

Summer Term 2025

Assignment on Computational Geometry - Sheet 1

Due Date 27. 04. 2025

Due by 27. 04. 2025 via email to ju_bo@uni-bremen.de

Exercise 1 (Balanced Quadtrees)

Suppose we make the balancing condition for quadtrees more severe: we no longer allow adjacent squares to differ by a factor two in size, but we require them to have exactly the same size. Is the number of nodes in the new balanced version still linear in the number of nodes of the original quadtree? If not, can you say anything about this number?

Exercise 2 (Range Queries)

Quadrees can be used to perform range queries.

- a) Describe an algorithm for querying a quadtree on a set P of points that outputs all points which are inside a given axis aligned rectangle.
- b) Determine the worst-case runtime for such a range query.

Exercise 3 (Quadrees and Binary Images)

We can also represent a binary image (an image where each pixel is either black or white) with a quadtree. Cells will be split until each cell contains only pixels of the same color.

Assume we have an image of size $2^n \times 2^n$ which contains only white pixels and exactly one black square of size $2^m \times 2^m$, $1 \leq m < n$.

- a) How many nodes does the quadtree contain in the best case?
- b) How many nodes does the quadtree contain in the worst case?
- c) How many nodes does the quadtree contain if the square is exactly centered?

Exercise 4 (Boolean Operations on Binary Images)

Assume we have a quadtree over a binary image as describes in exercise 3.

Describe an algorithm to perform a boolean operation of your choice (or, and, xor) on the image which uses the quadtree as an acceleration data structure.