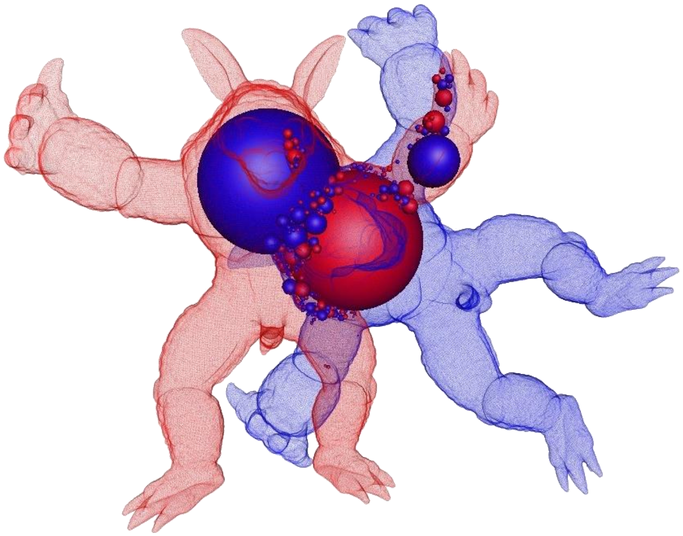
Bremen

Computational Geometry

Of Collisions and Spheres

and their Application to Computer Graphics and Beyond



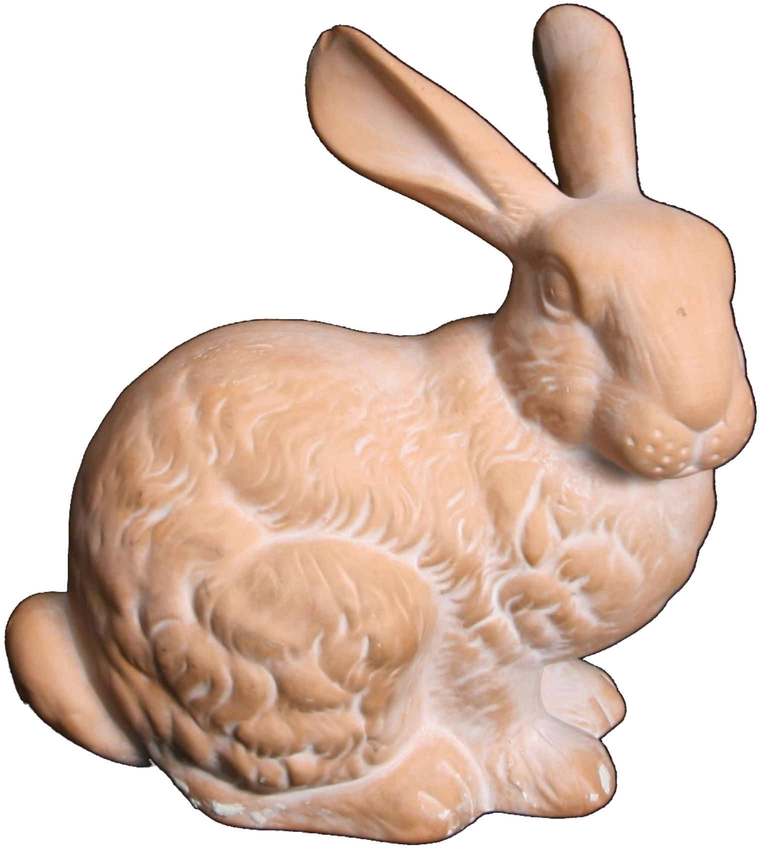
Motivation

[Rise of the Tomb Raider]

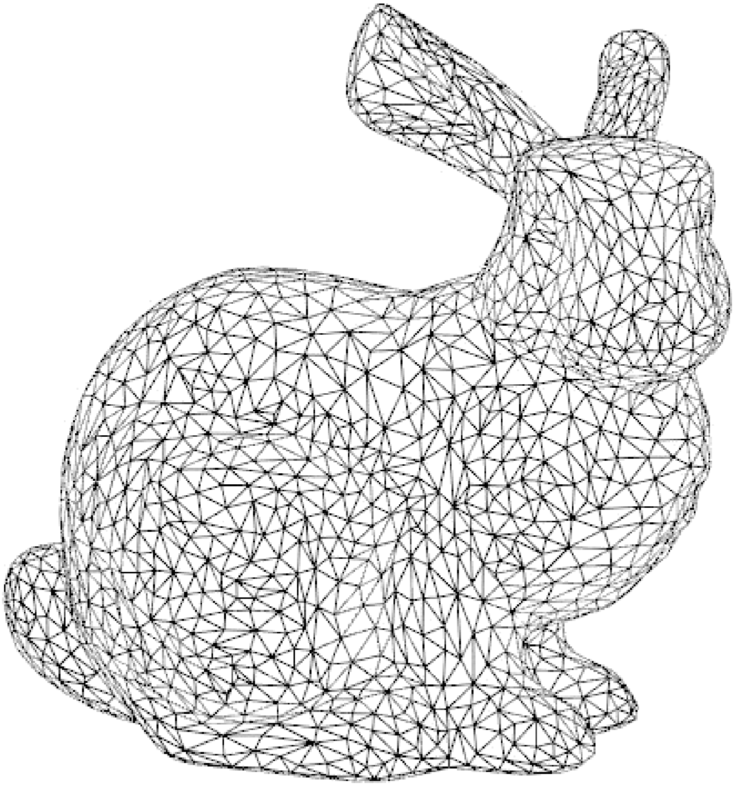
[RTT Deltagen 12]

Motivation

Collision Detection Basics



Collision Detection Basics



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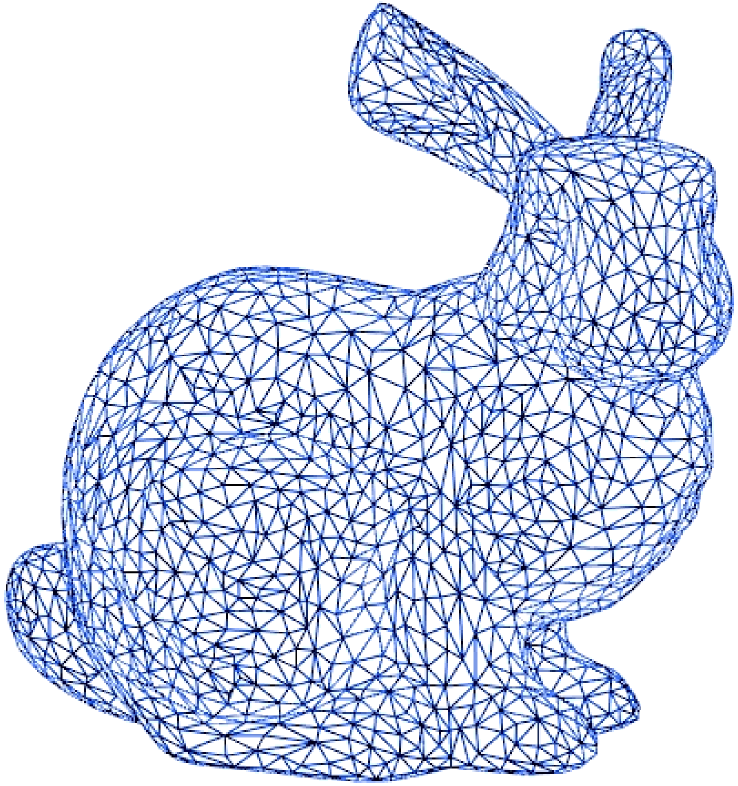

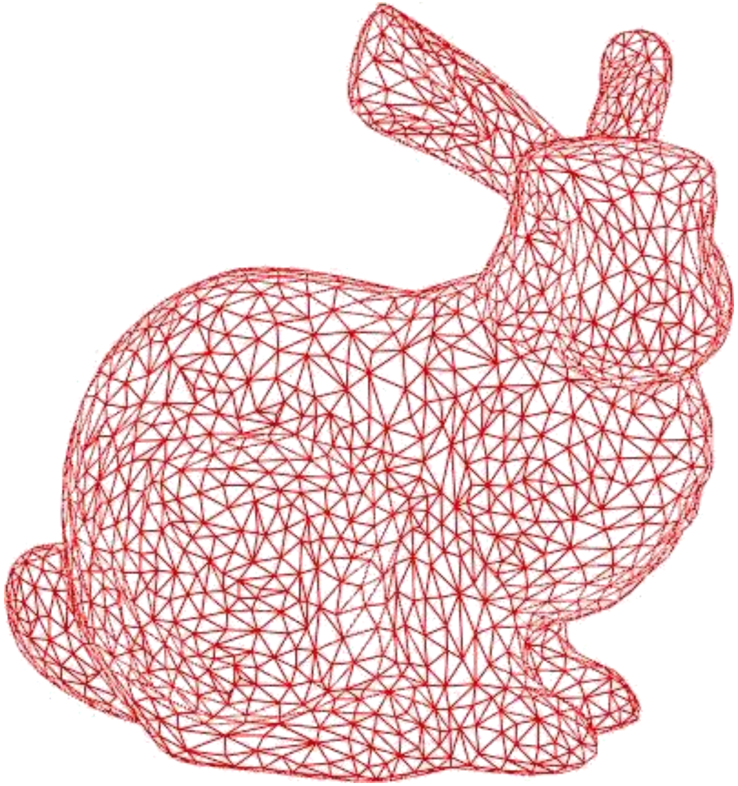

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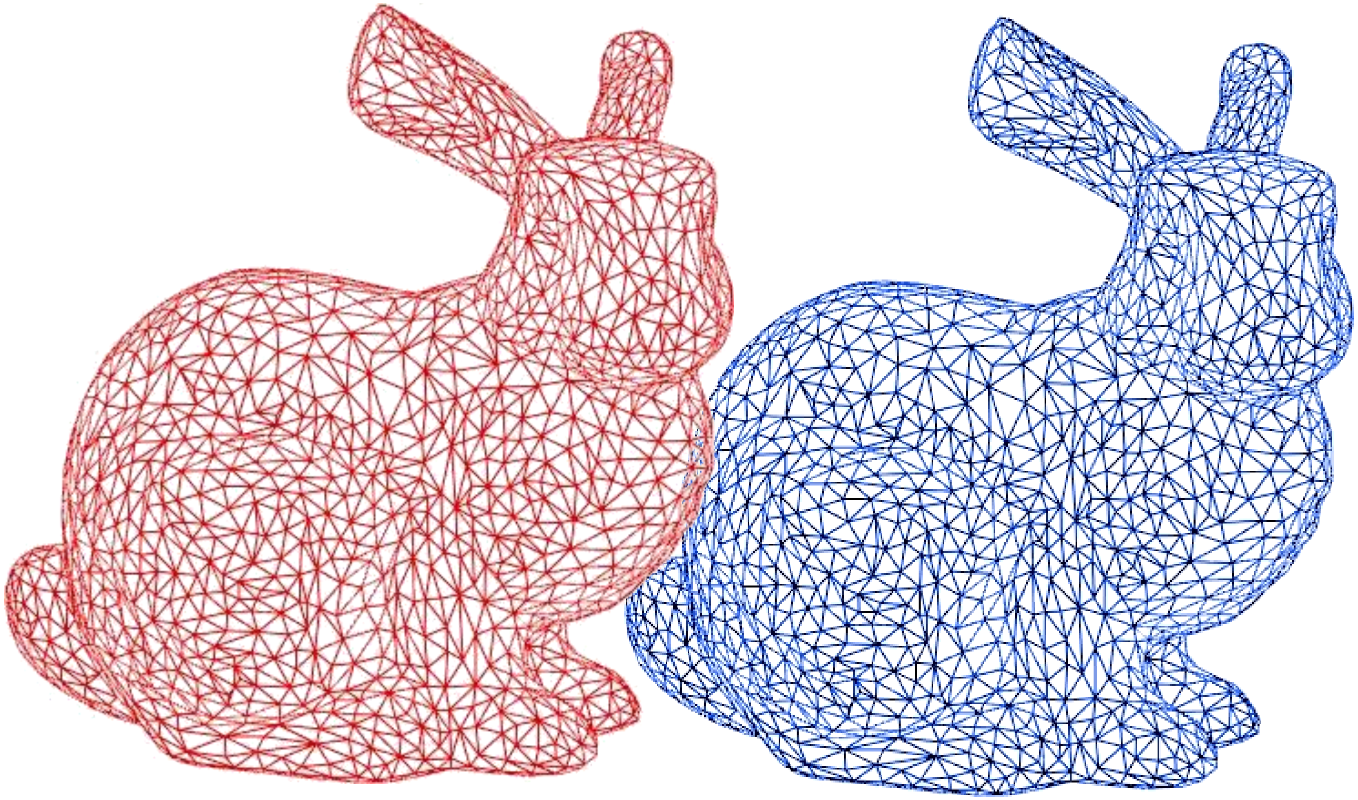

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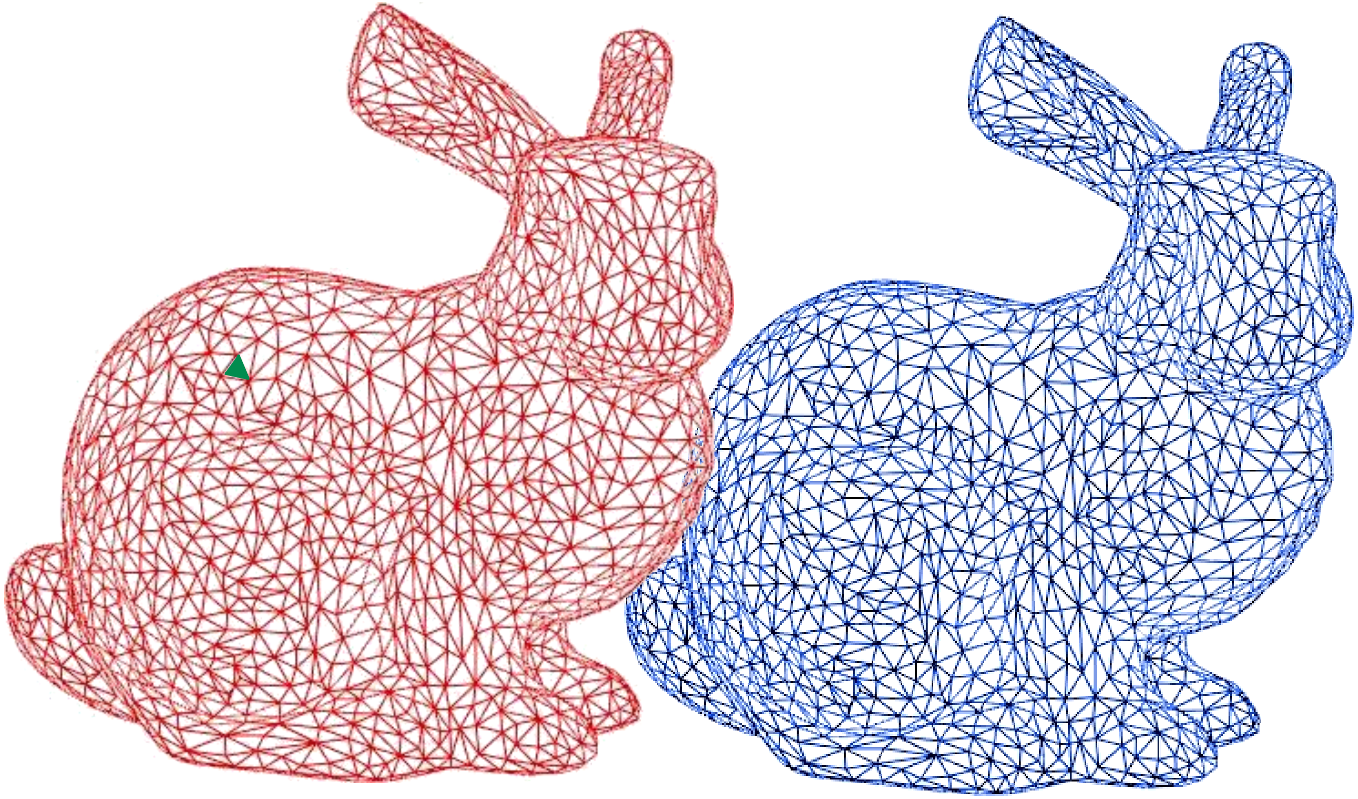

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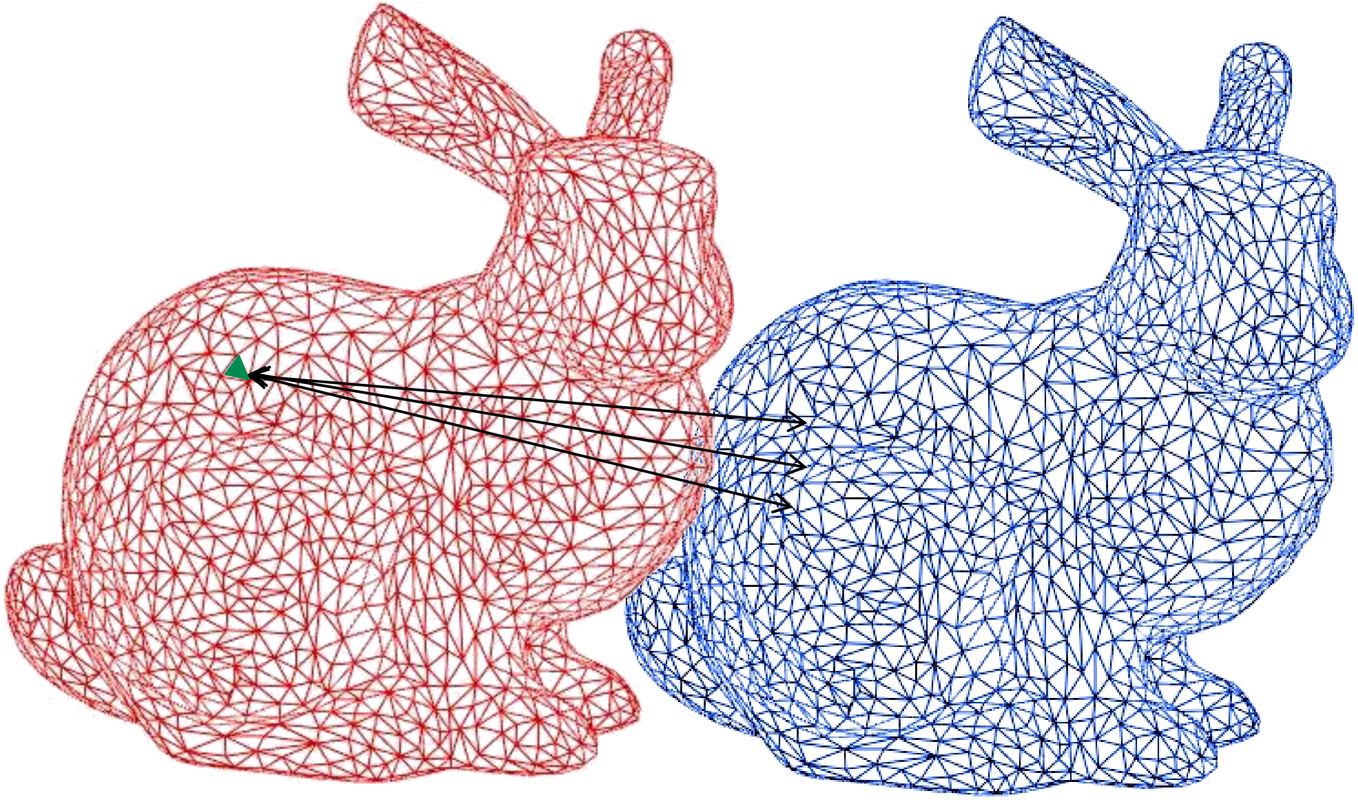

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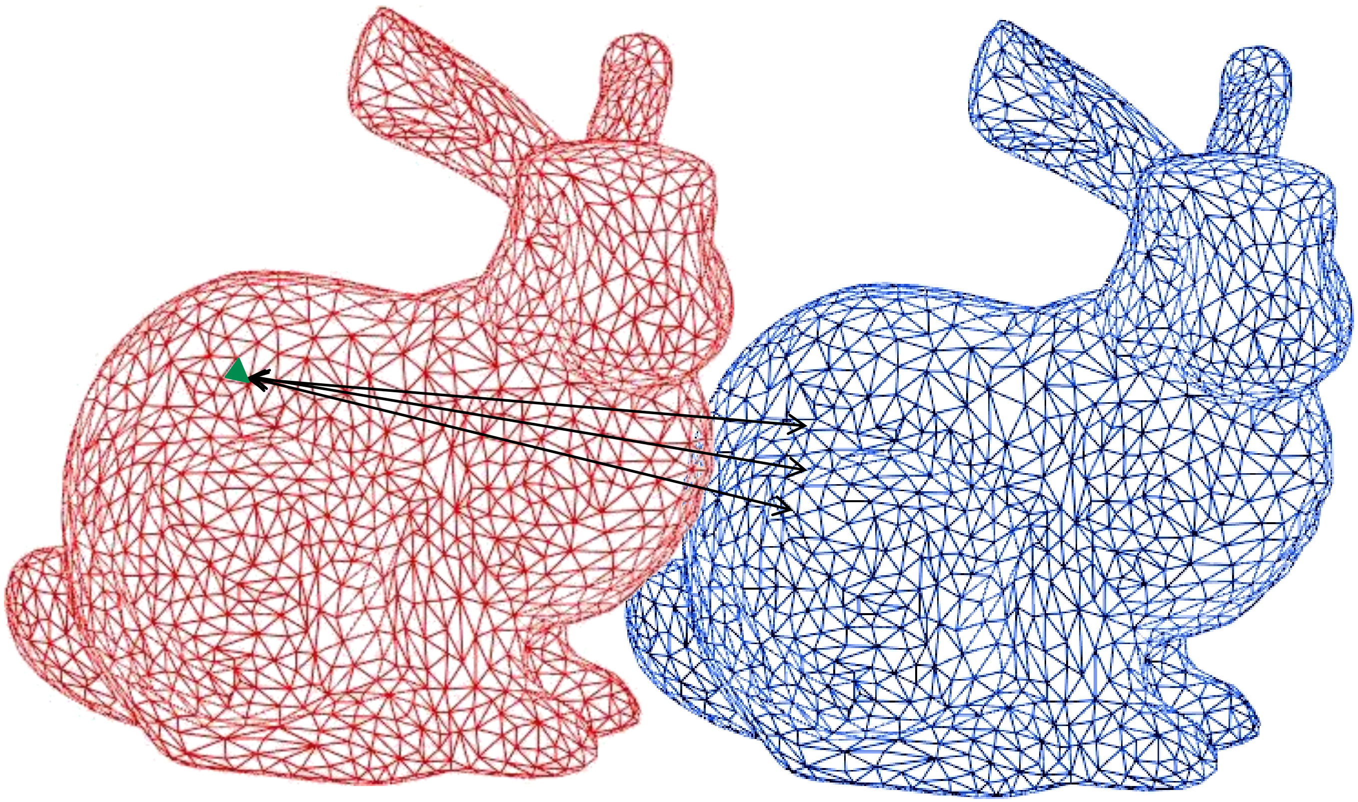
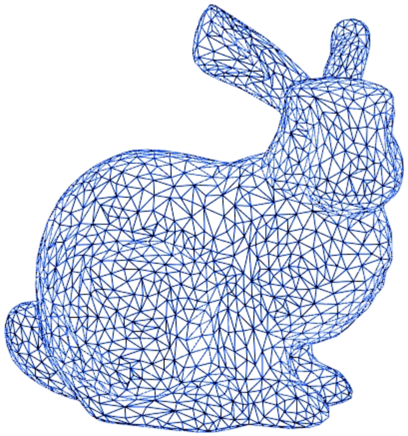


$$O(n^2)$$

Bounding Volume Hierarchies



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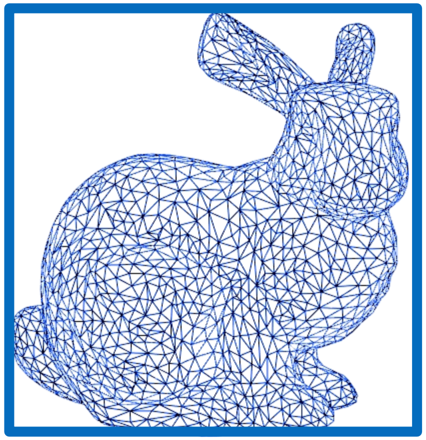

