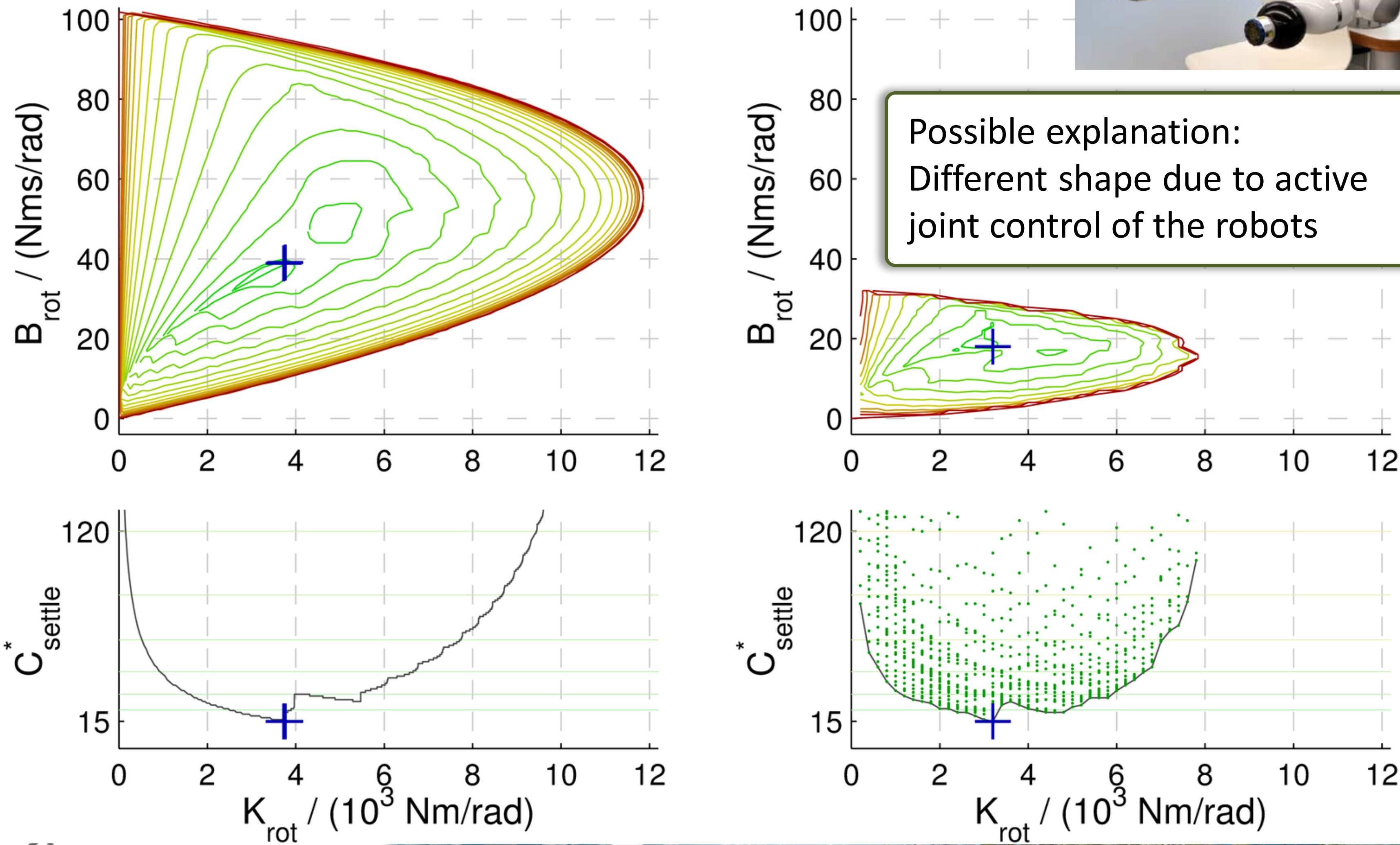
$t_{\mathrm{d}} = 2$ ms

$b_{\mathrm{rot}} \leq 0.1$ Nms/rad

$I = 0.19$ kg m^2

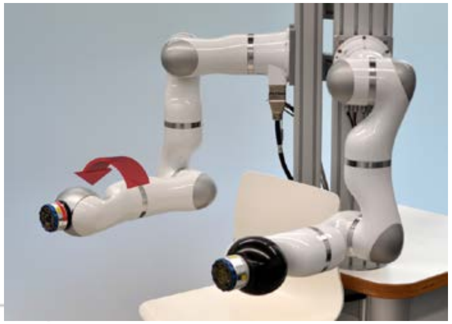
Experiments on the LWR
Results: 2% Settling Time

