Bounding Boxes Hirachy BVH's Homog.

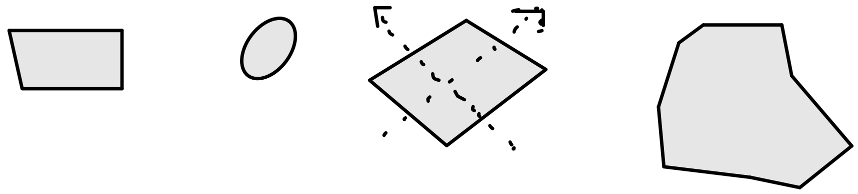
Def. Given set S of obj: If $|S| \le K$, then $BVH(S) :=$ leaf node b, storing S.
If $|S| > K$, then $BVH(S) :=$ node v, with $n(v)$ many children $v_1, \dots, v_n$, where $v_i := BVH(S_i)$, and $S_1, \dots, S_n$ partition S (i.e., $\bigcup_i S_i = S$)

Every node v in the BVH stores $BV(v)$ enclosing all obj's in S, ie $\forall o \in S$: $o \subseteq BV(v)$

Layerd BVH: where $\forall$ children v_i of v: $BV(v_i) \subseteq BV(v)$.

Wrapped BVH $\forall$ leaves v_i of v: $S(v_i) \subseteq BV(v)$

Types of BV's: HABB OBB DOP

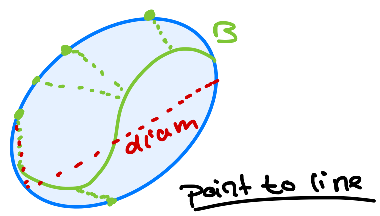


Def. Tightness

Let B be a BV, G = some Geometry, $G \subseteq B$

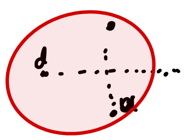
Def. directed Hausdorff distance

$$h(B,G) = \max_{b \in B} \min_{g \in G} d(b,g)$$



[btw: $h(B,G) \ne h(G,B)$]

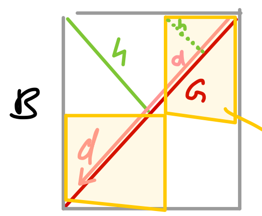
Def. diameter $diam(G) := \max_{g,f \in G} d(g,f)$



Tightness $\tau := \dfrac{h(B,G)}{diam(G)}$

[just one possible notion of tightness.]

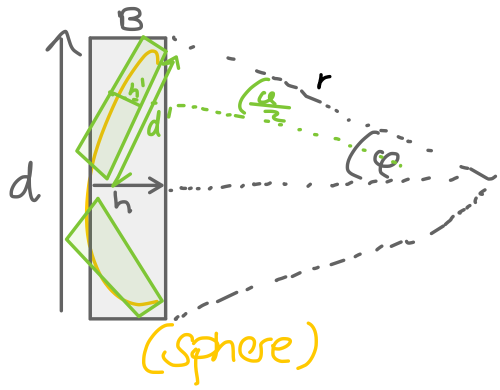
Note: Thightness of AABB-BVH's depends on "nice" orientation of the geometry



$$\tilde{\tau} = \frac{h}{d}$$

$$\tau' = \frac{h'}{d'} = \frac{h/2}{d/2} = \tau$$

Tightness of OBB-BVH's



$$h = r(1-\cos\varphi)$$

$$d = 2r\sin\varphi$$

$$\tau = \frac{1-\cos\varphi}{2\sin\varphi} \longrightarrow 0 \text{ as } \varphi \to 0$$

i.e. tightness of OBB depends on curvature of the geometry

Taylor..

$$\tau = \frac{1-\left(1-\frac{1}{2!}\varphi^2+\frac{1}{4!}\varphi^4-\dots\right)}{2\left(\varphi-\frac{1}{3!}\varphi^3+\dots\right)} \approx \frac{\frac{1}{2}\varphi^2}{2\varphi} = \frac{1}{4}\varphi \in O(\varphi)$$

corresponds to depth in tree

$\Rightarrow$ tightness of OBB's increases with level of the nodes in an OBB-BVH.

