

08.05.19 Collision Detection (uploaded slides)

The Sausage Conjecture

Volume of a sausage packing


$$\begin{aligned} V_{\text{sausage}} &= V_{\text{cylinder}} + 2 V_{\text{halfsphere}} \\ &= \pi h r^2 + \frac{4}{3} \pi r^3 \\ &= 2\pi((n-1)r^2 + \frac{4}{3} \pi r^3) \end{aligned} \quad (1)$$

Volume of Tetrahedron

$$V_T = \frac{\sqrt{2}}{12} a^3$$


(2)

Radius of inscribed sphere

$$\begin{aligned} r &= \frac{\sqrt{6}}{12} a \\ \Rightarrow a &= 2\sqrt{6} r \end{aligned} \quad (3)$$



$x = \text{number of spheres}$
in the row

Tetrahedron with x spheres in the row

$$a = \frac{1}{2}((x-1) + \sqrt{6}) \cdot r \quad (4)$$

$$(4) \text{ in } (2) \Rightarrow V_{\text{spheres}} < \frac{\sqrt{2}}{12} \cdot ((x-1) + \sqrt{6}) \cdot r^3 \\ < \frac{2(x-1 + \sqrt{6})^3 \sqrt{2} \cdot r^3}{3}$$

n = numbers of spheres with
length x spheres in one row (Tetrahedron)

$$n = \sum_{i=1}^x \sum_{j=1}^i j = \frac{x(x+1)(x+2)}{6} \quad (5)$$

$$(5) \text{ in } (1): V_{\text{sphere}} = \frac{x(x+1)(x+2)-2}{3} \pi r^3$$