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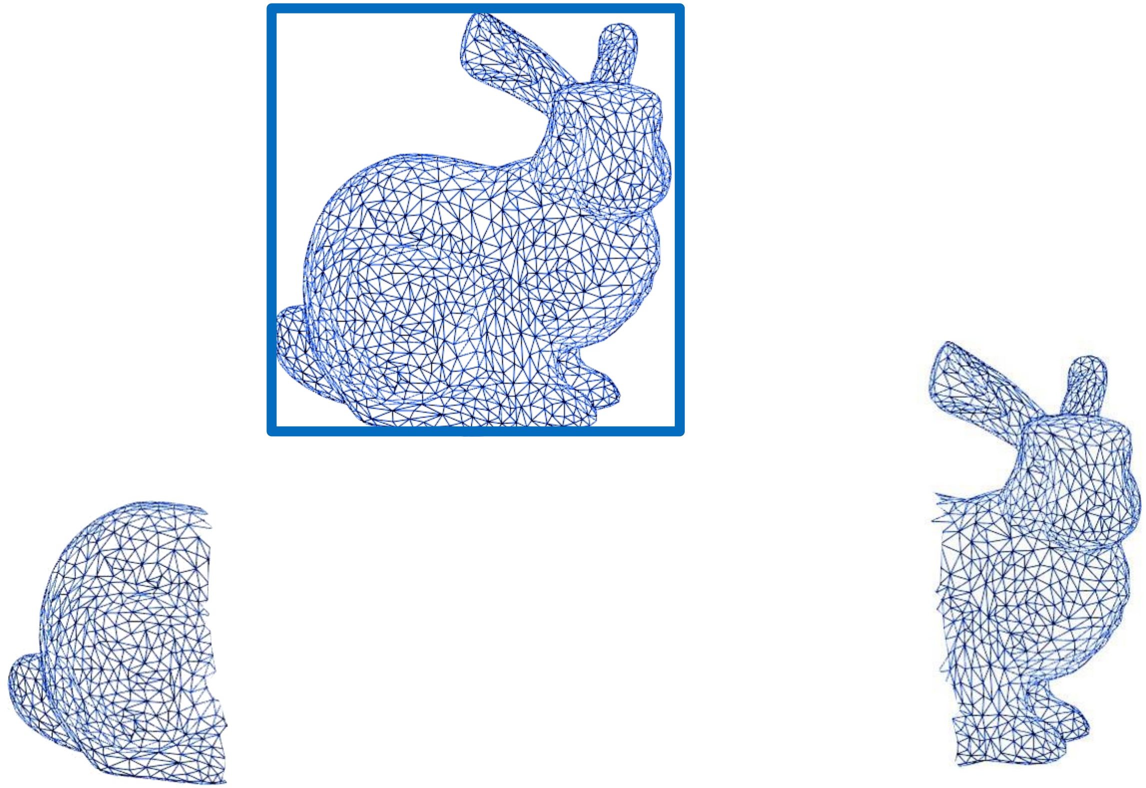

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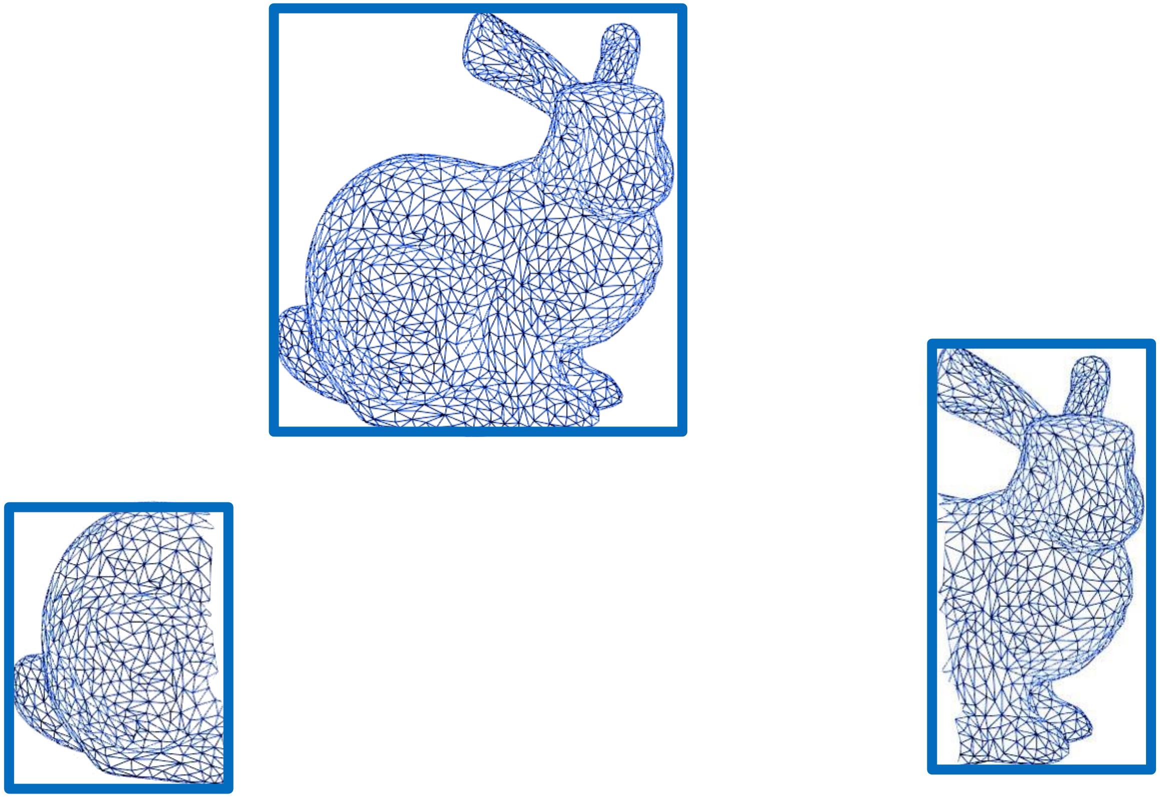

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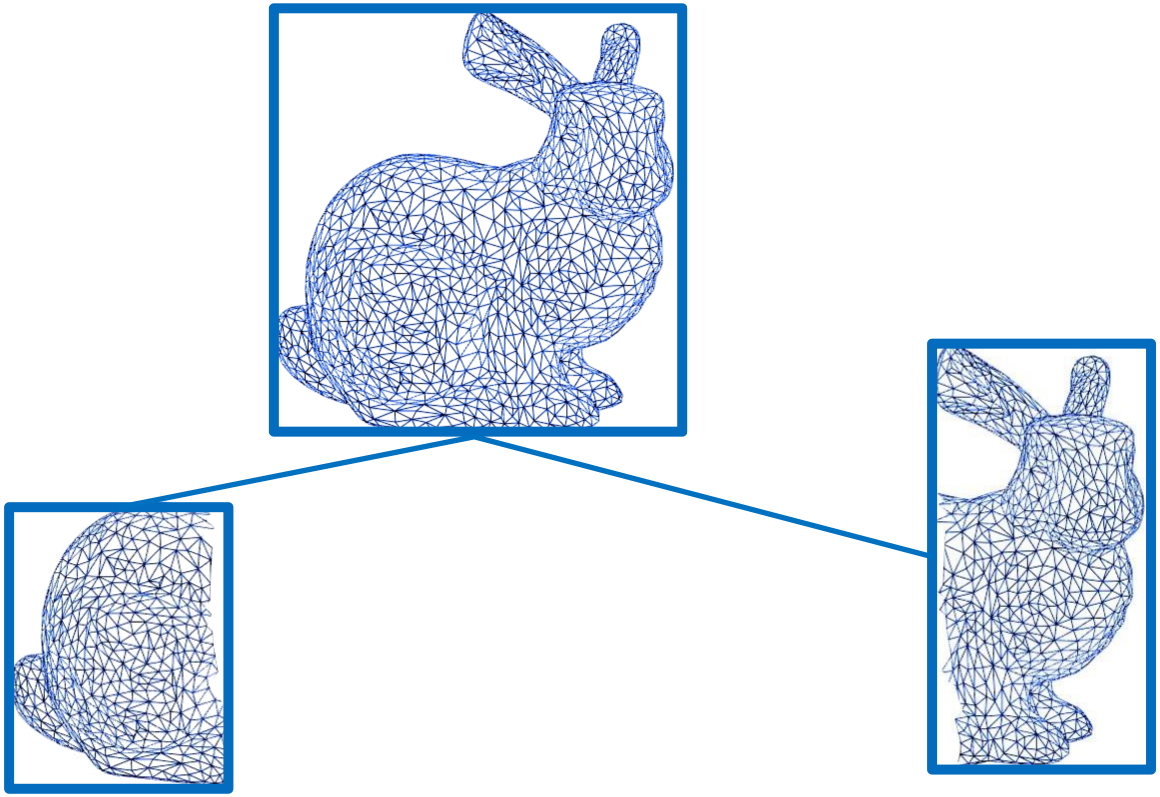

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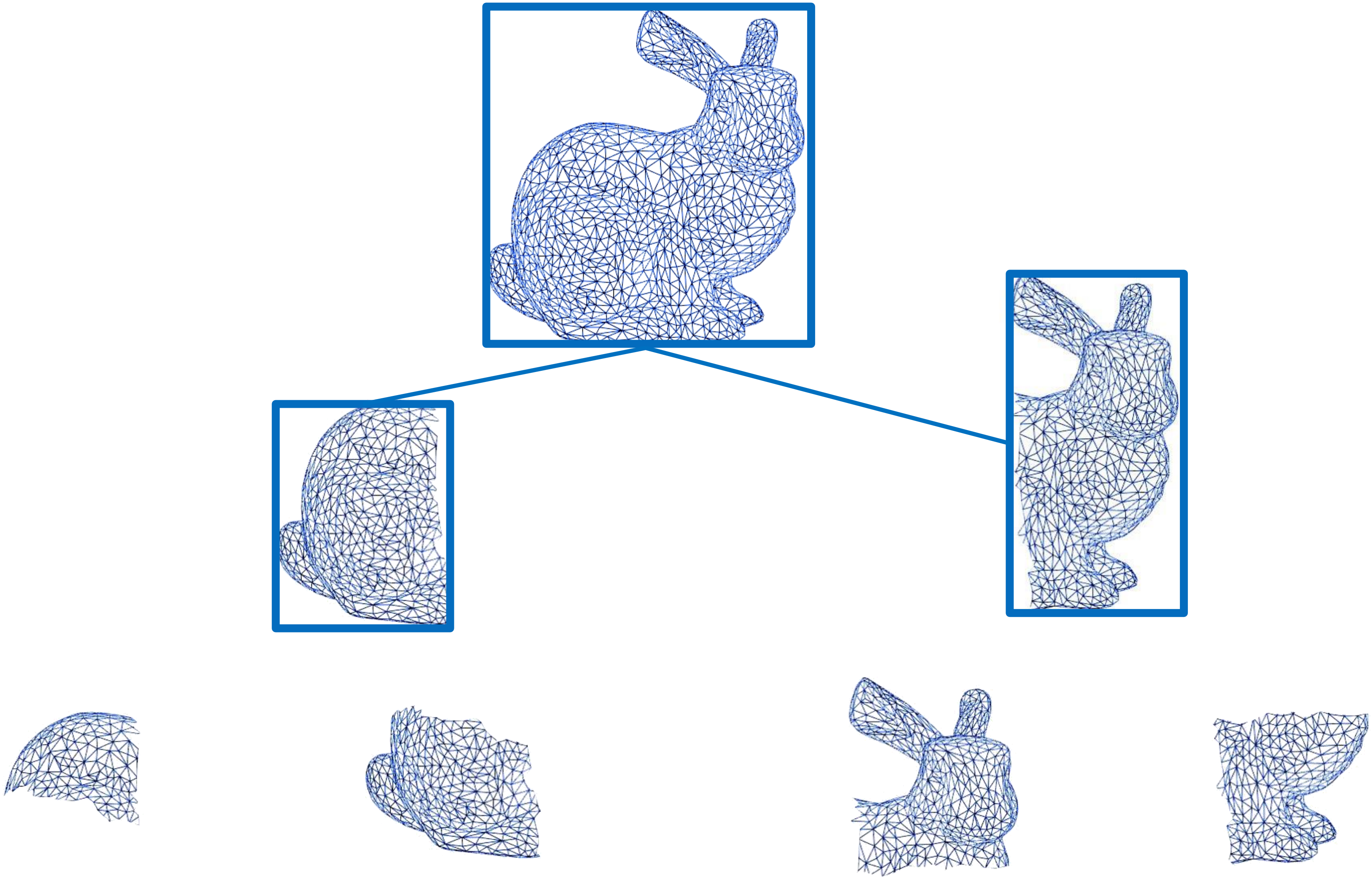

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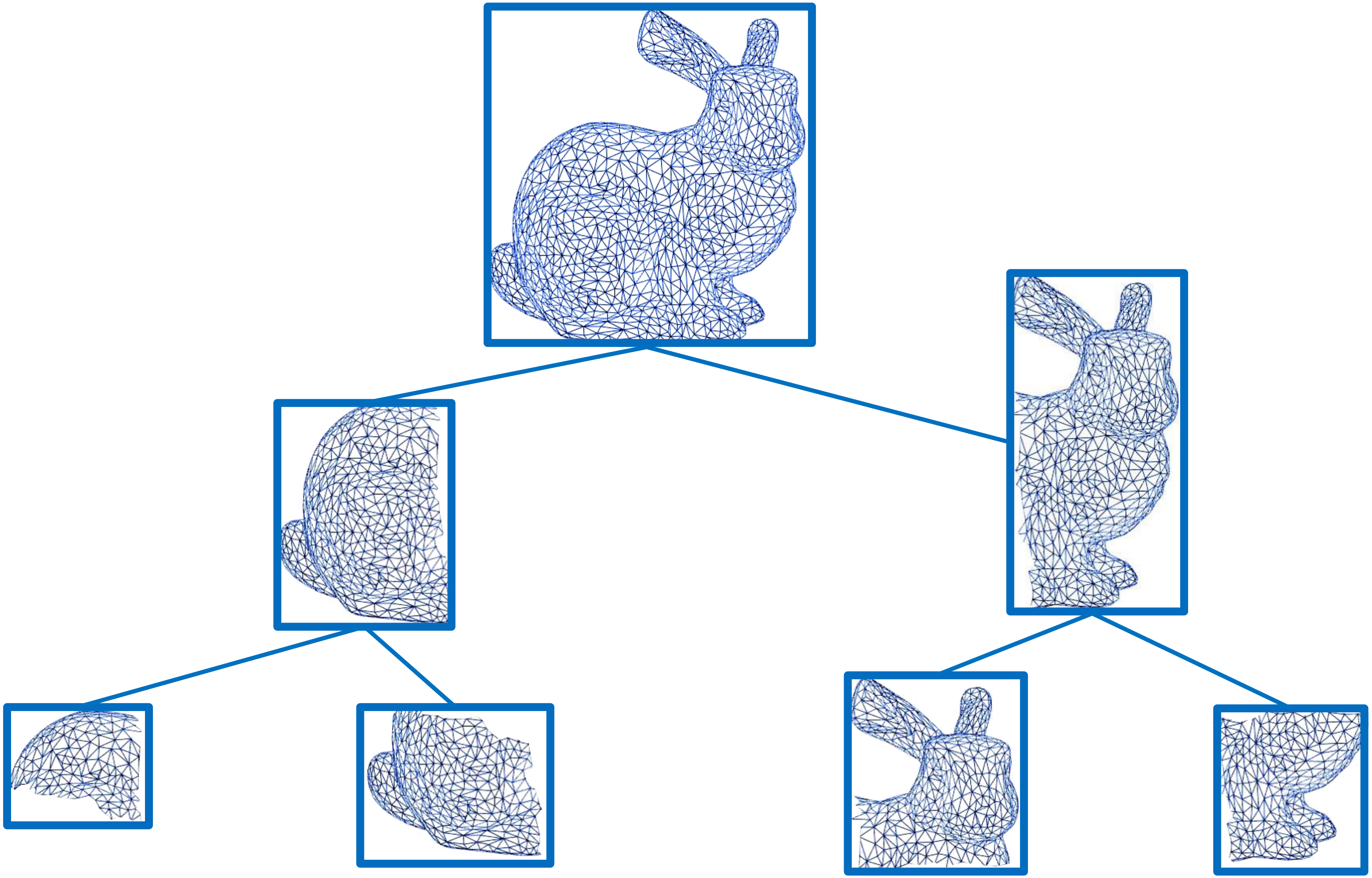
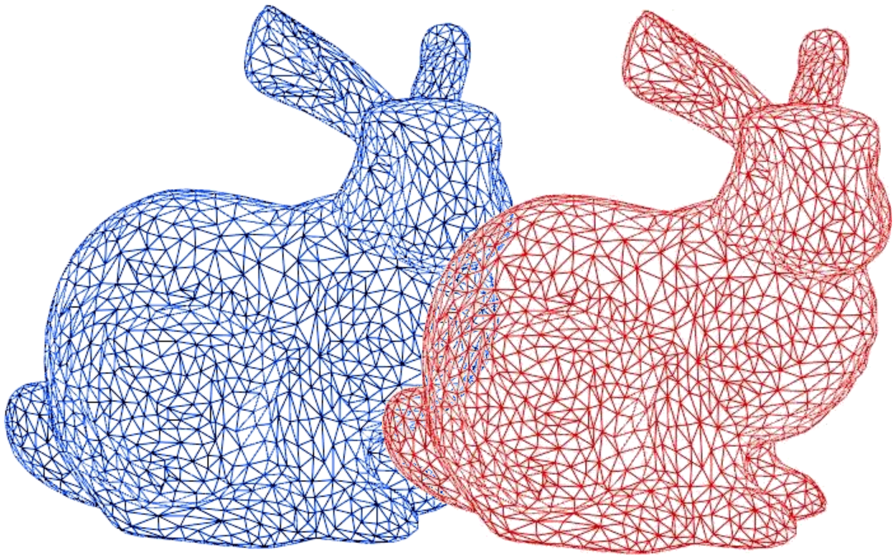


BVH Traversal



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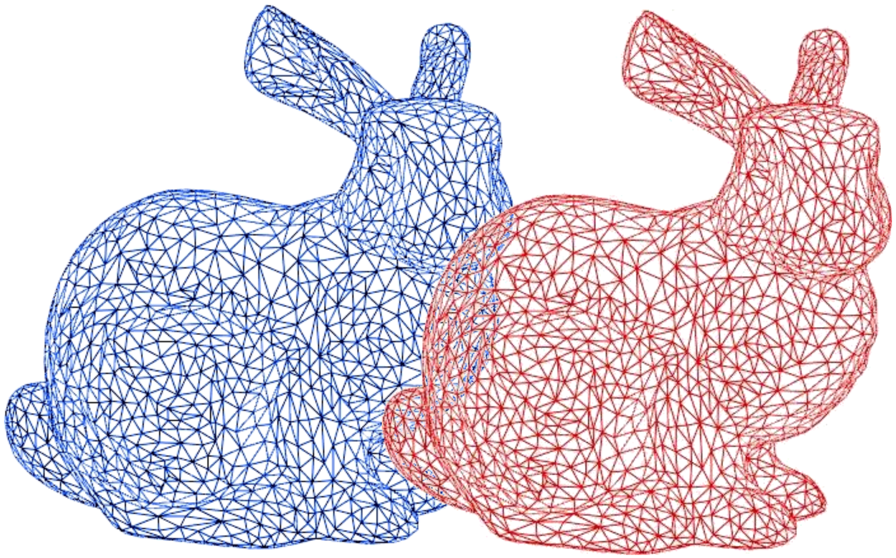

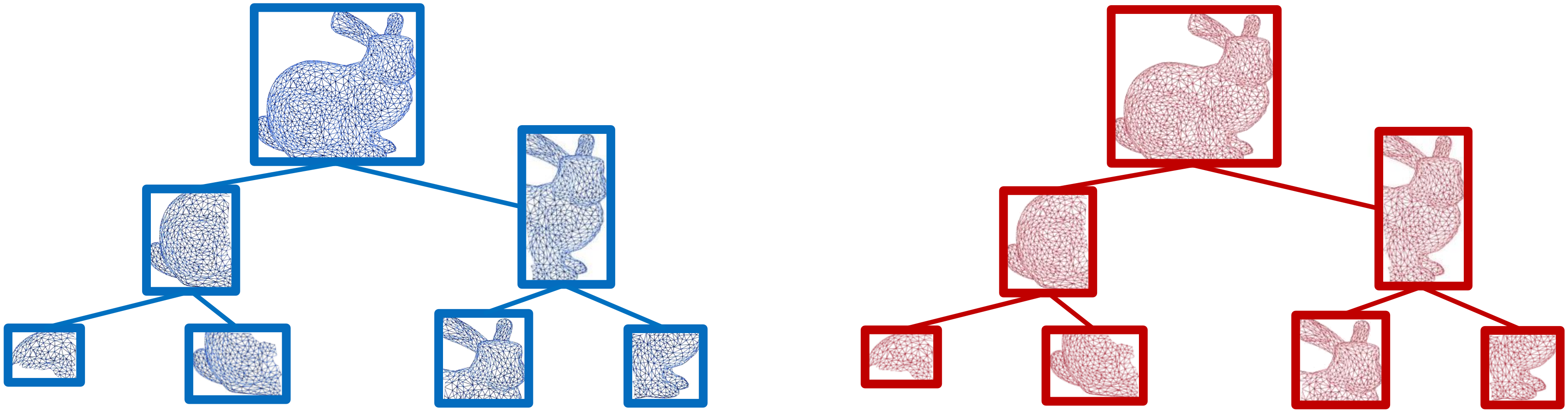

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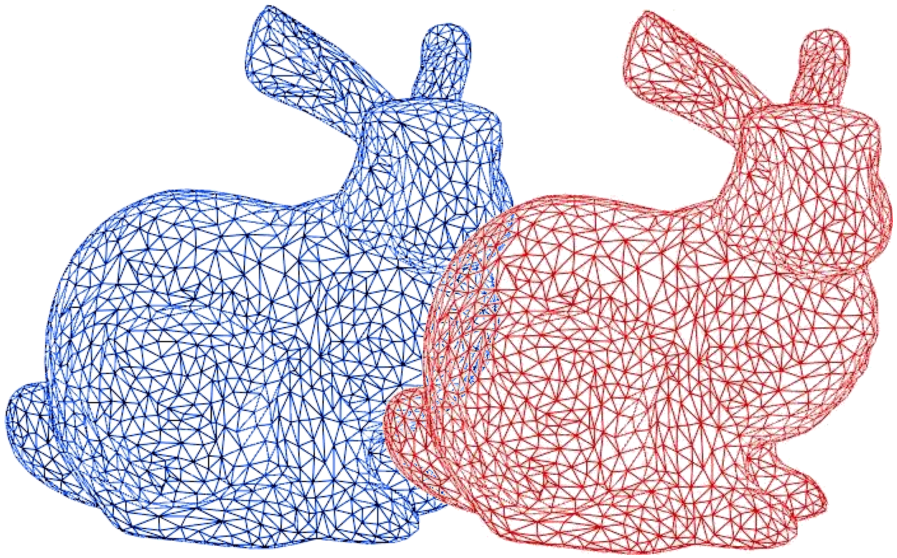