Construction:

Insertion strategy:

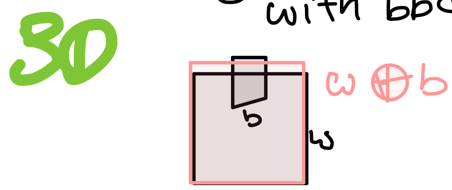
Start with empty BVH
Given set S of elementary BV's
while $|S| \geq 1$.
choose next $b \in S$,
start with $v :=$ root,
while v is not leaf:
choose child w of v, s.t. inserting b in BVH(v) decreases quality of BVH(V) the least.
$V := w$
....
at the leaf v: replace v by inner node v' with children v, b

Measure for quality: depends on the query!

Example: ray casting

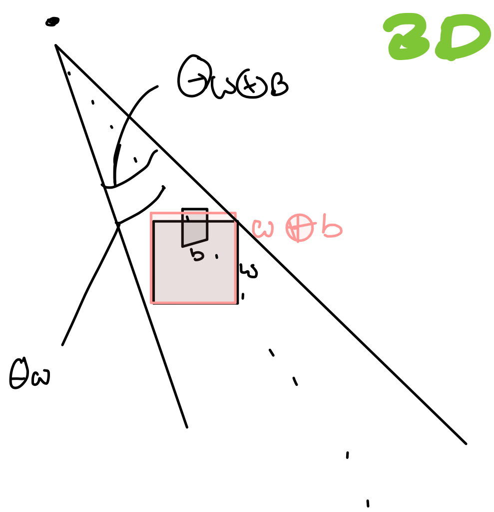
choose child w, s.t. area(bbox($w \oplus b$)) = min

merge bbox(w) with bbox(b)

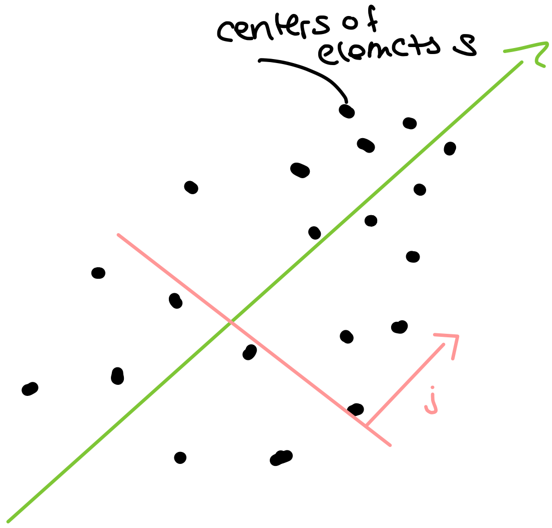


Surface Area Heuristic (SAH)

Reason: $\Pr[\text{ray hits } w \oplus b \mid \text{hits } w] \approx \dfrac{\Theta_{w+b}}{\Theta_w} = \dfrac{\text{area}(\text{bbox}(w+b))}{\text{area}(\text{bbox}(w))}$



Top-Down-strategy:



Needs "good" partitioning of S

Apply SAH. partition $S = S_1 \cup S_2$

Define cost

$$C(S_1, S_2) = \frac{\text{area}(bv(S_1))}{\text{area}(bv(S))} \cdot C(S_1)\,|S_1| + \frac{\text{area}(bv(S_2))}{\text{area}(S)} \cdot \underbrace{C\ S_2}_{S_2}\ |S_2|$$

How to find S_1, S_2 s.t. $C(S_1, S_2) = \min$?

Pragmatic algo.

calc PC1
sort S along PC1

find $$\min_{j=0,\dots,n} \left\{ C\left(\underbrace{\{s_0, \dots, s_j\}}_{S_1}, \underbrace{\{s_{j+1}, \dots, s_n\}}_{S_2} \right) \right\}$$