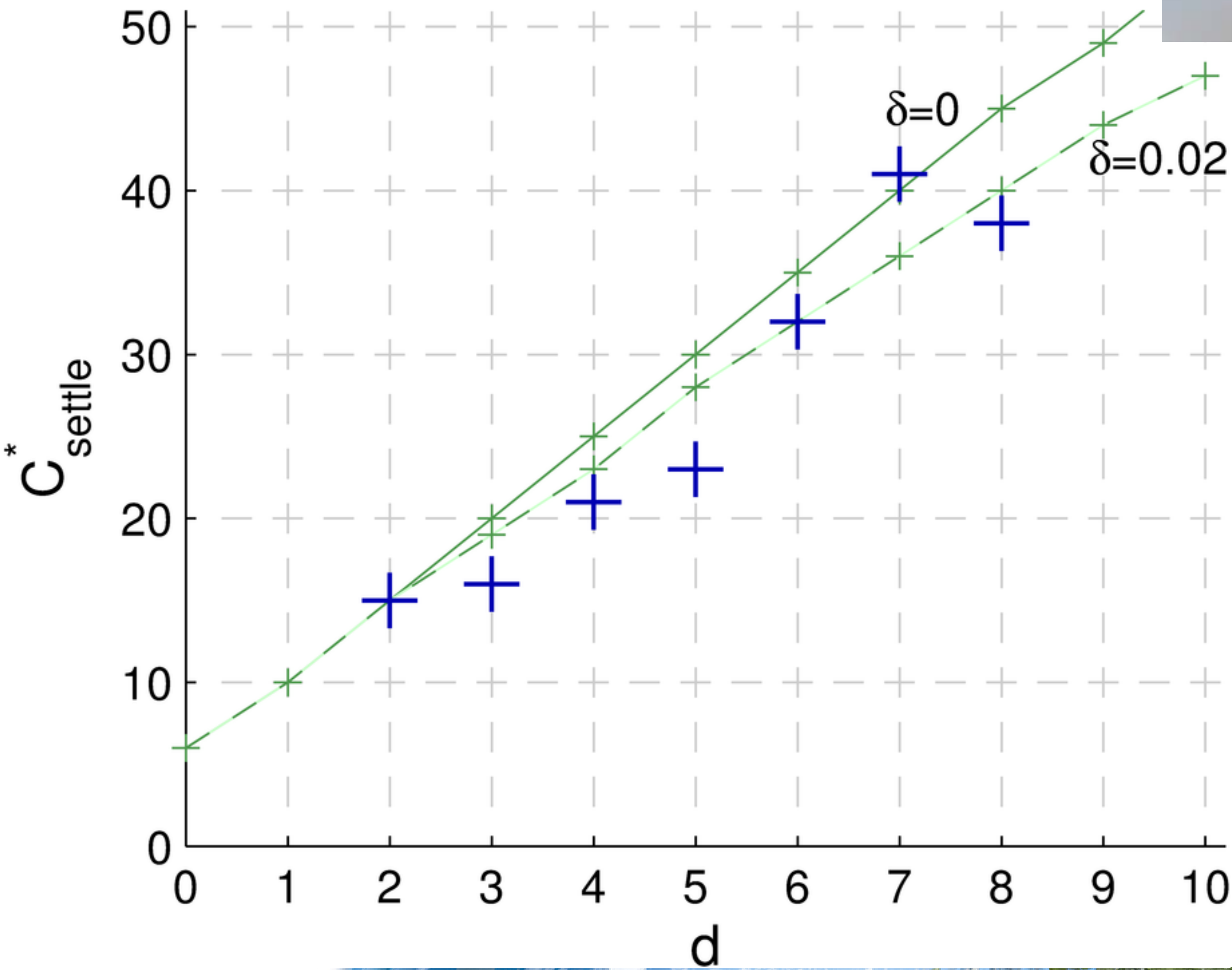




Possible explanation:
Different shape due to active joint control of the robots

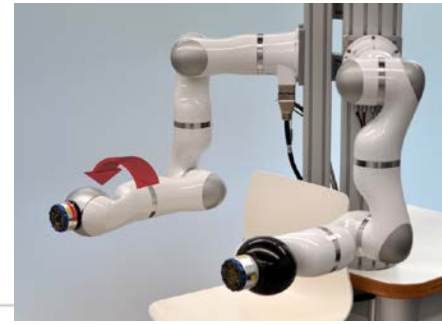
Experiments on the LWR
Results: 2% Settling Time