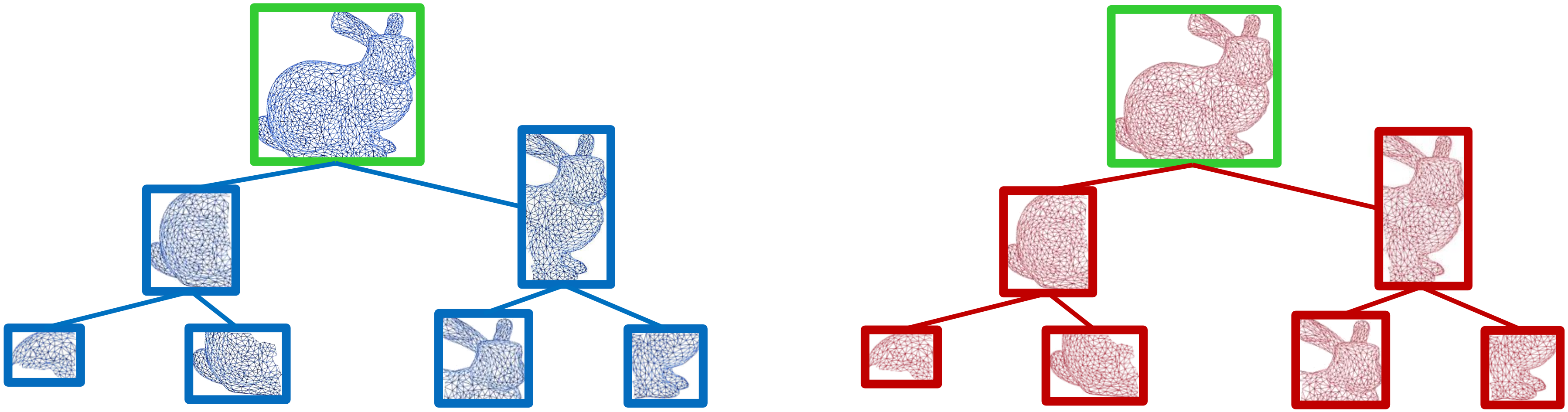

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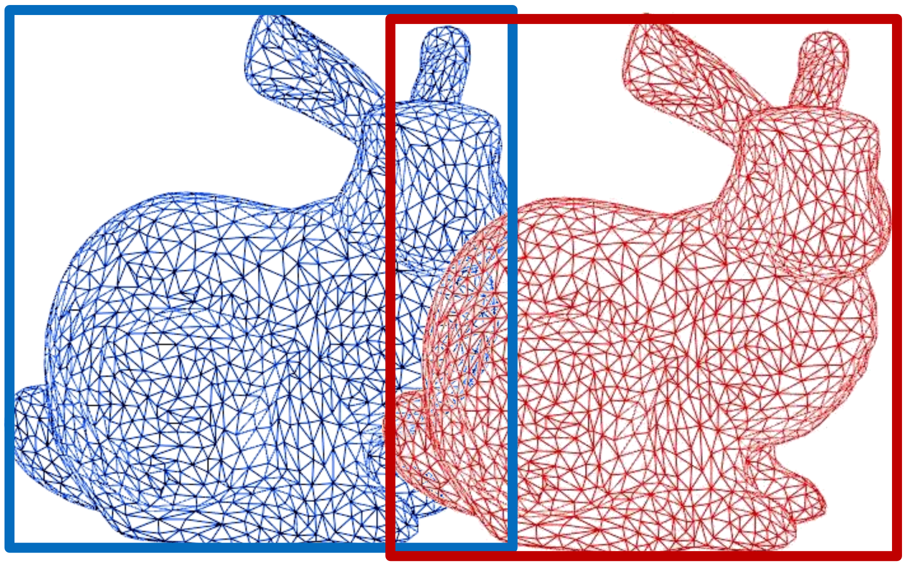

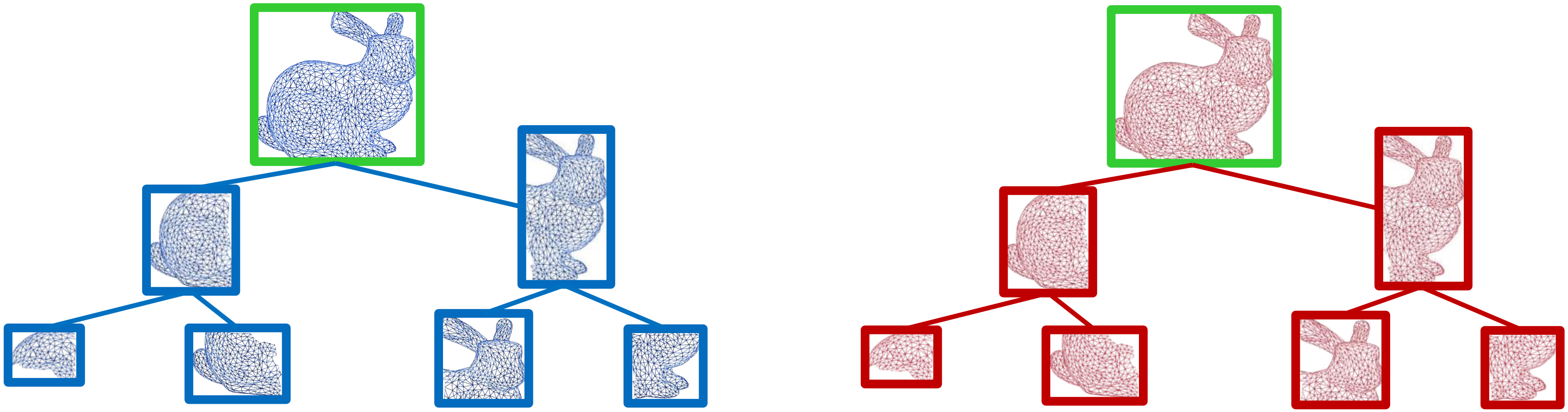

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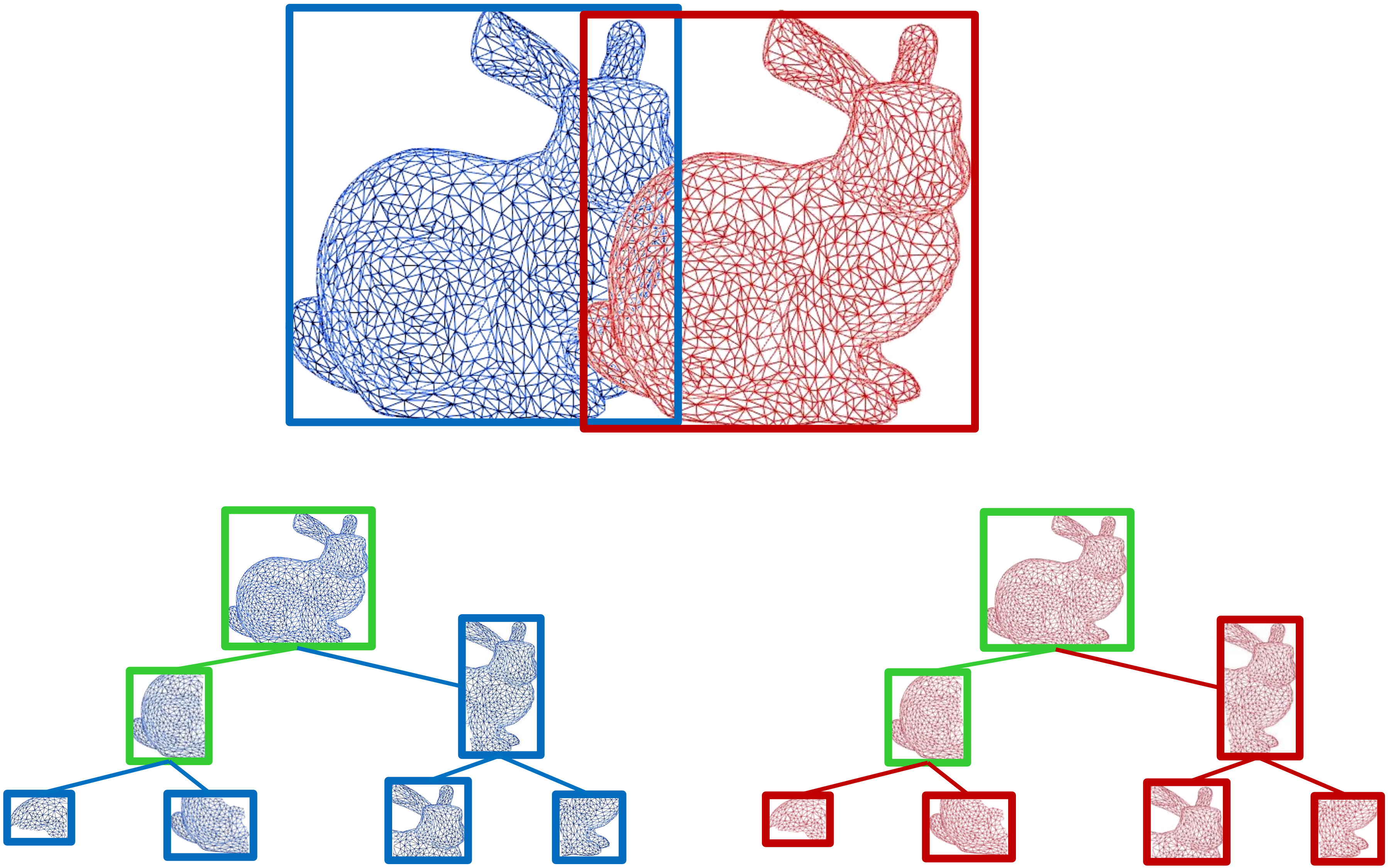

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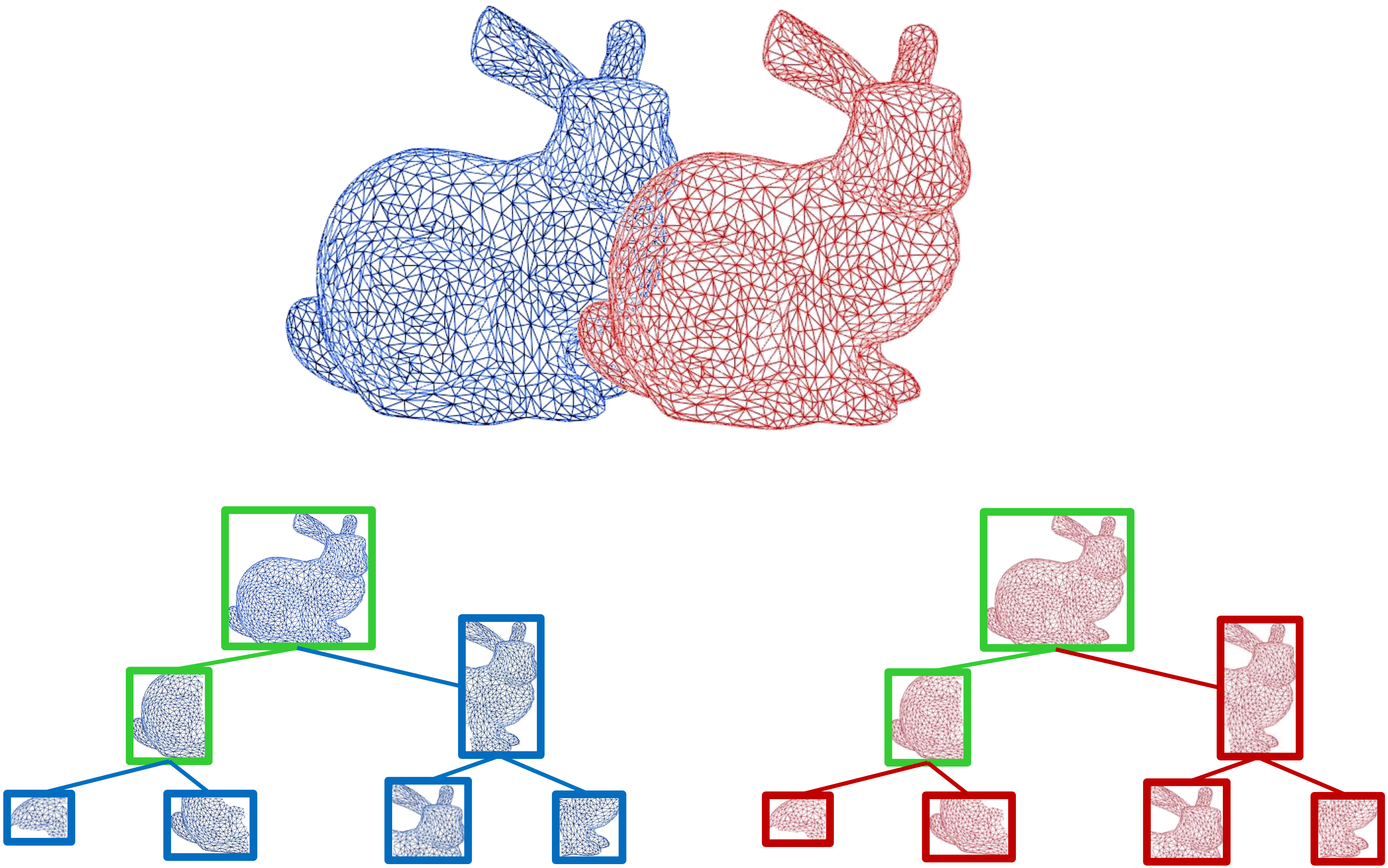

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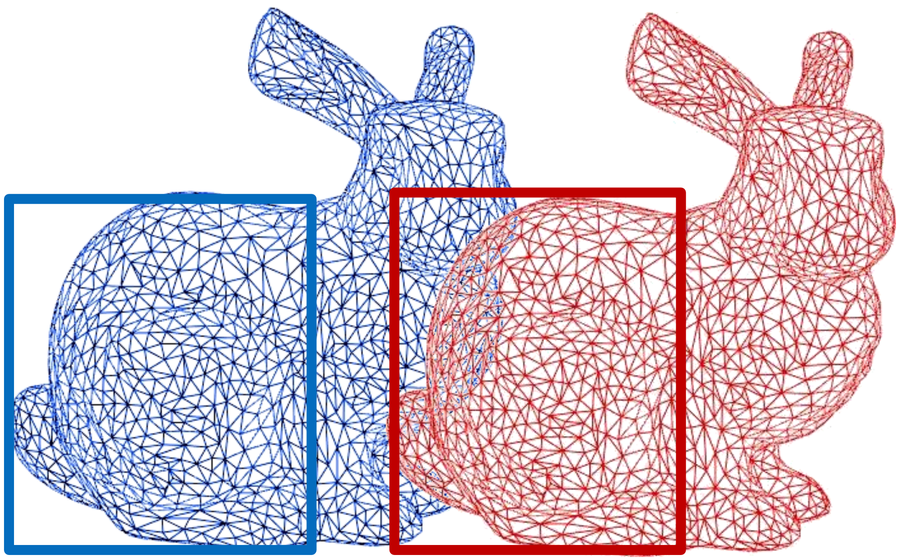
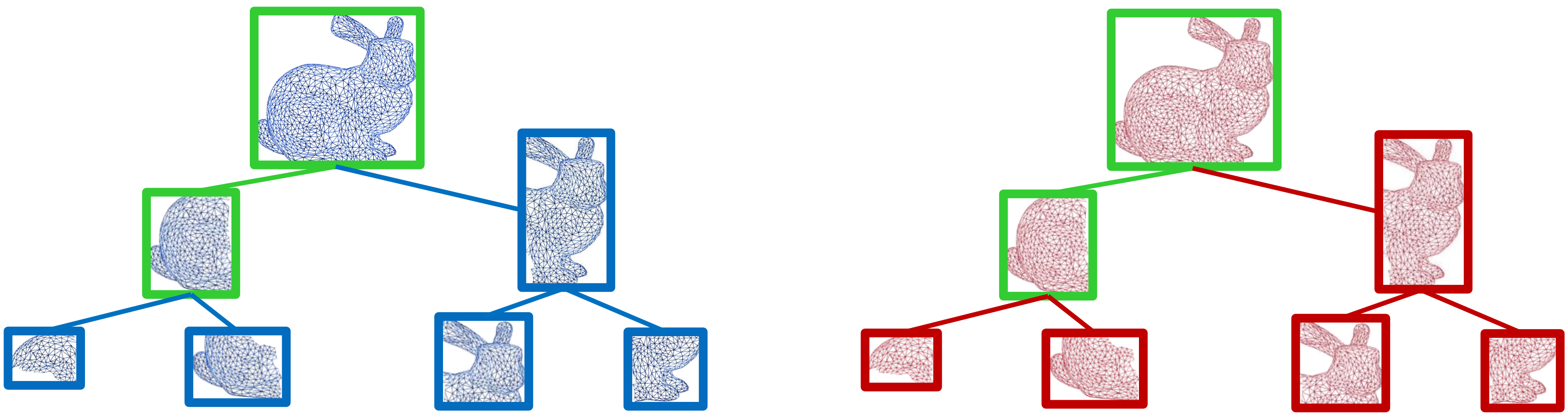


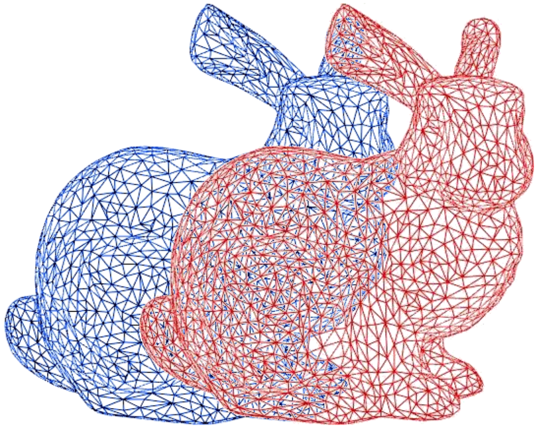


Simultaneous Recursive BVH Traversal

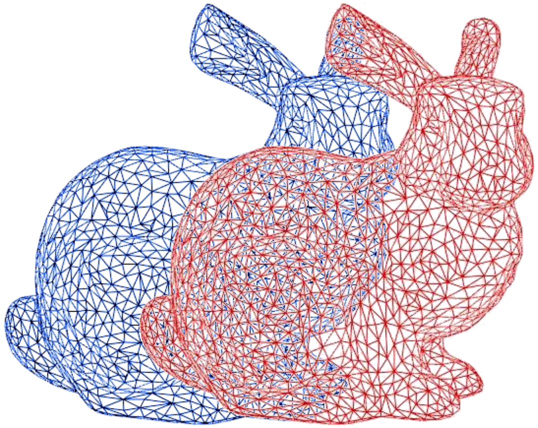
Simultaneous Recursive BVH Traversal

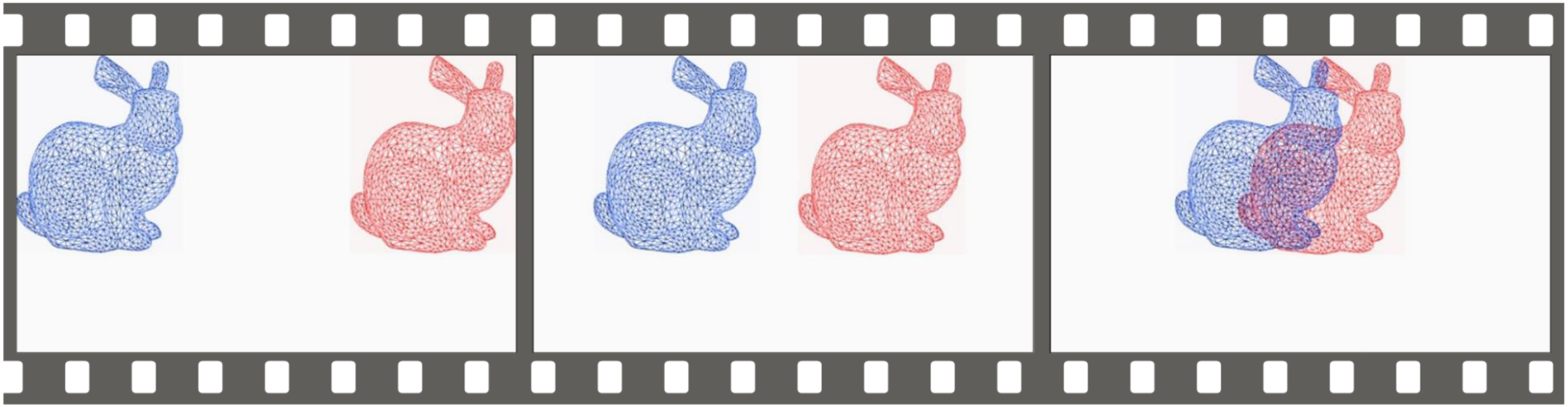
```
bool checkCollision( BV A, BV B )
  if A and B are Leaves then
    return checkPolygons(Polygon of A, Polygon of B)
  else
  forall the Children Ai of A do
    forall the Children Bi of B do
       if( overlap (Ai , Bi))
         return checkCollision(Ai , Bi)
  return false
```

Discrete Collision Detection

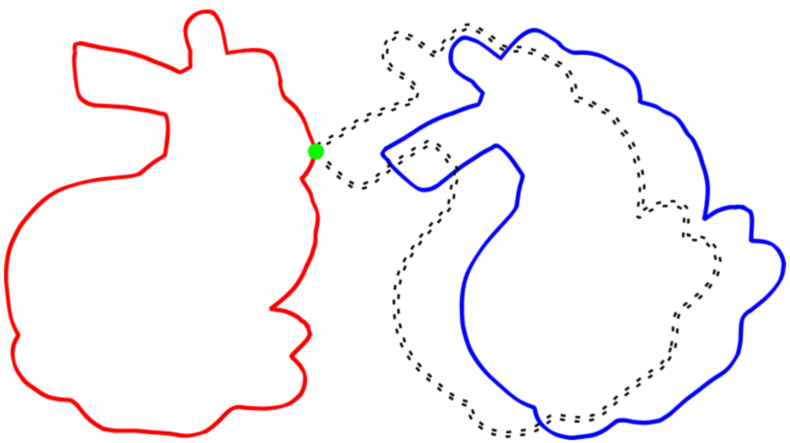


Discrete Collision Detection



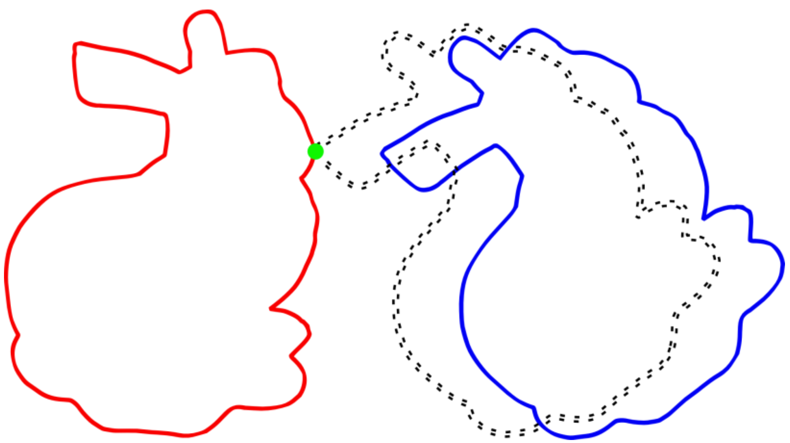


Penetration Measures

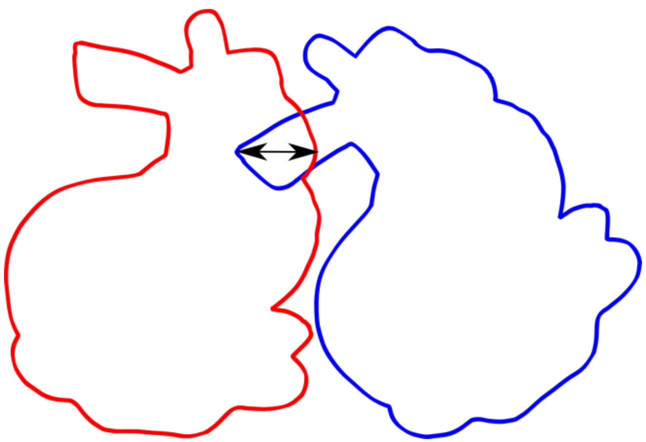


Continuous collision detection

Penetration Measures

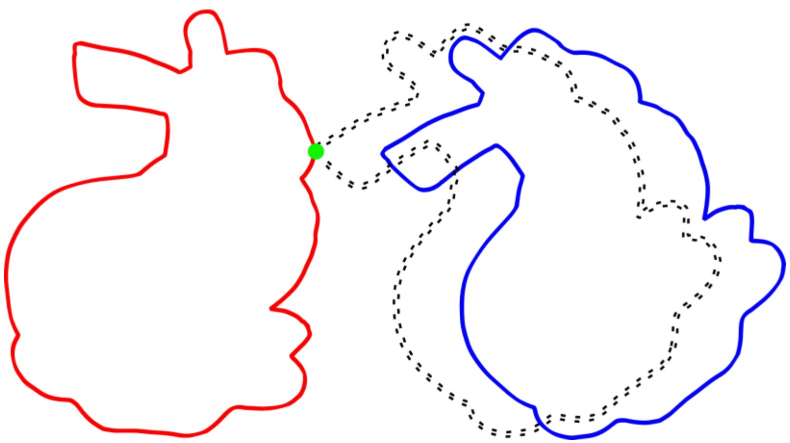


Continuous collision detection



Translational penetration depth

Penetration Measures



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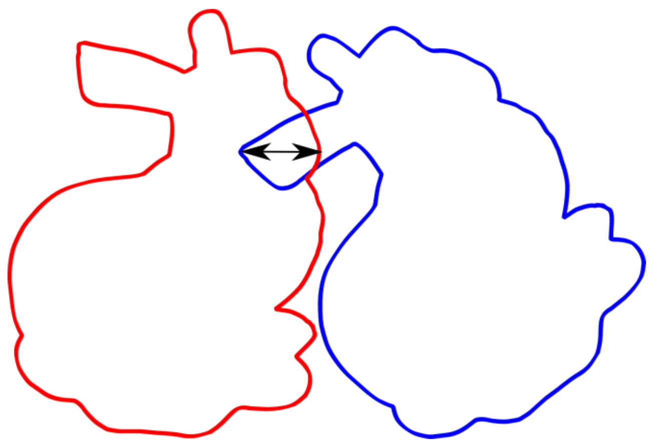