Complexity: $T(n) = 2T\left(\frac{n}{2}\right) + O(n \log n)$

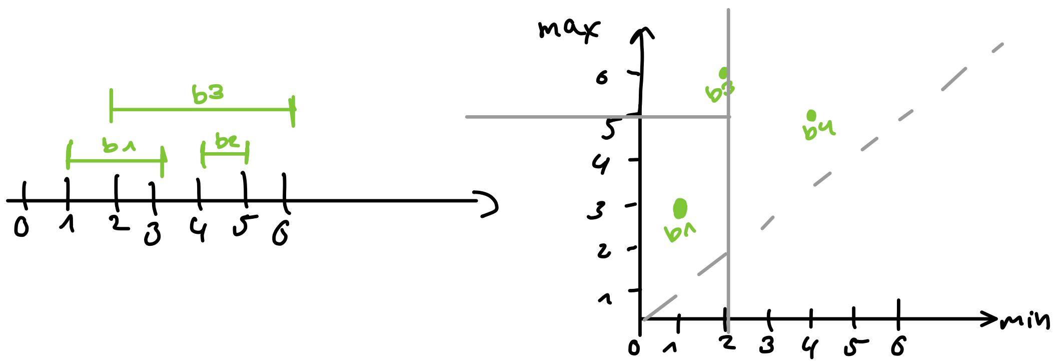
$$= O(n \log^2 n)$$

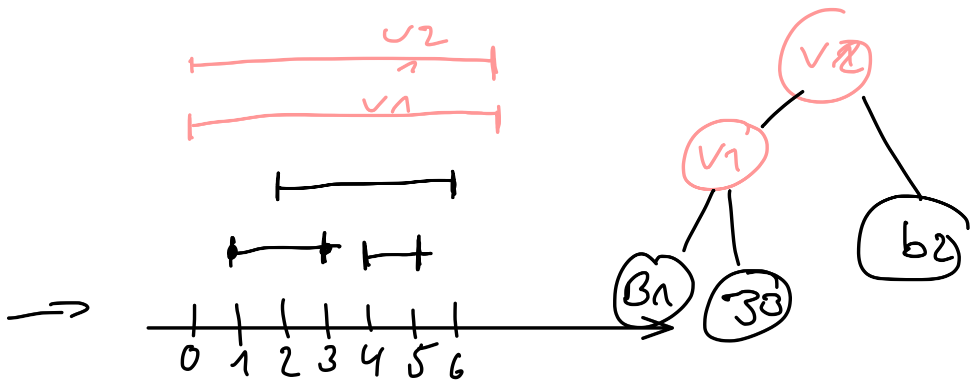
Configuration space strategy

Consider AABB's $b_i \in S$ as points $p_i \in \mathbb{R}^{2D}$

Build KD-tree over those pts

Transform kd-tree into AABB-BVH:
leaf p_i in kd-tree $\longrightarrow$ leaf b_i in AABB-tree
inner node v in kd-tree $\longrightarrow$ inner node v in AABB tree
with $bbox(v) = bbox(v_1, v_2)$ ← children
i.e. layered hierachry





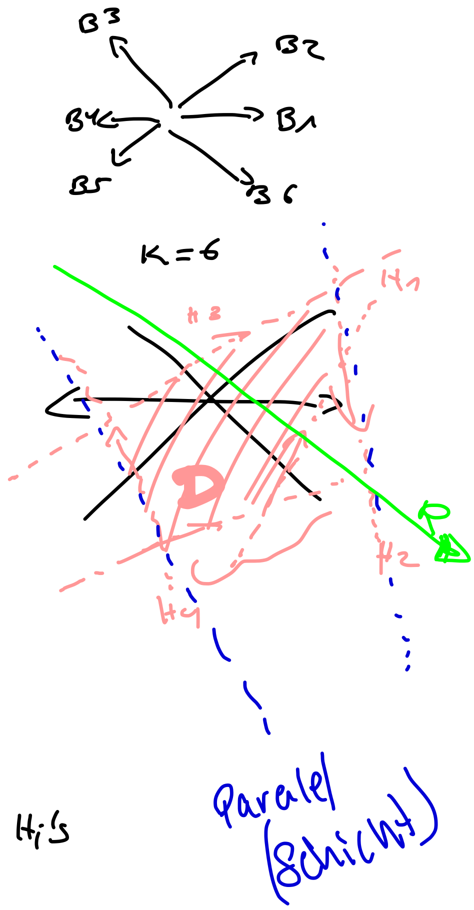
Theorem (wo proof)
Rectangle intersection query in $\mathbb{R}^d$ can be answered in time $O(n^{1-\frac{1}{d}} + k)$.

In 3d. $O(n^{(2/3)})$

k-DOPS (Discrete Oriented Polytope)

choose set $B = \{B_1, \dots, B_k\}$ fixed, $k \geq 2d$, $B_i \in \mathbb{R}^d$

with condition $B_{k+i} = -B_i$



Define. DOP $D := \bigcap_{i=1}^{k} H_i$,

halfspaces $H_i : x \cdot B_i - d_i \geq 0$

$\rightarrow D = (d_1, \dots, d_k)$

1. Application: ray Tracing

intersect ray $x = \vec{p} + t\vec{r}$ with all H_i's

$\Rightarrow t_i = \dfrac{d_i - B_i p}{B_i \cdot r}$ $\rightarrow$ 8k FLOPS per DOP

precomputed for ray

Note. K-DOPS are generalizations of AABB's