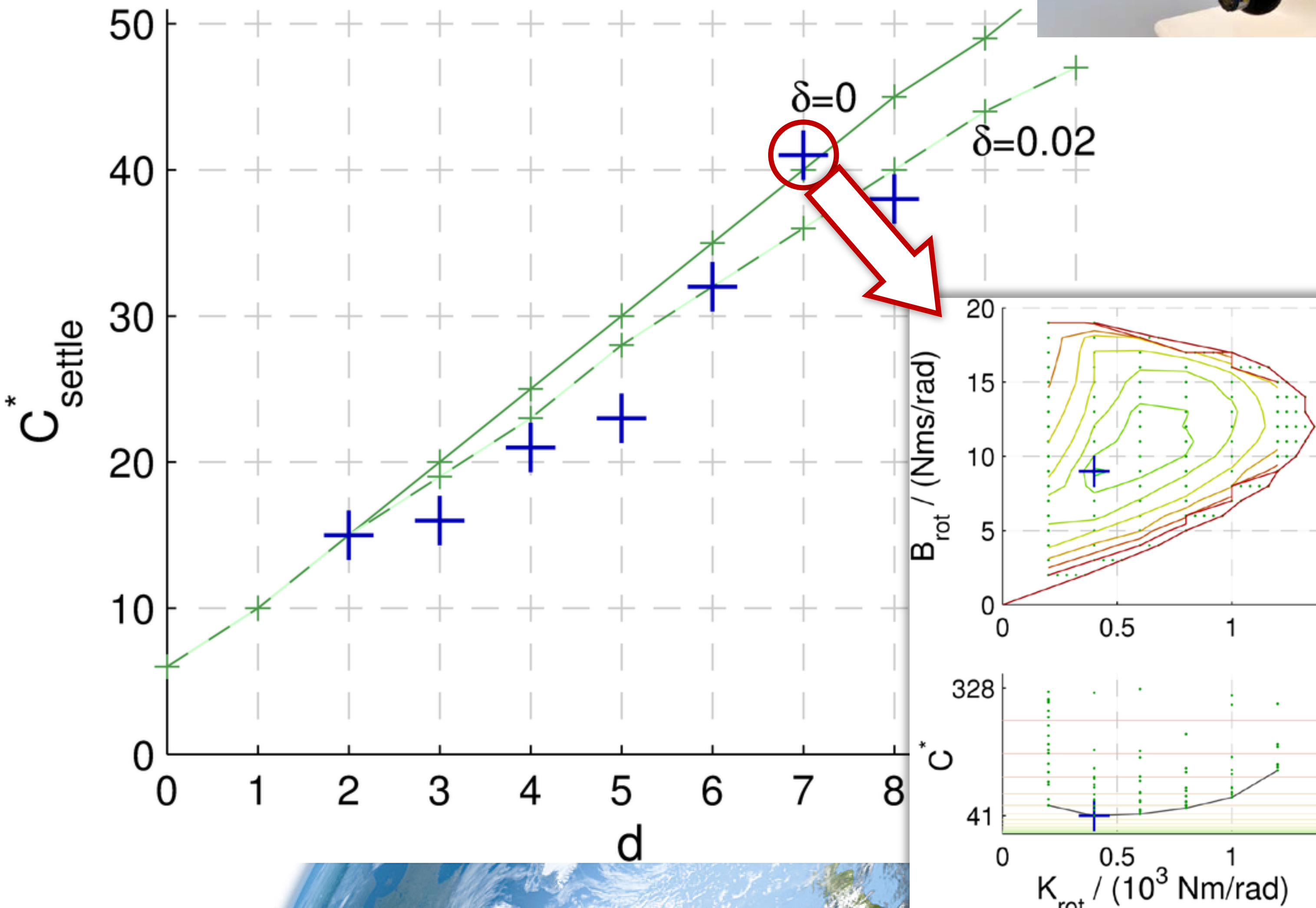




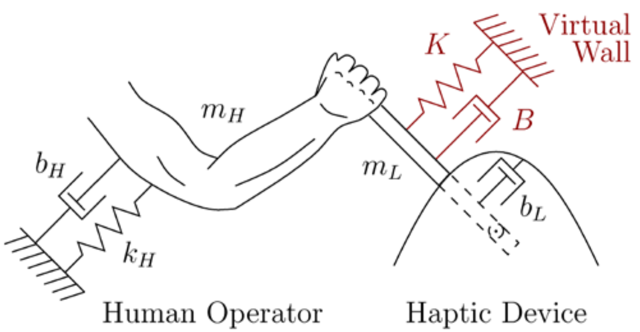
Experiments on the LWR

Results: 2% Settling Time





Summary

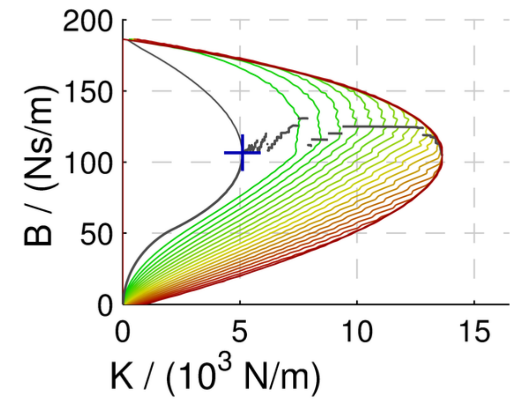


Stability Analysis

- Exact discrete-time equivalent
- Stability boundaries in normalized parameter plane
- Effect of human operator mainly by its mass contribution

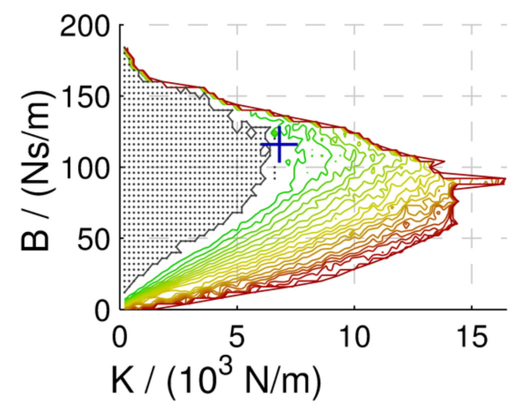
Control Design

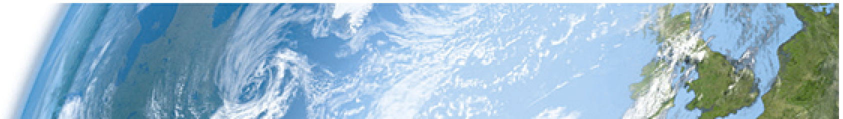
- Sub-region without overshoot inside the stable region
- Prediction function for the optimal performance
  (e.g. useful for cost-benefit assessment)
- Assessment and comparison of haptic devices
  (e.g. LWR vs. Falcon)
- (Passivity prevents optimal performance)



Experiments

- Theoretical analysis is suitable to predict the optimal performance, but not the exact shape of the boundaries
- Stabilizing effect of the human operator was confirmed