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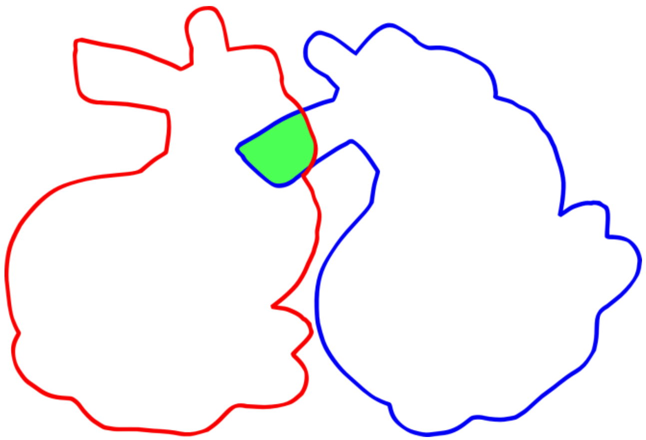
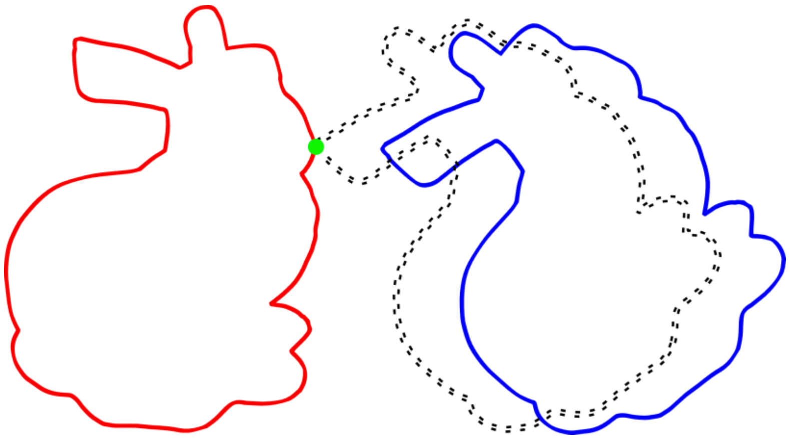


Penetration volume - “the most complicated yet accurate method” [Fisher and Lin, 2001]

Penetration Measures



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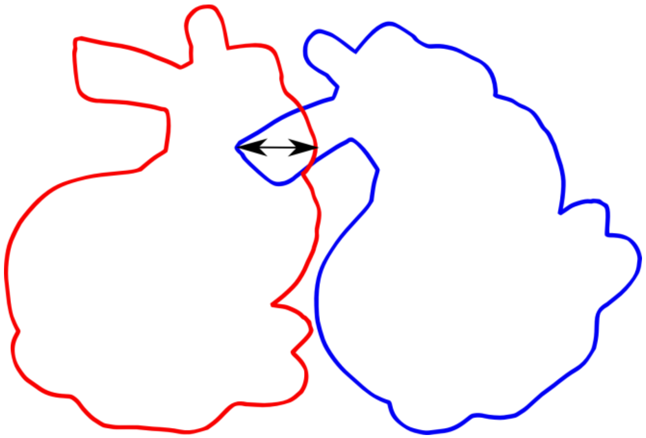

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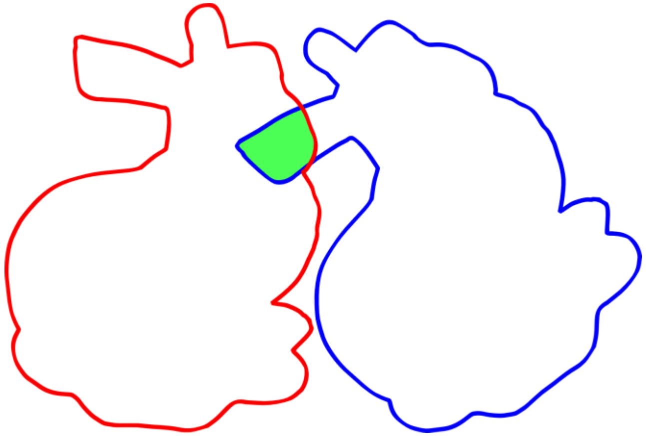
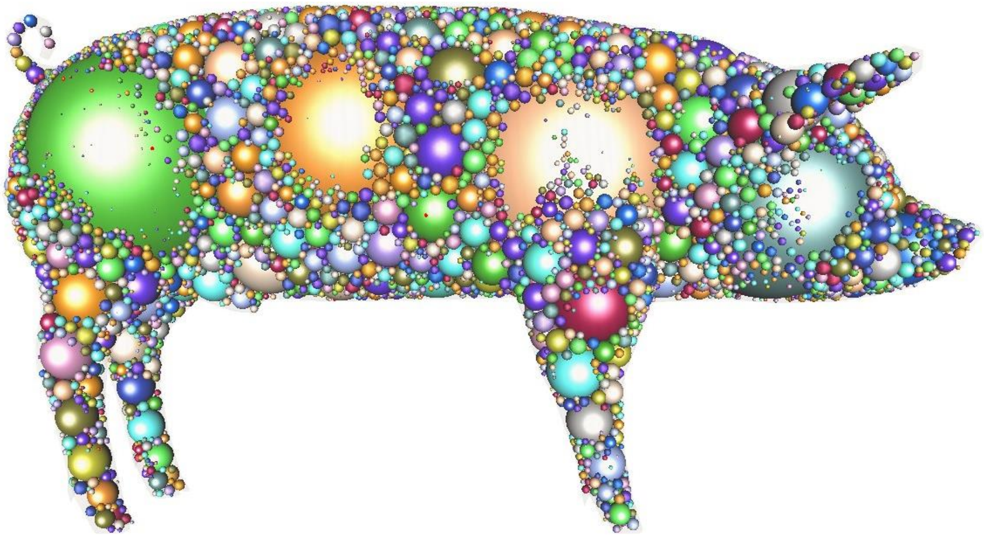
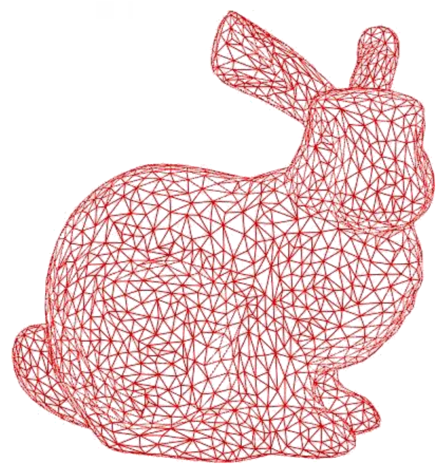


Penetration volume - “the most complicated yet accurate method” [Fisher and Lin, 2001]

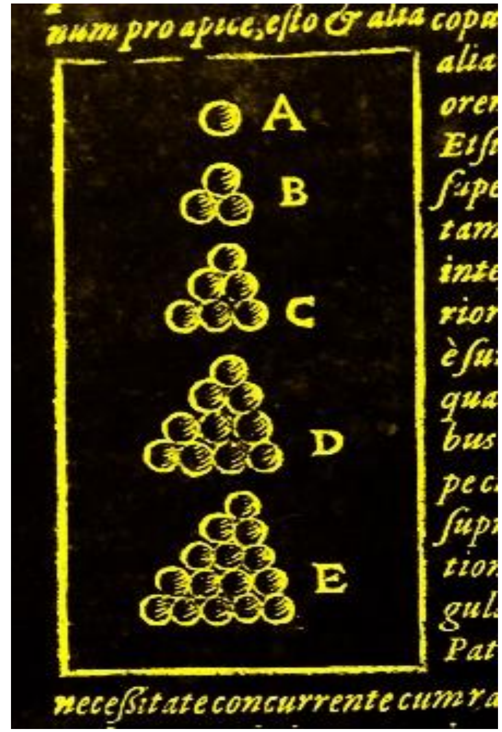
- Fill the object
  - from the *inside*
  - with *non-overlapping* spheres
- Build sphere hierarchy on *inner* spheres



How to get the Spheres into the Object?

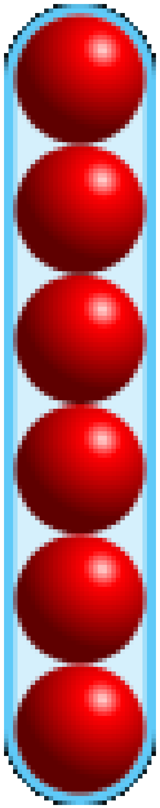


A brief History of Sphere Packings



Johannes Kepler (1571 – 1630)

Excurse: The Sausage Conjecture



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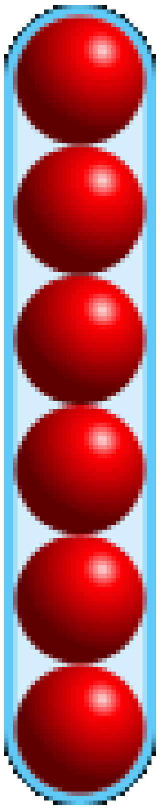

Sausage

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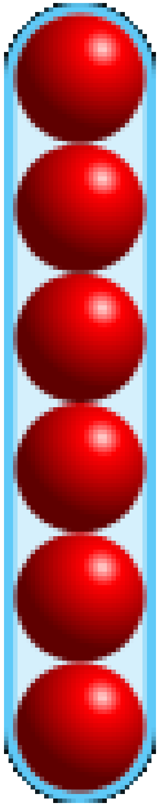

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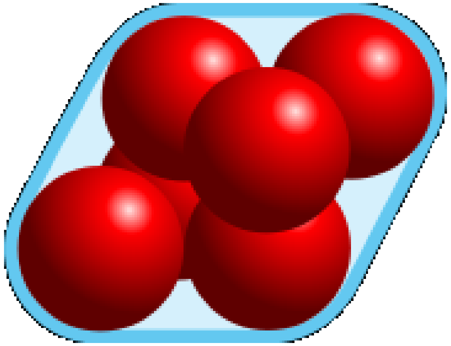
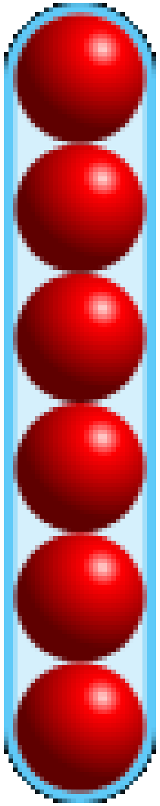
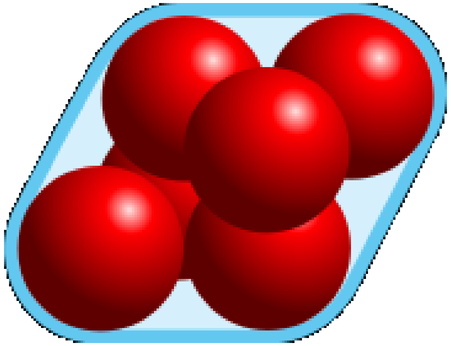


Cluster

Excurse: The Sausage Conjecture


:optimal for n=3 and 4





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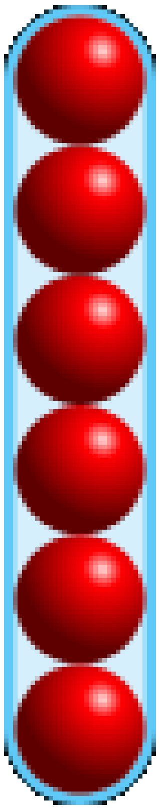

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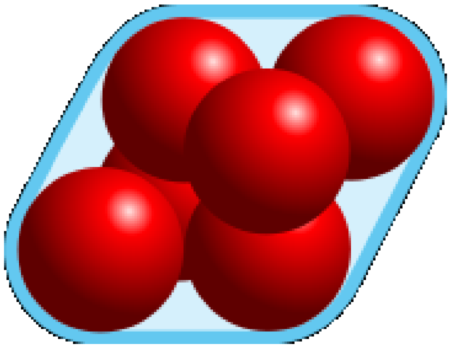
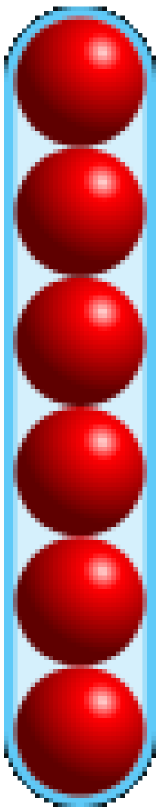


Cluster


:optimal for n=3 and 4


:optimal for n<56?

Excurse: The Sausage Conjecture



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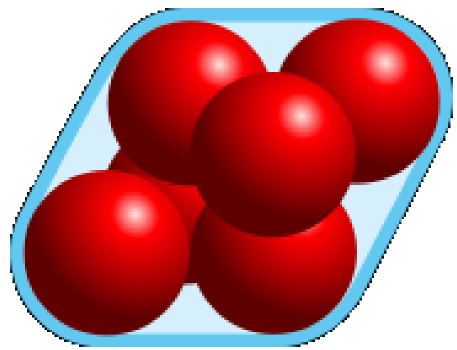

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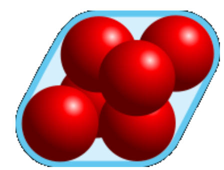

optimal for n=56, 59, 60, 61, 62, 65

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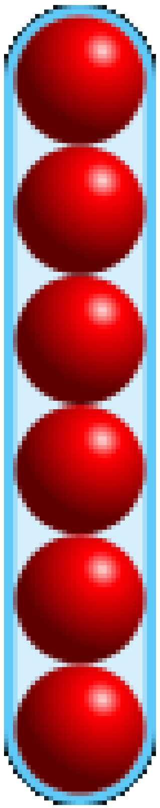

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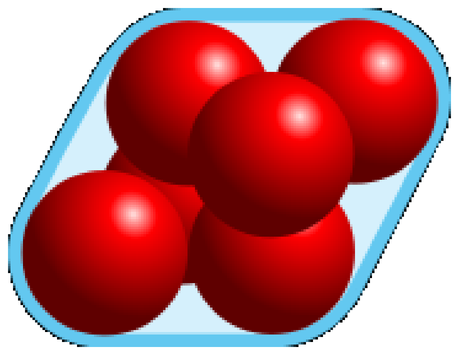

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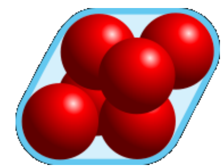
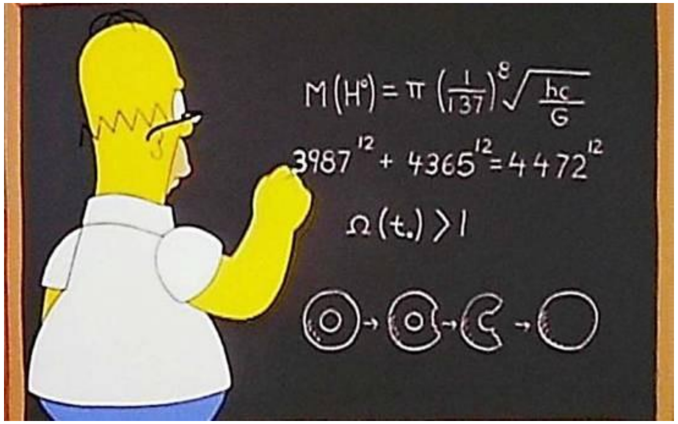


optimal for n=56, 59, 60, 61, 62, 65

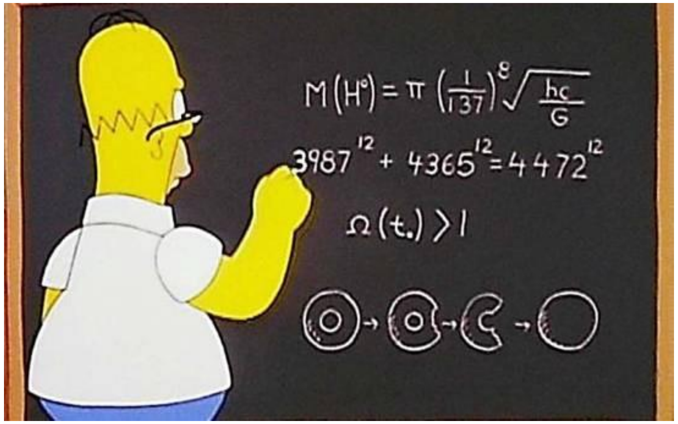
Nobody knows how they look

More Sausage Catastrophes



More Sausage Catastrophes

- For dimension d=4: cluster becomes optimal for n>375370?



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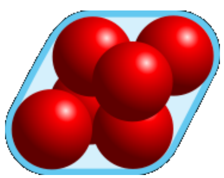
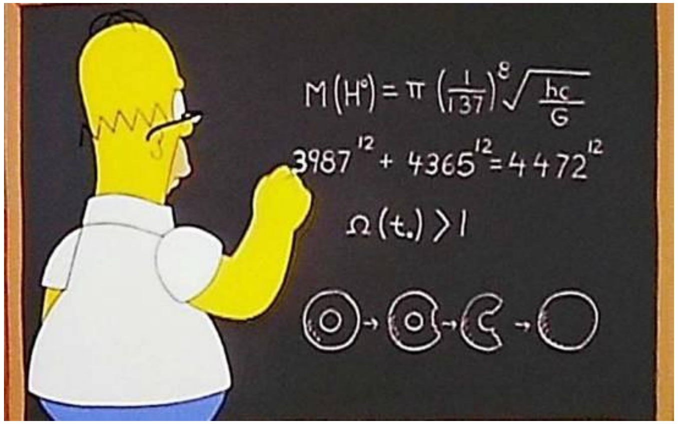
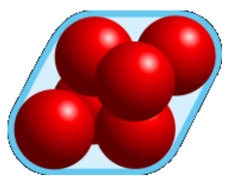
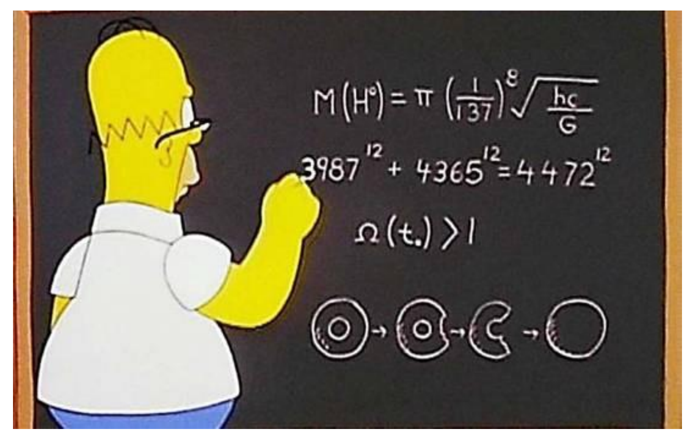




More Sausage Catastrophes

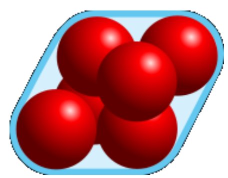
- For dimension d=4: cluster becomes optimal for n>375370?
- For d<11 the optimum packing is always a cluster or a sausage, but never a pizza!

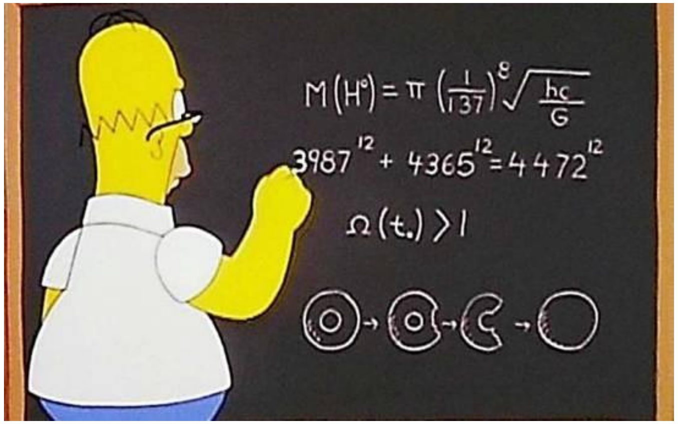




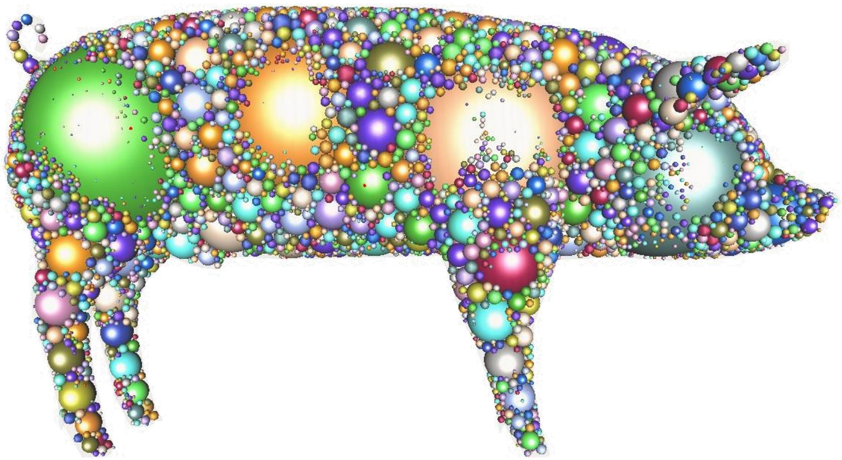
More Sausage Catastrophes

- For dimension d=4: cluster becomes optimal for n>375370?
- For d<11 the optimum packing is always a cluster or a sausage, but never a pizza!
  - Note, this is not true for other convex shapes
  - E.g. it is possible to find for each d>2 a convex shape where the pizza is the closest packaging



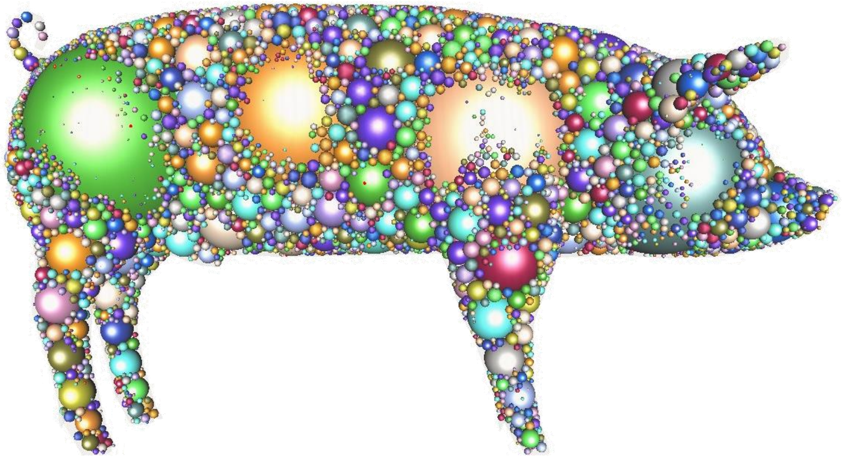


Our Goal



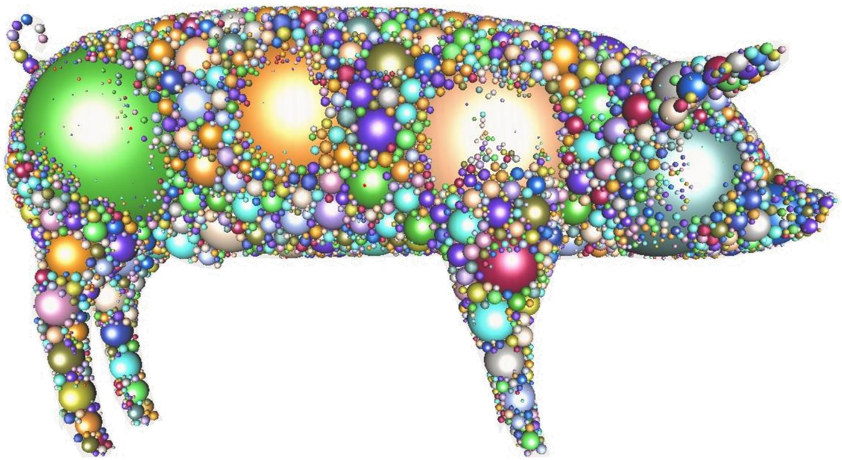
Our Goal

- Feasible Sphere Packing
  - All spheres are inside
  - Spheres do not overlap



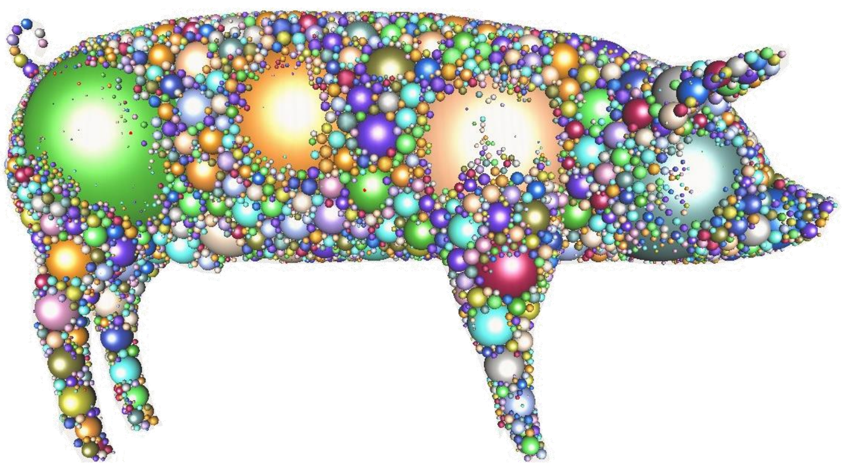
Our Goal

- Feasible Sphere Packing
  - All spheres are inside
  - Spheres do not overlap
- Polydisperse packing
  - Space filling

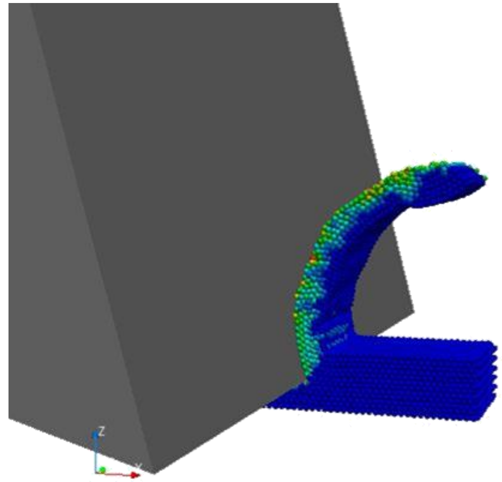


Our Goal

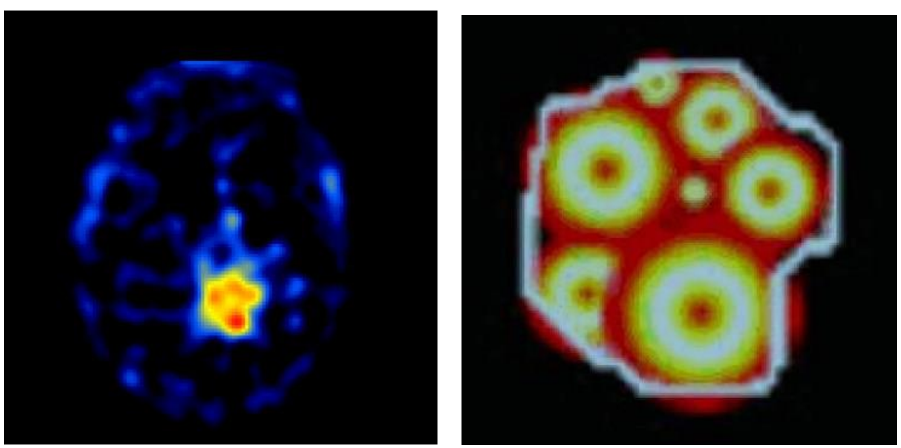
- Feasible Sphere Packing
  - All spheres are inside
  - Spheres do not overlap
- Polydisperse packing
  - Space filling
- Arbitrary objects
  - Arbitrary object representations (Polygonal, NURBS, CSG, point clouds,…)



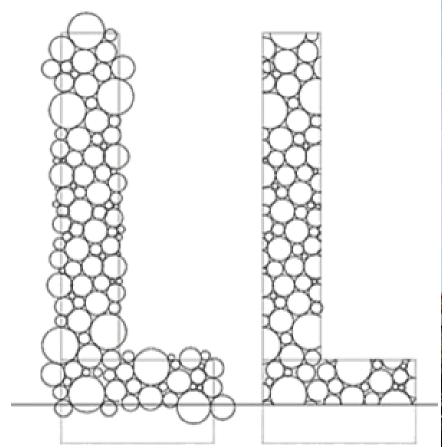
Applications of Polydisperse Sphere Packings



P. Eberhard et al, 2009

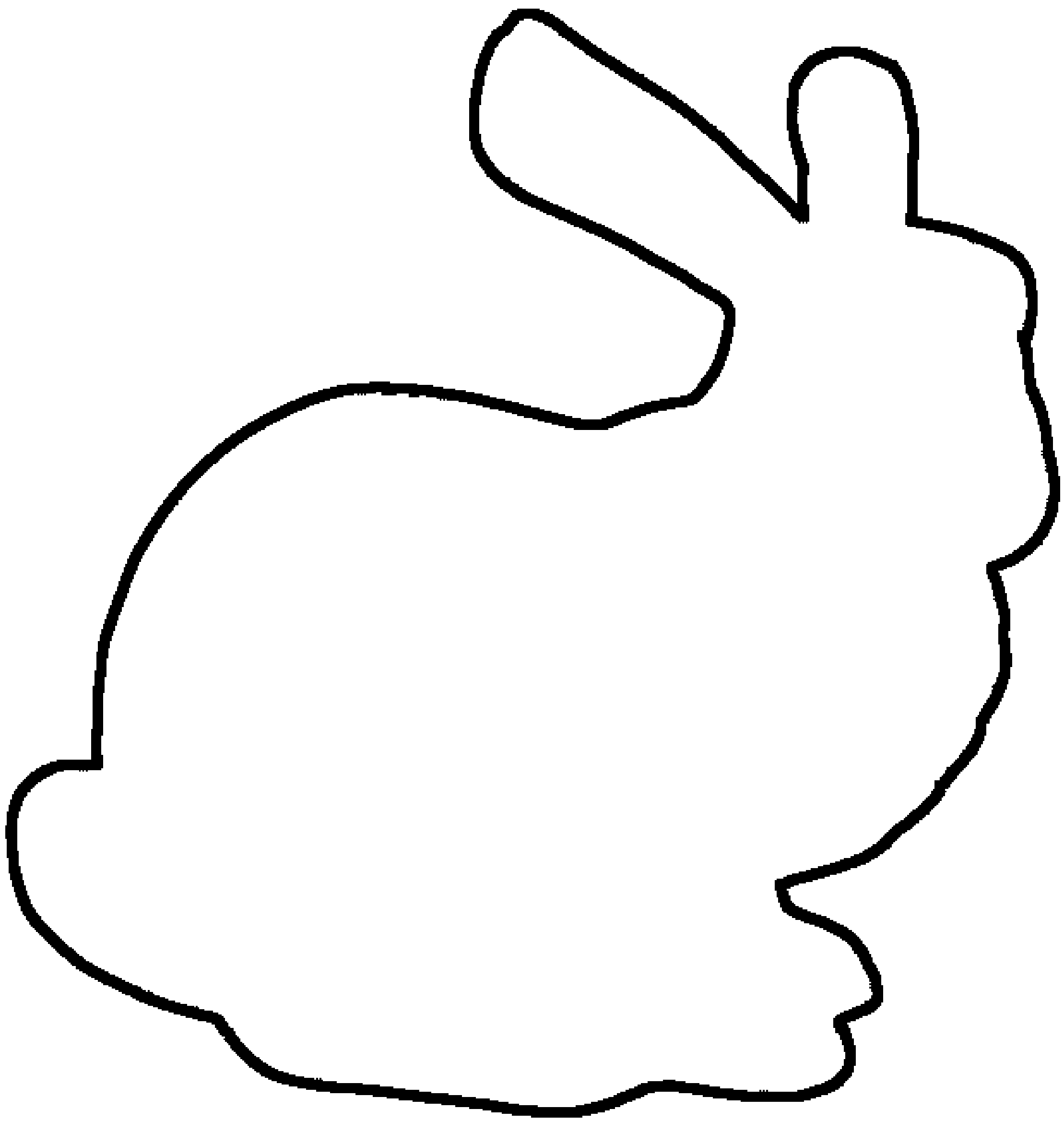


Long Yun et al, 2002



smo architektur, 2006

Basic Idea



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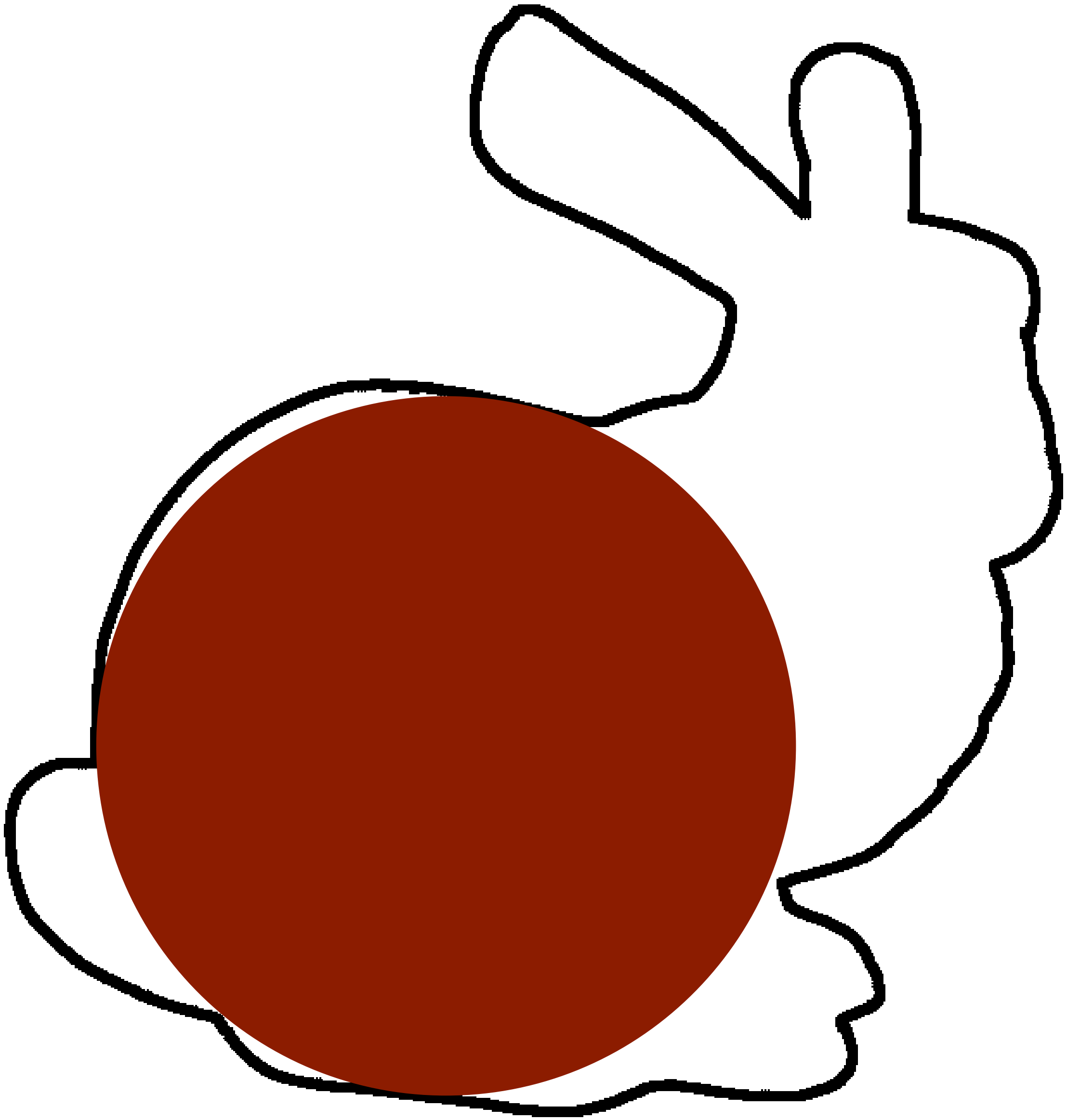

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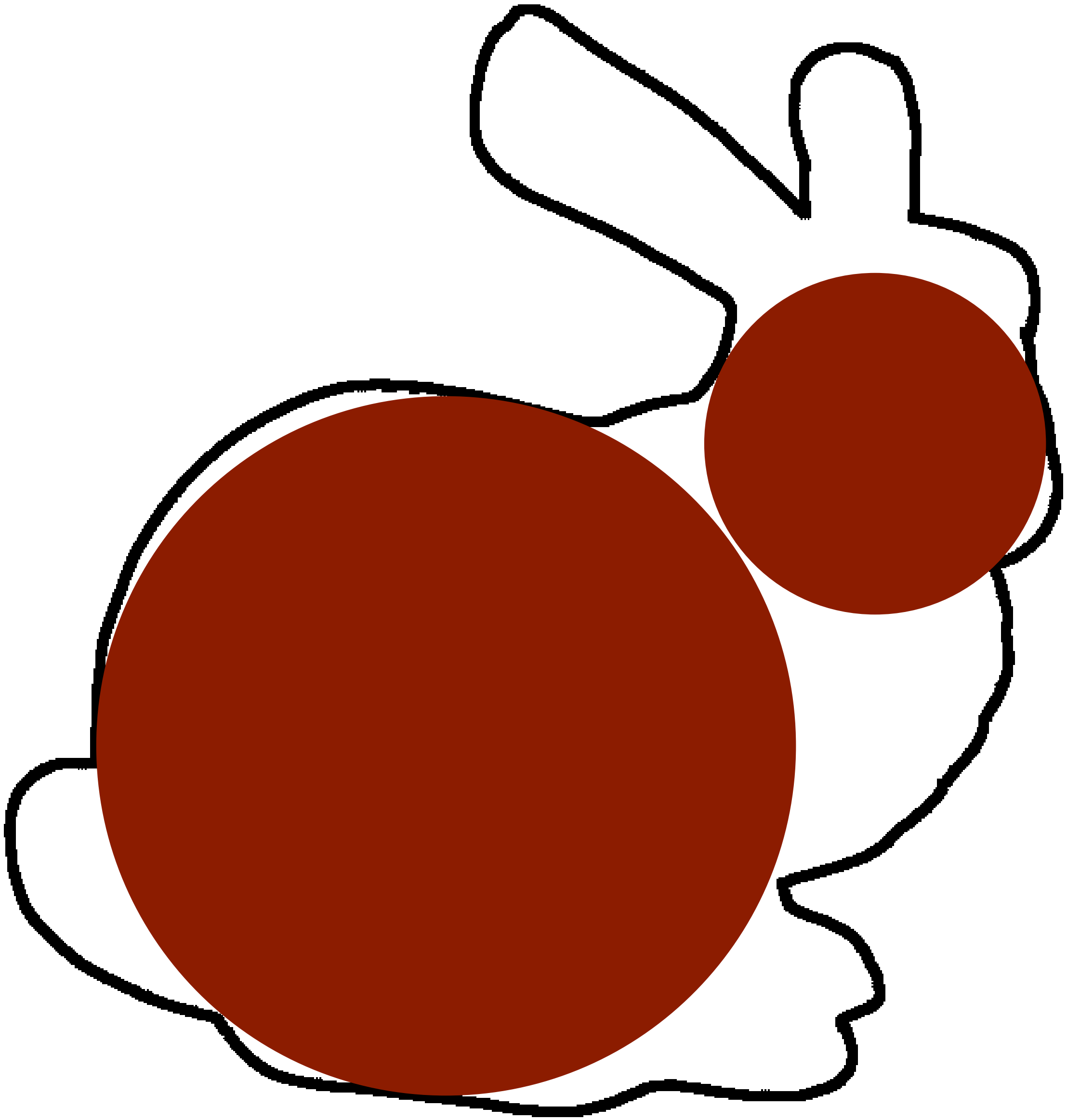

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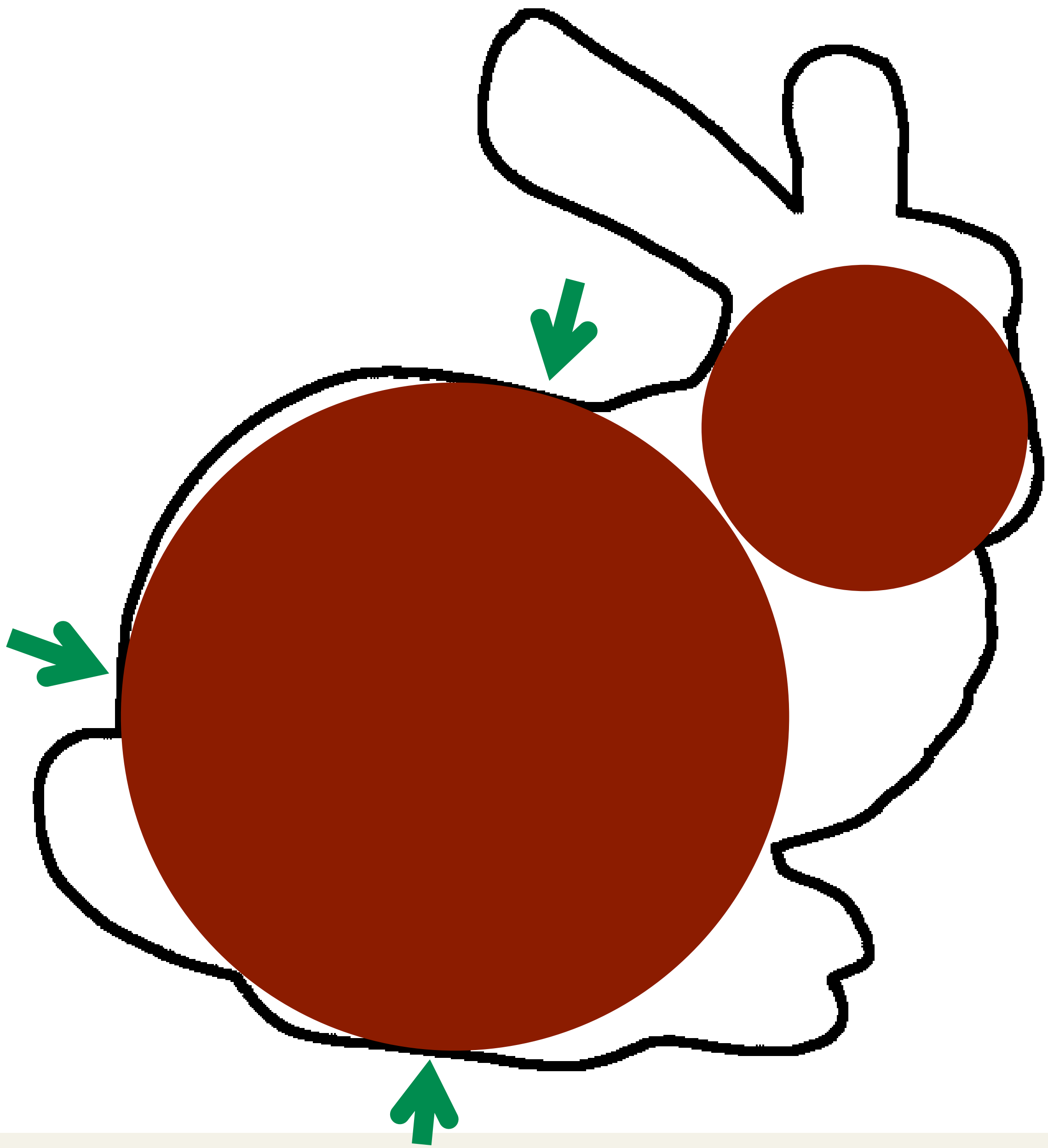

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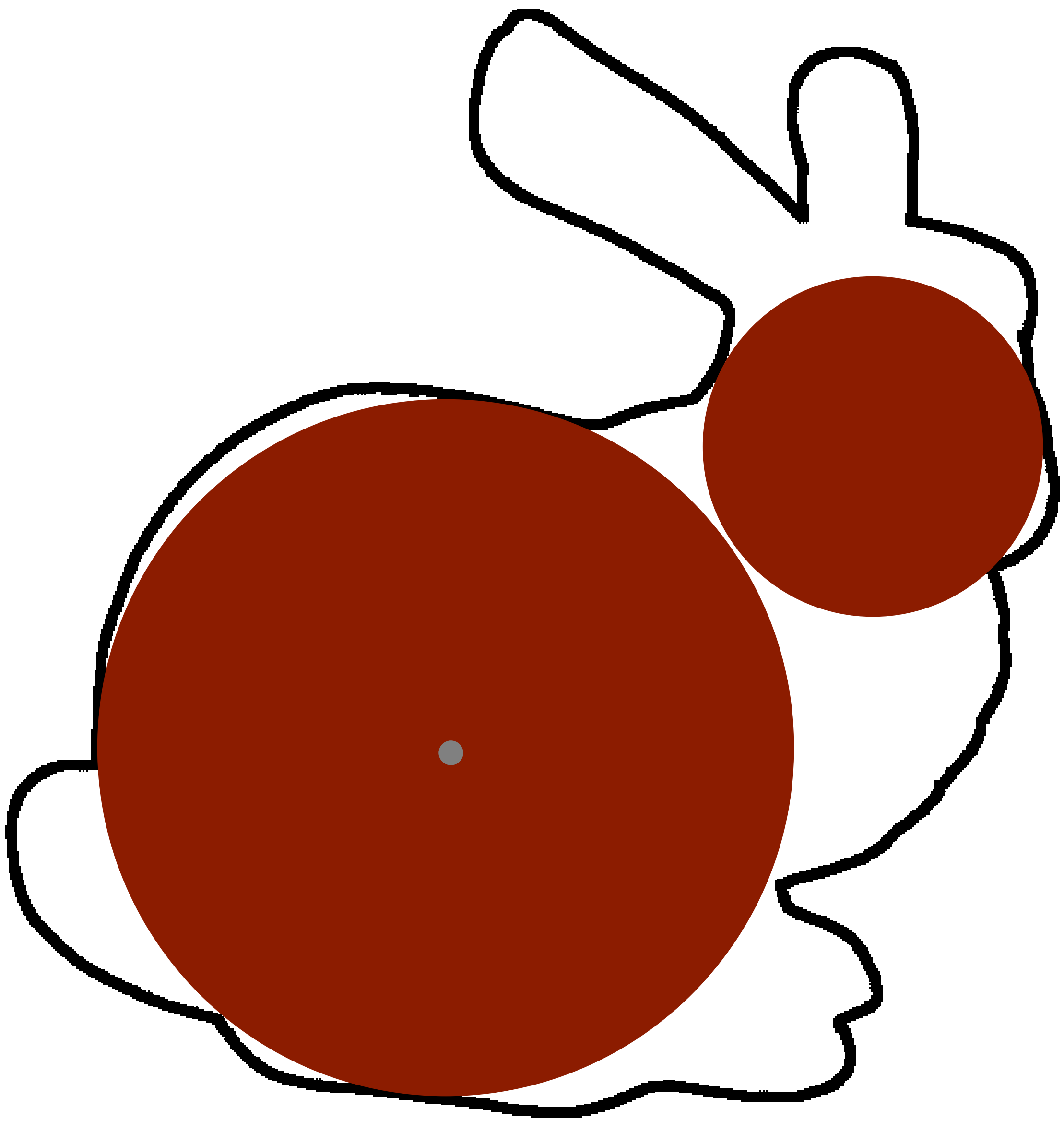