Future Work

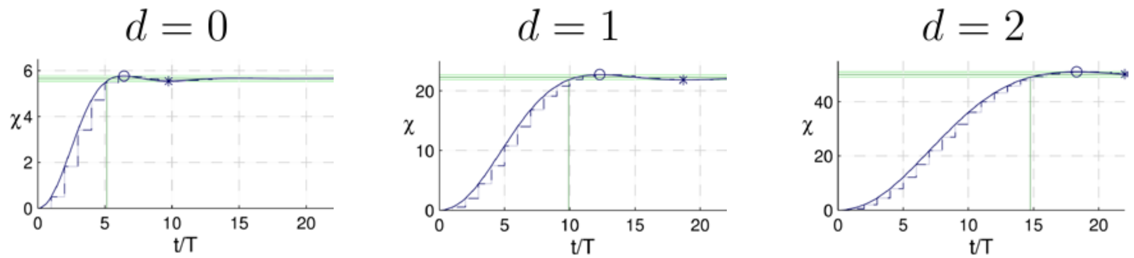
- **Analyze more comprehensive and realistic models**
  - Velocity filtering
  - Mechanical compliance
  - Nonlinearities
  - More complex virtual environments

- **Use different analysis methodologies and controllers**
  - Time-variant control
  - H-infinity
  - Lyapunov
  - Fractional-order controllers

- **Psychophysical studies**
  - Which control design approach *feels* most realistically?

Take-Home Messages

- **Rule of Thumb for the Optimal Performance**
  *"Each sampling period of additional time delay causes the optimal settling time (with a 2% tolerance band) to increase by approximately **five** sampling periods."*



- **Effect of Time-Discretization**
  "Effect of discrete-time sampling corresponds to a delay of one whole sampling period T in terms of optimal cost."

$$T_{\mathrm{eff}} = (d+1)T$$

Thank you for your attention!