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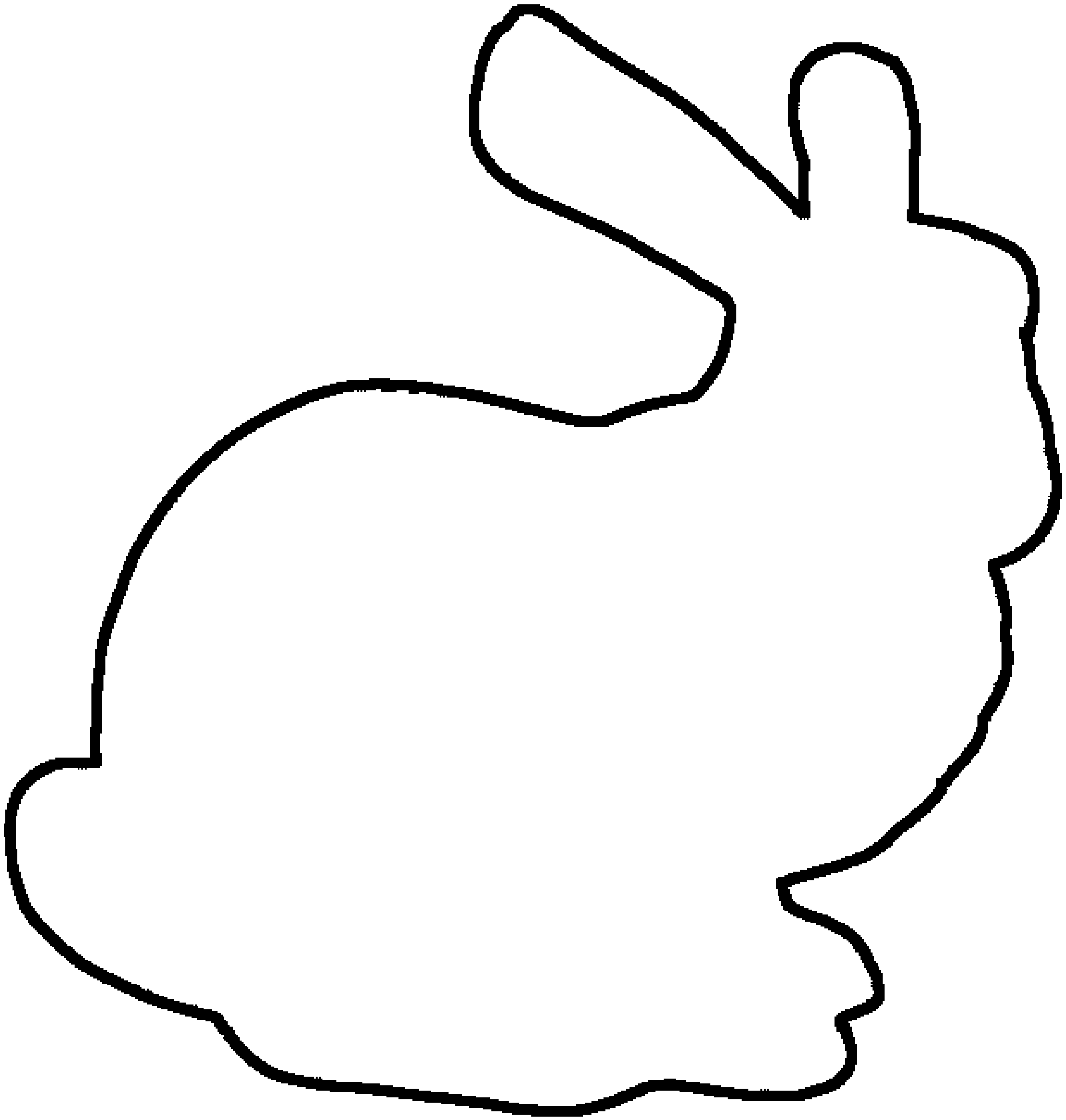

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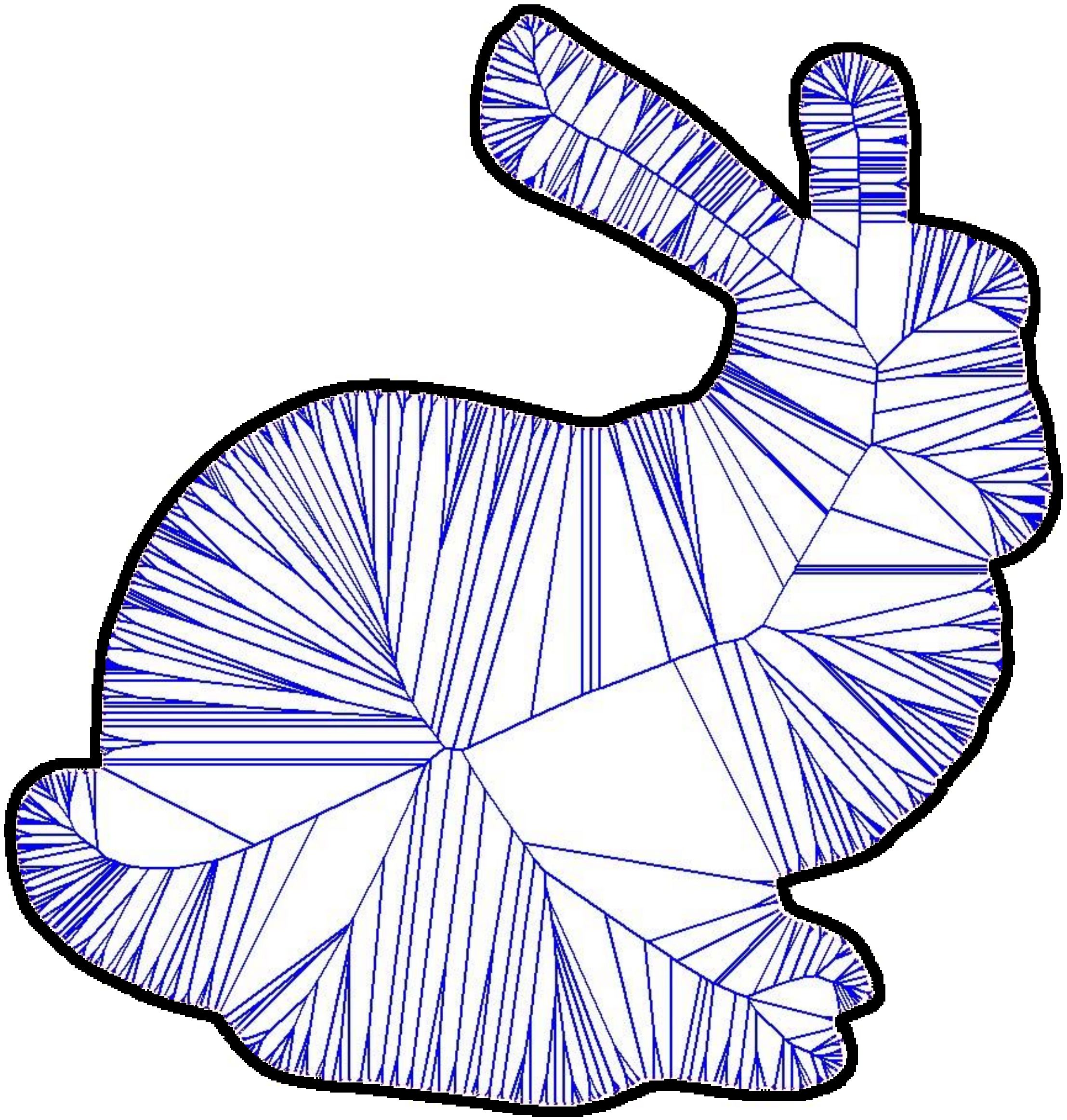

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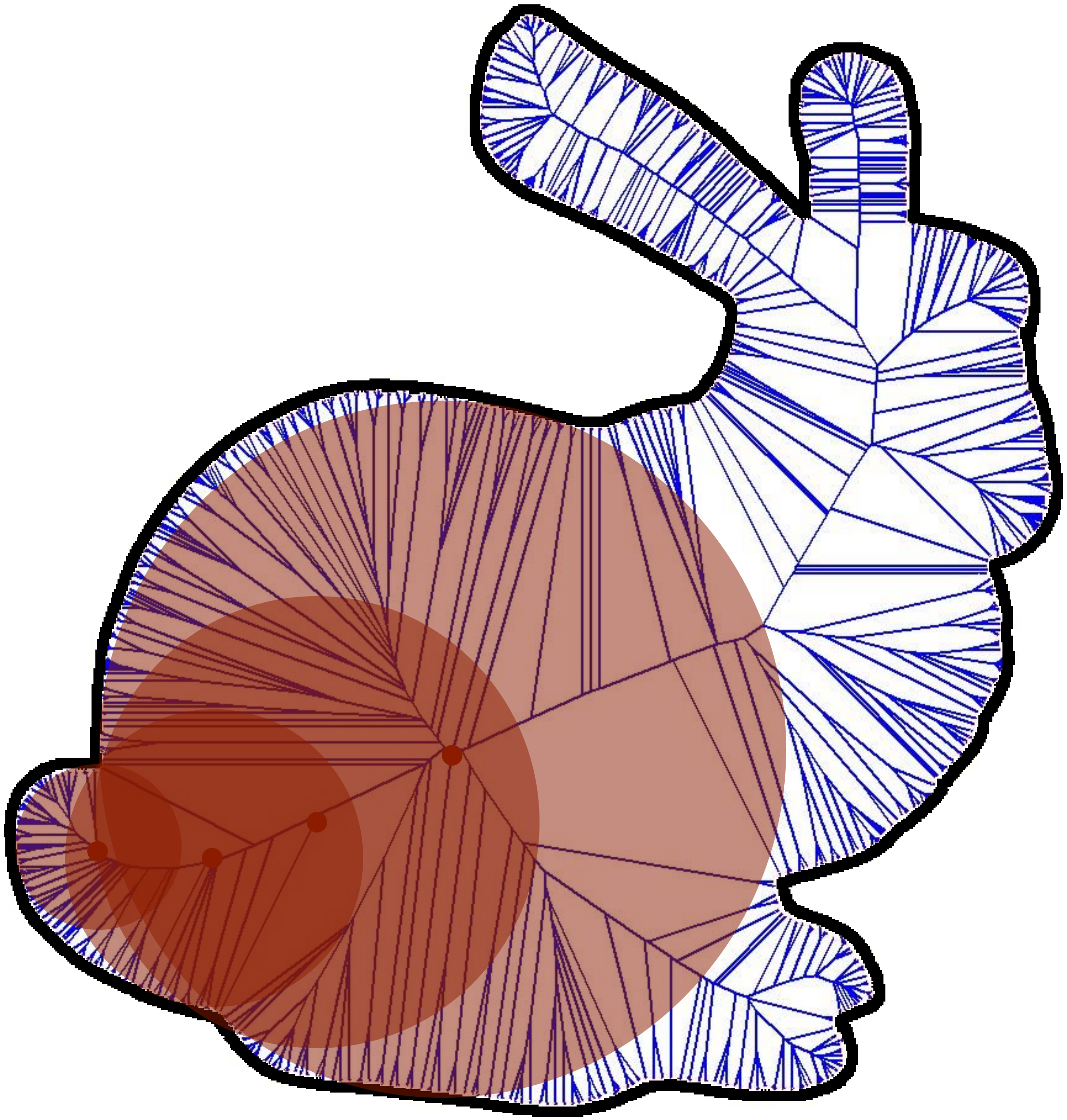

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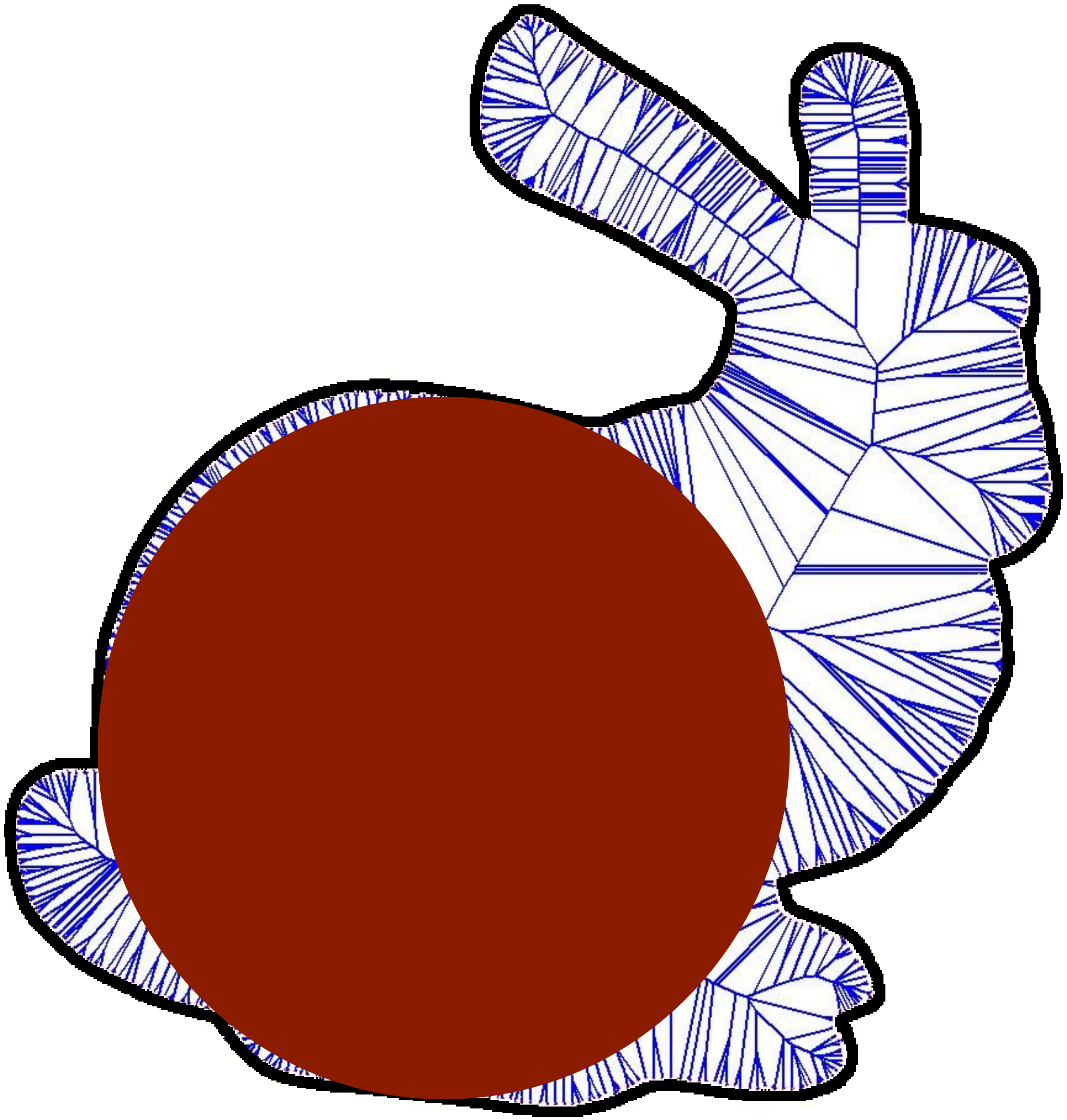

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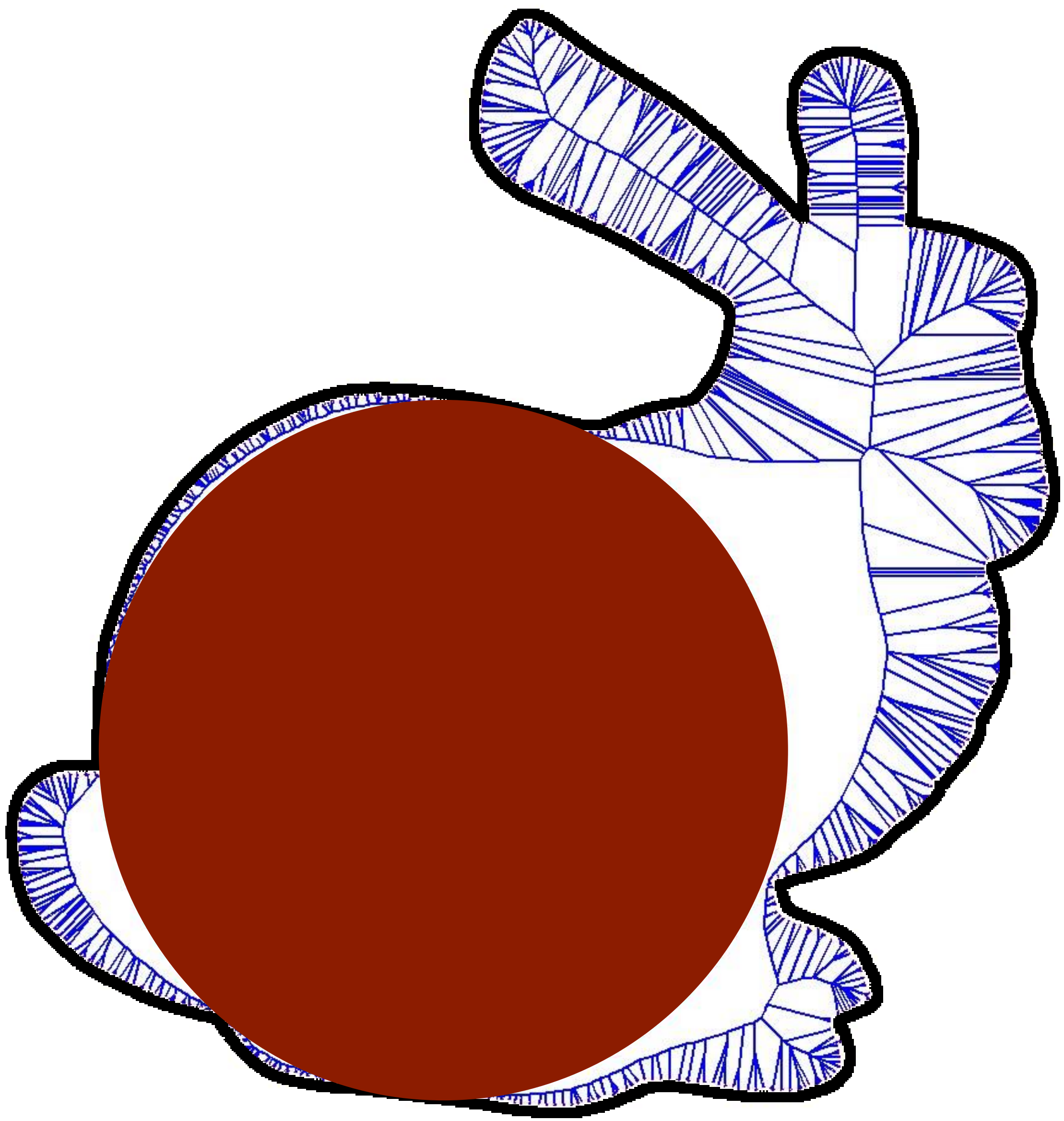

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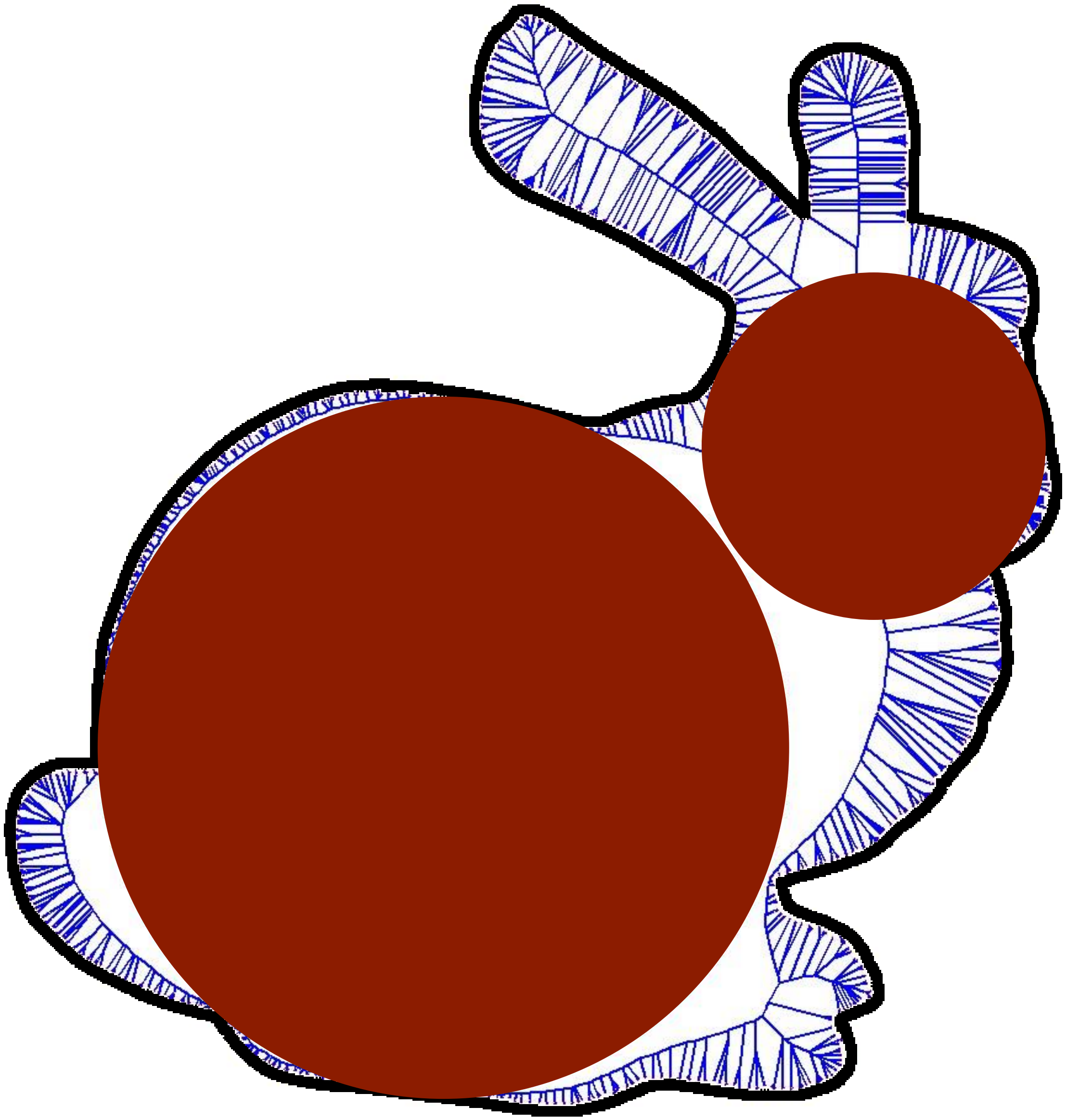

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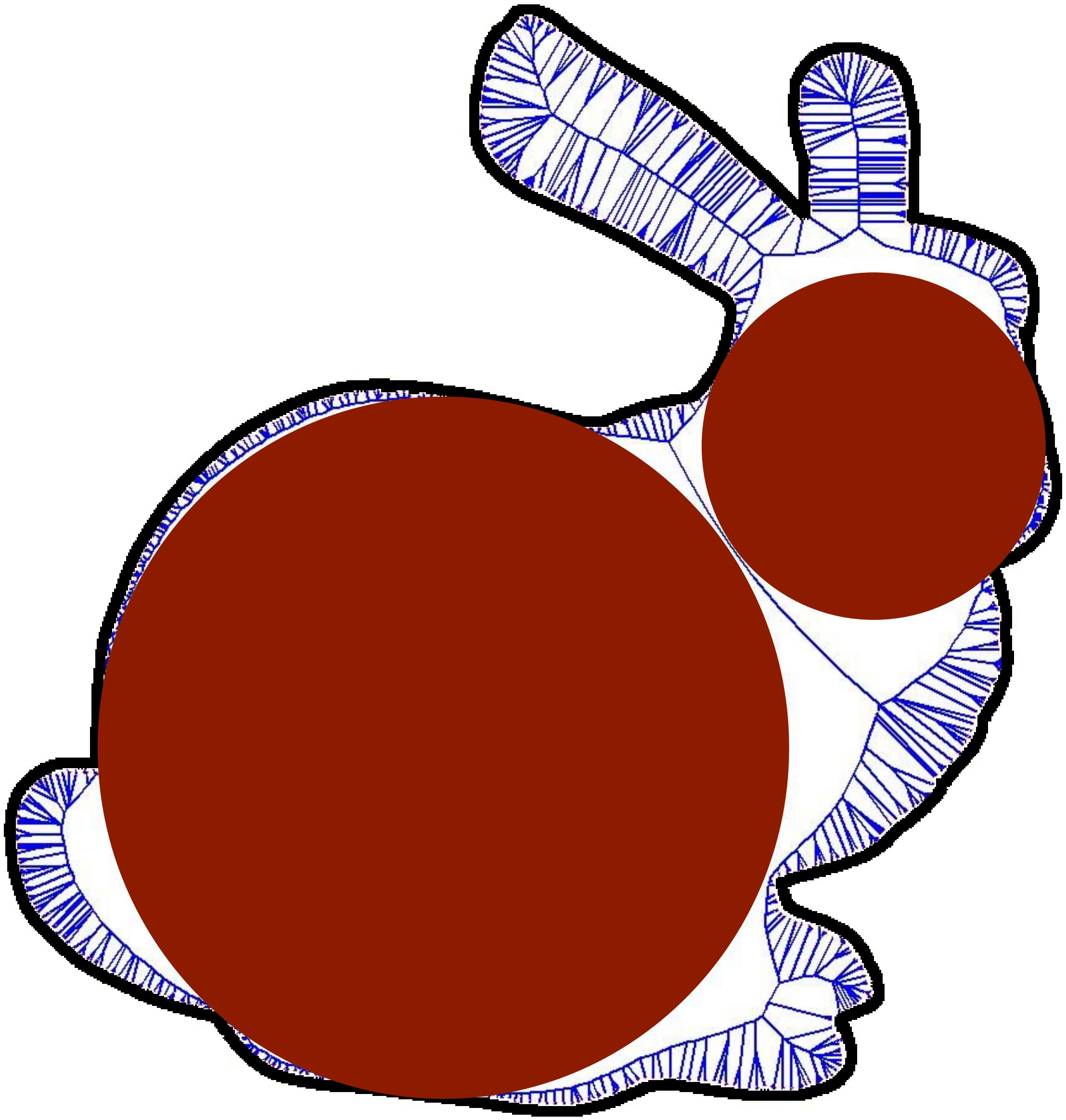

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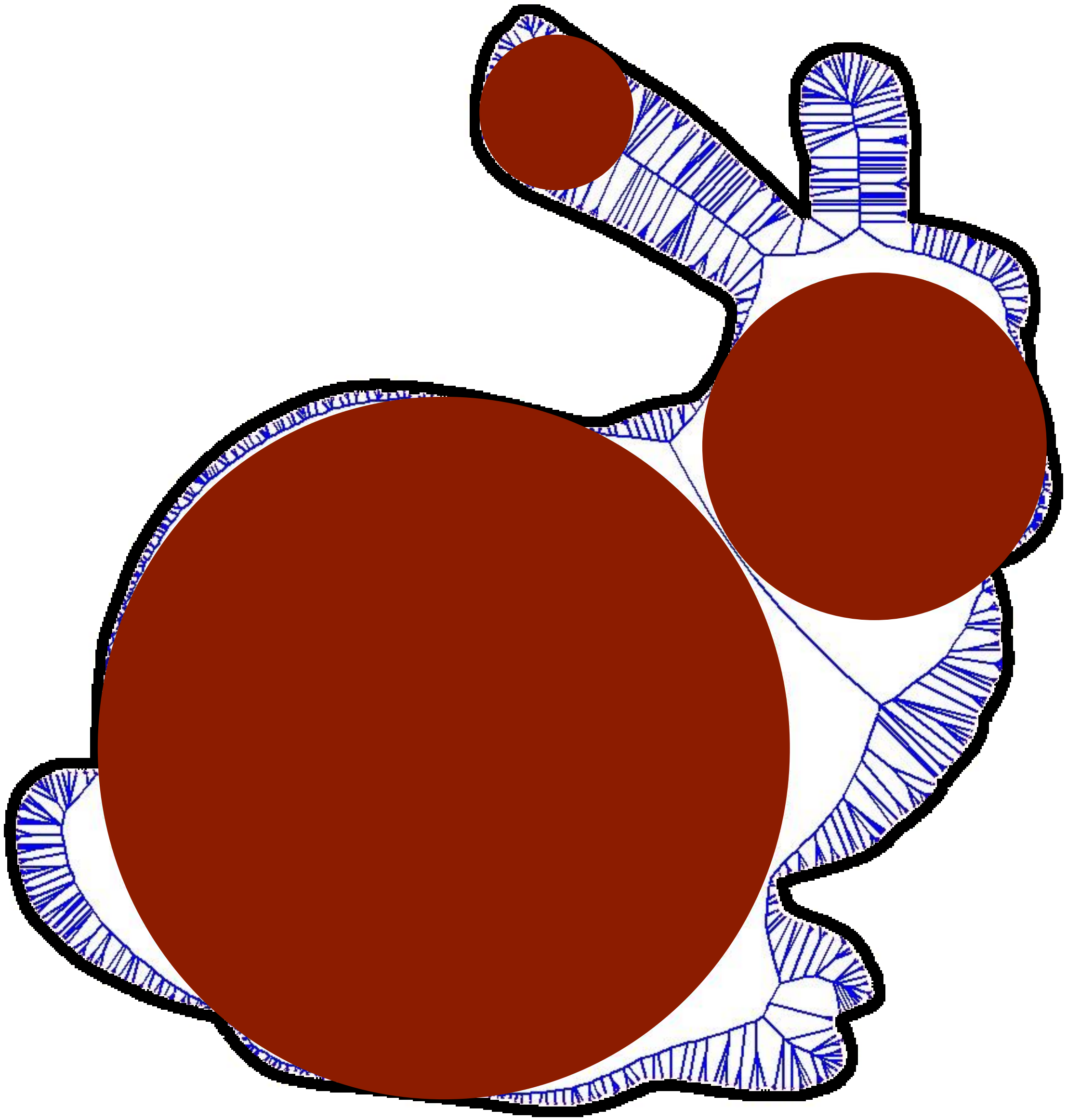

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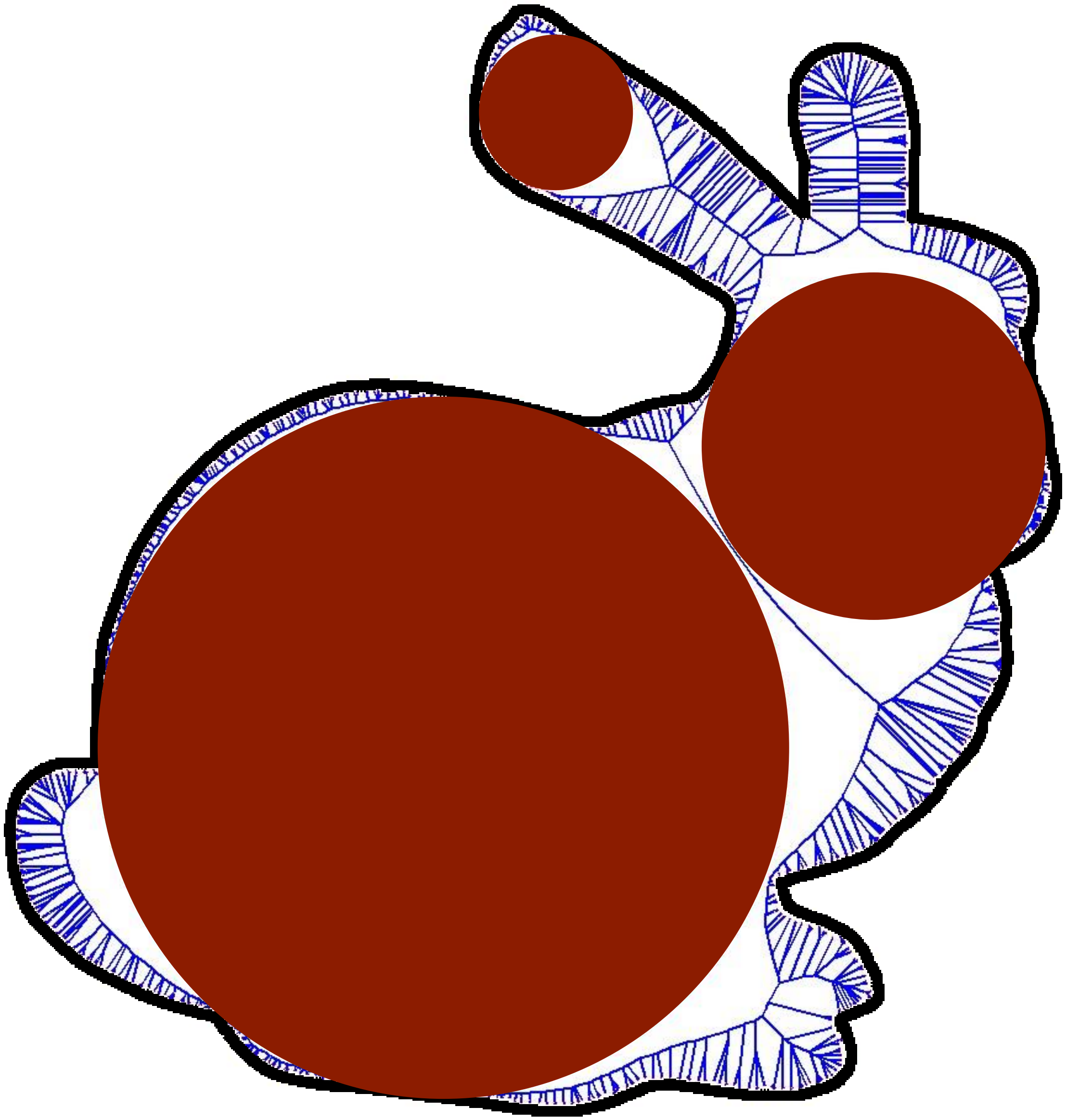

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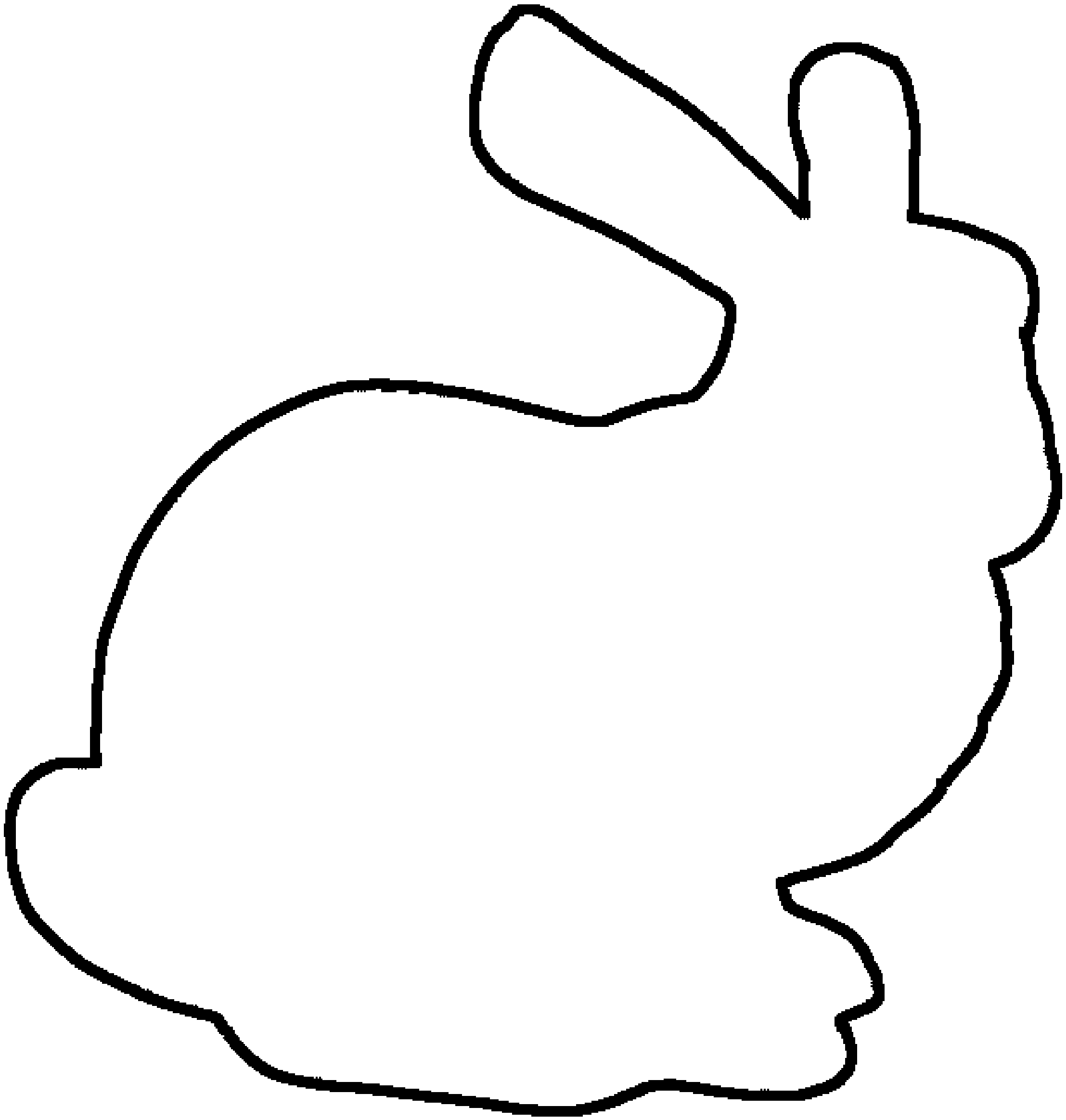

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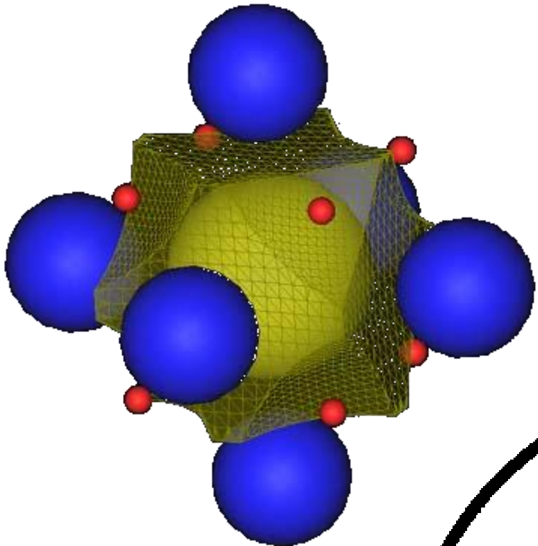
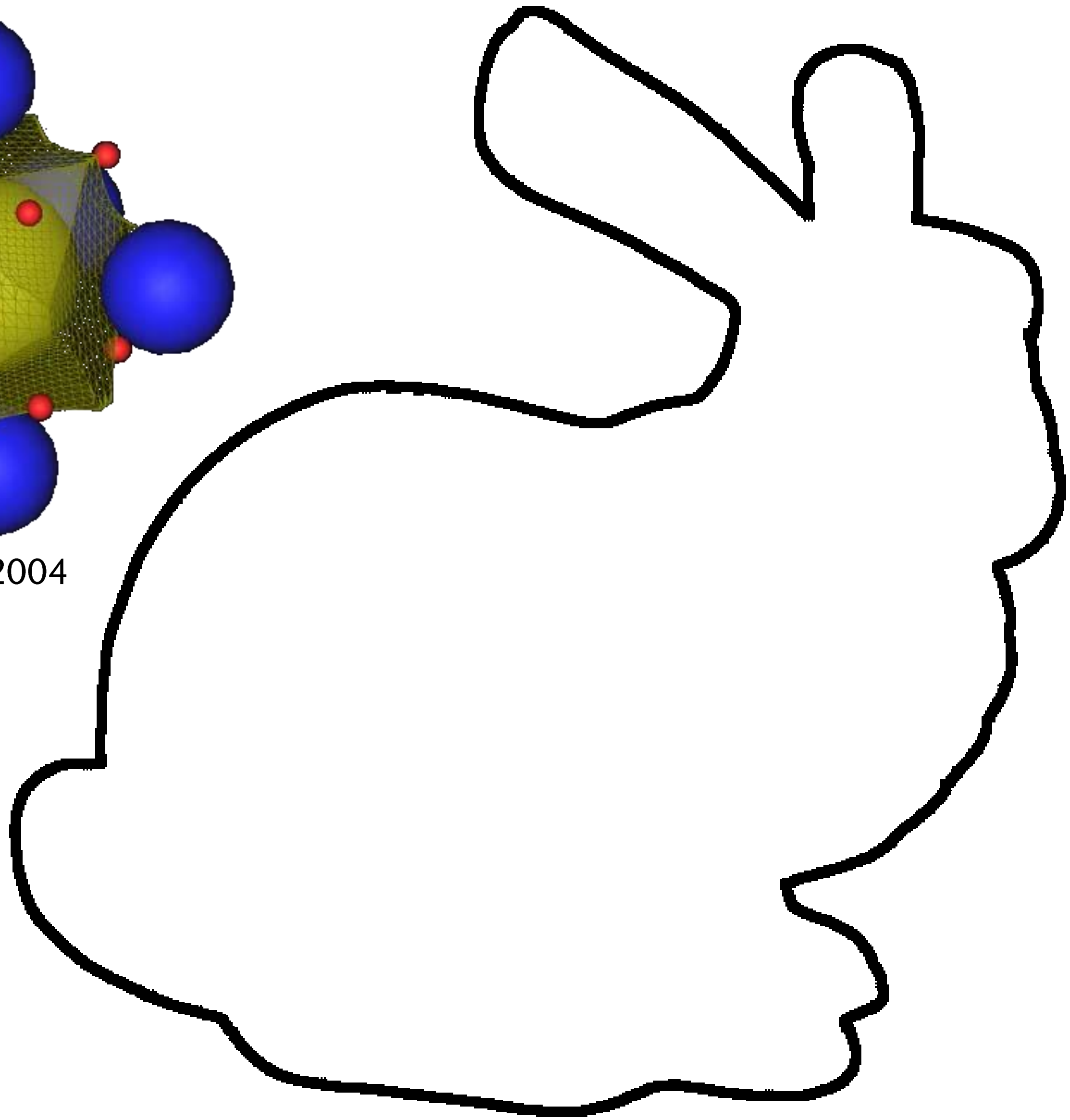


Cho et al., 2004



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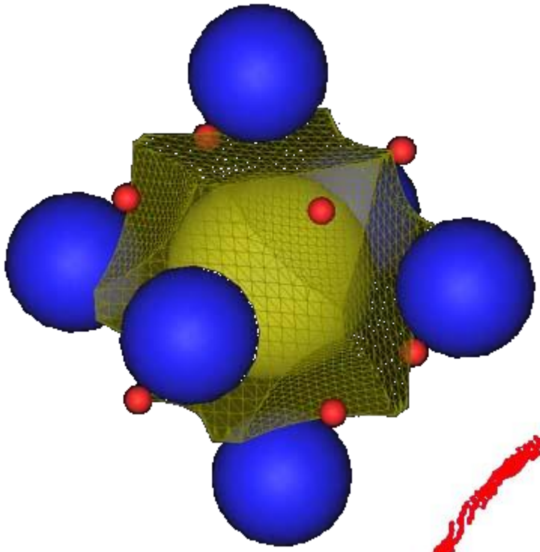
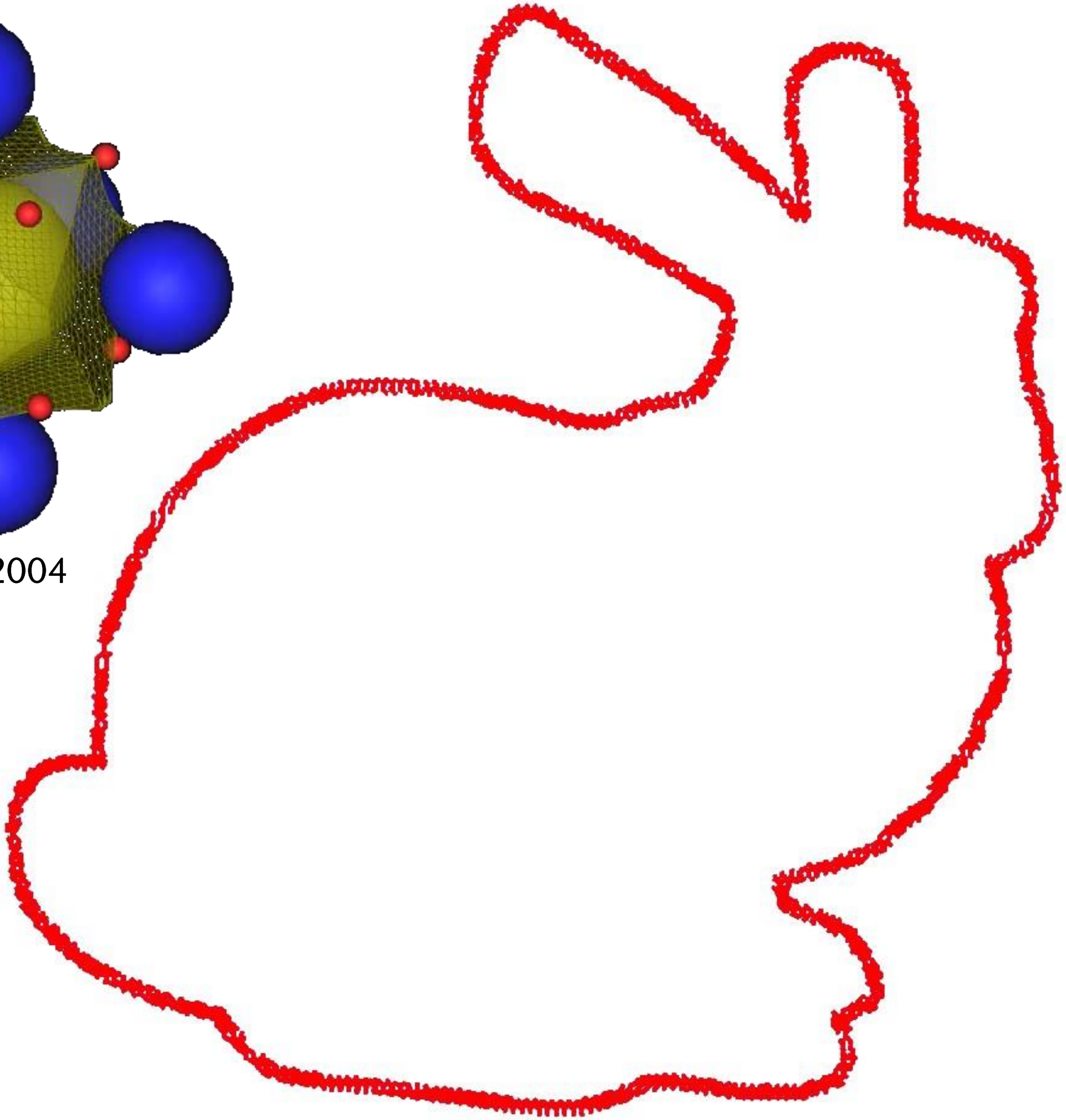


Cho et al., 2004



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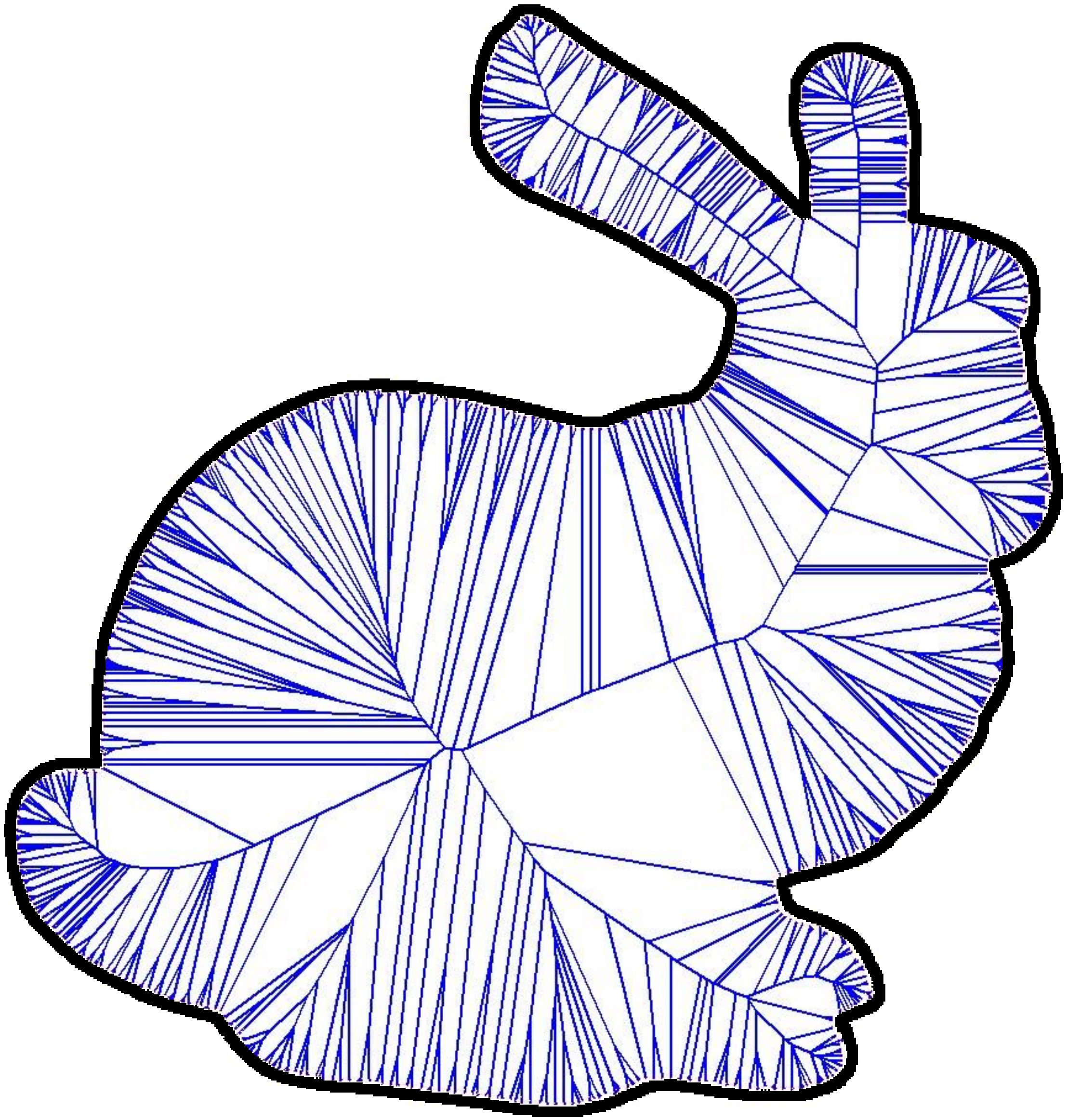

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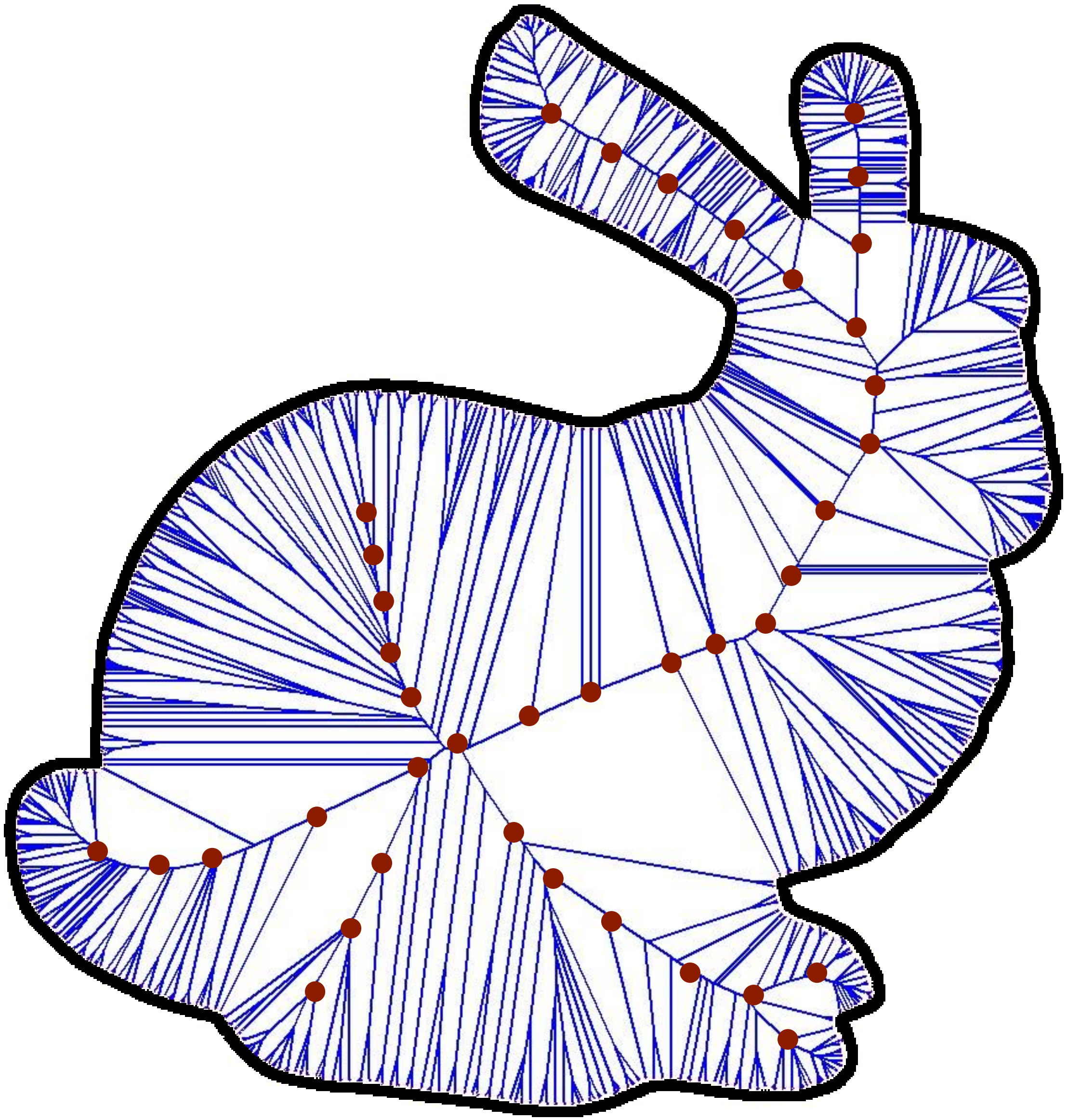

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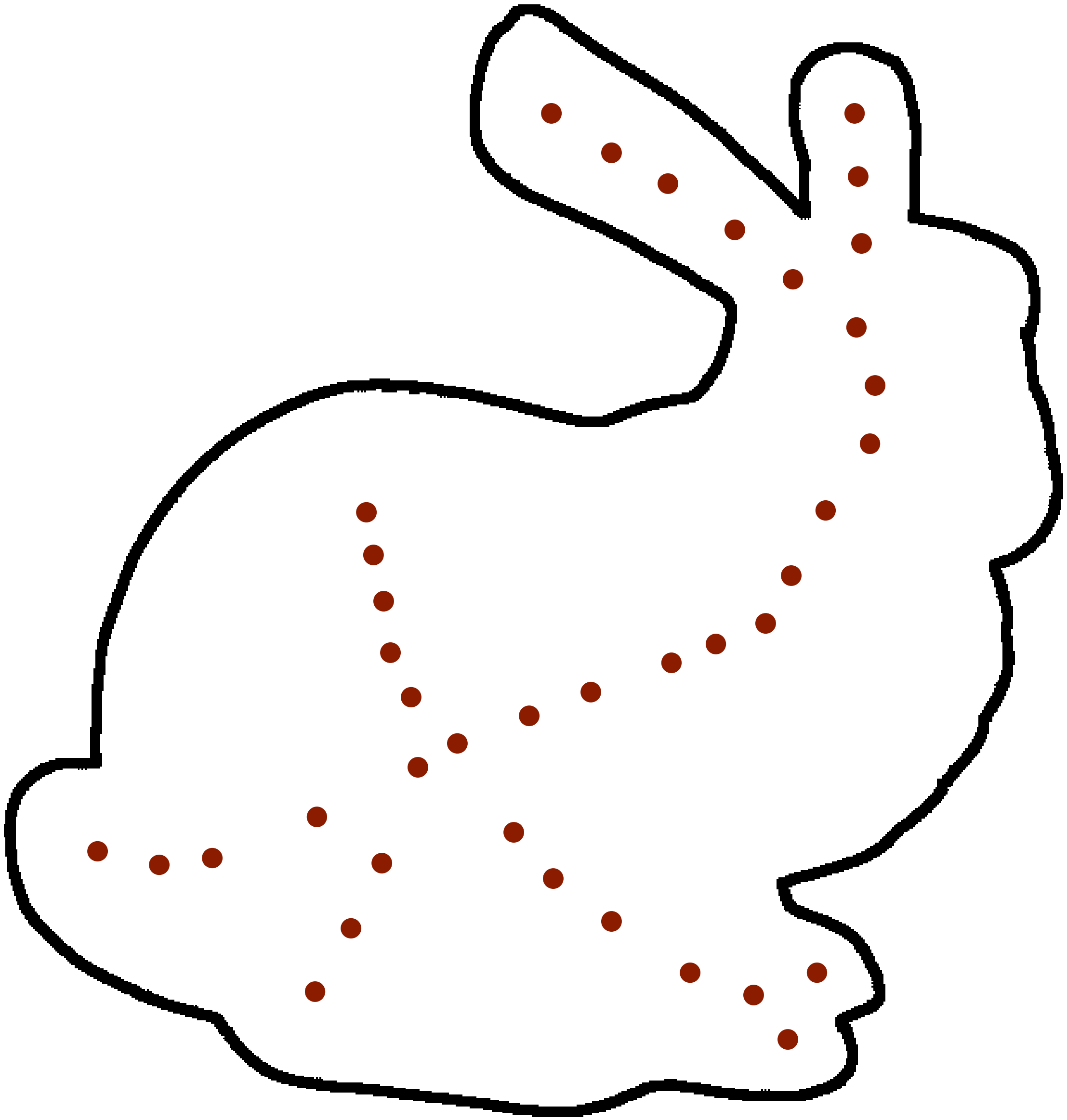

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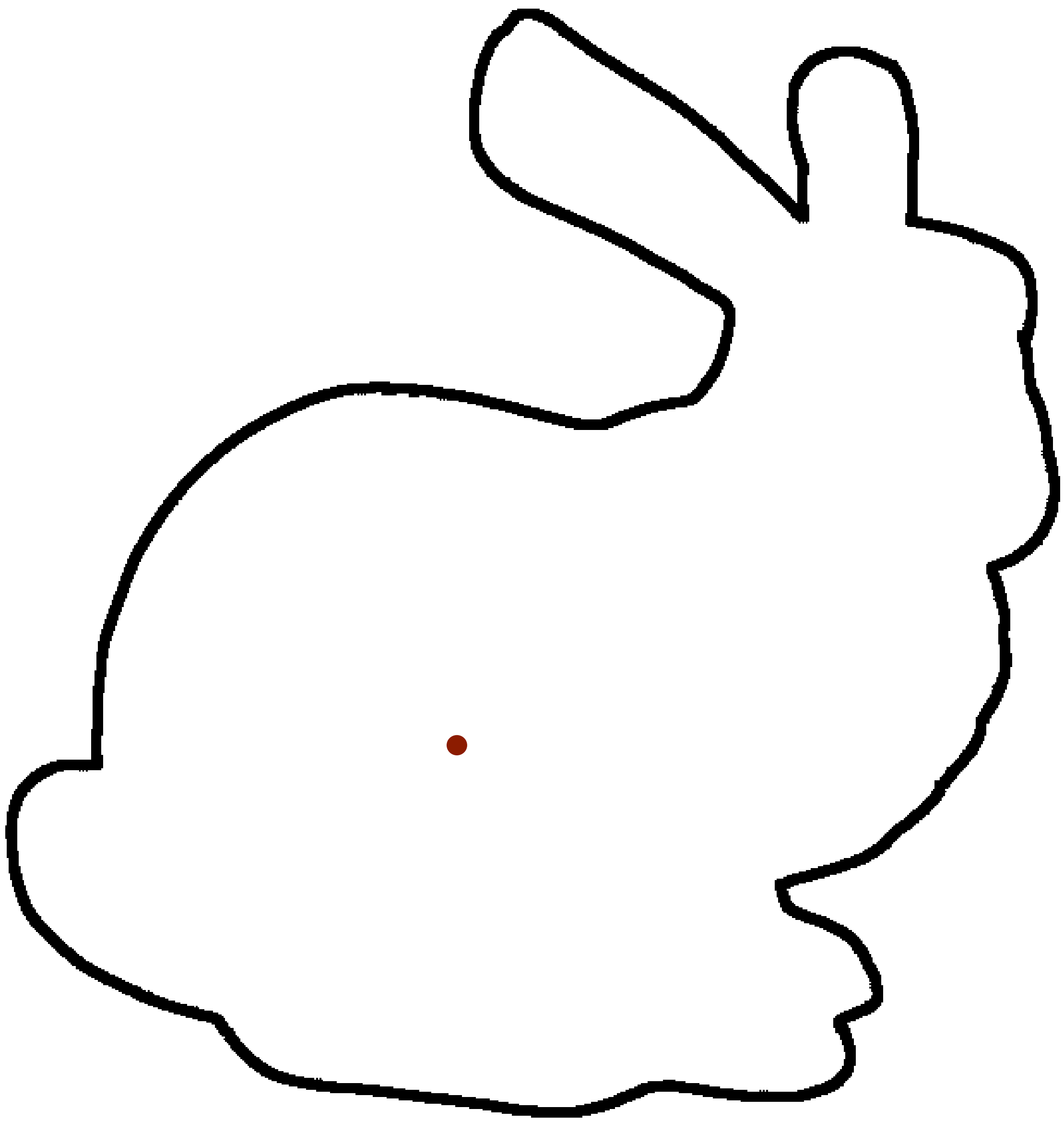

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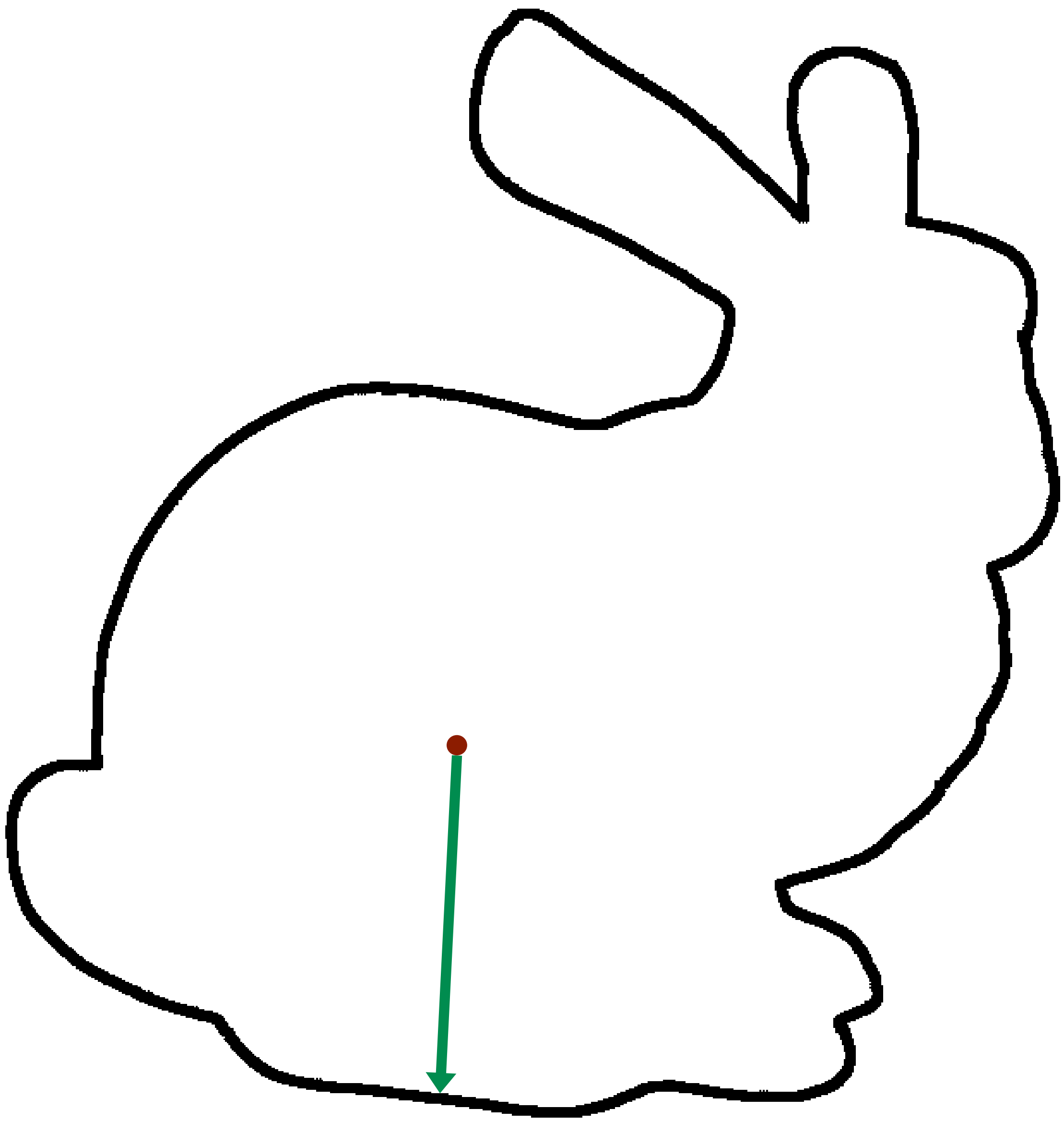

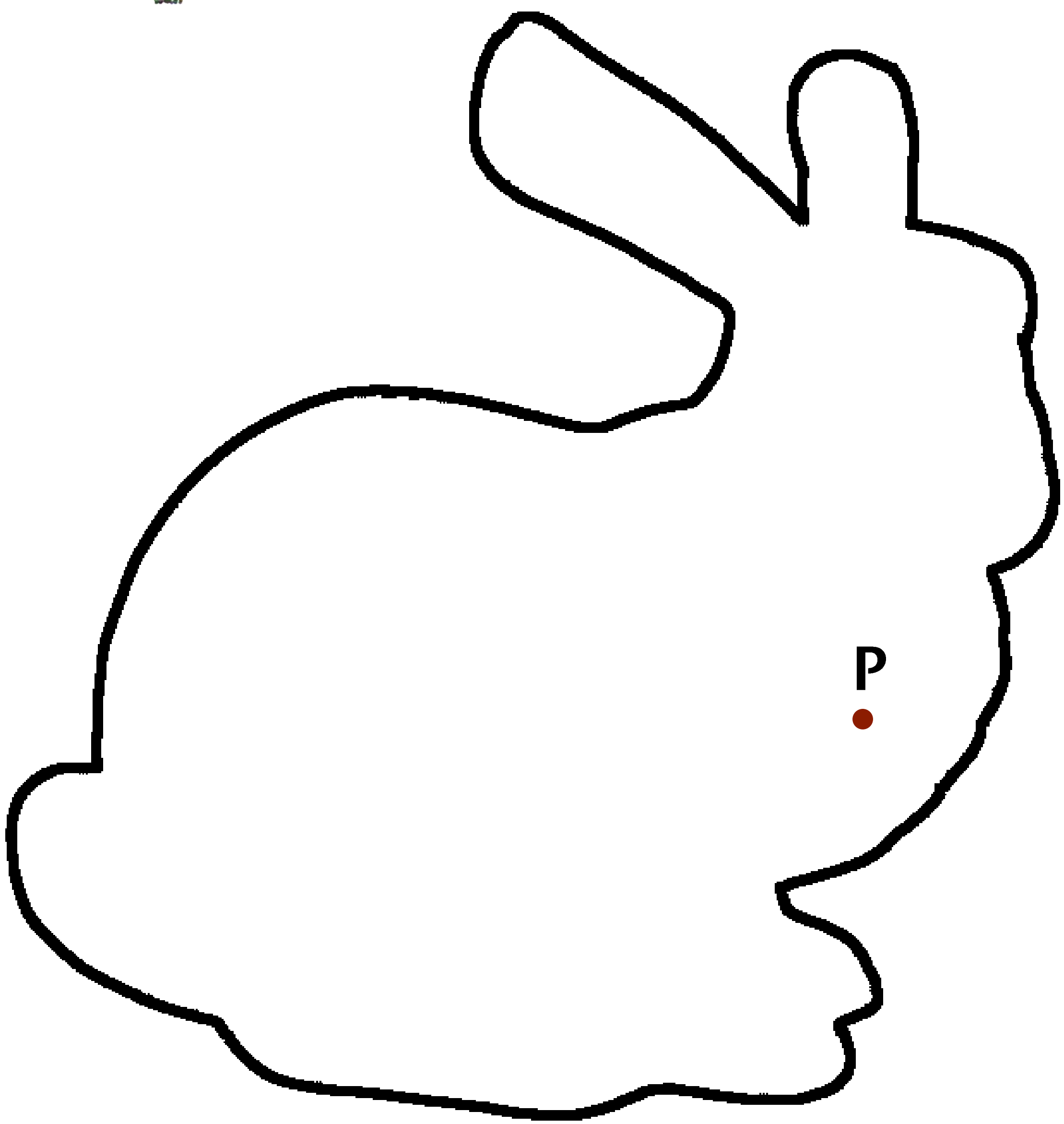
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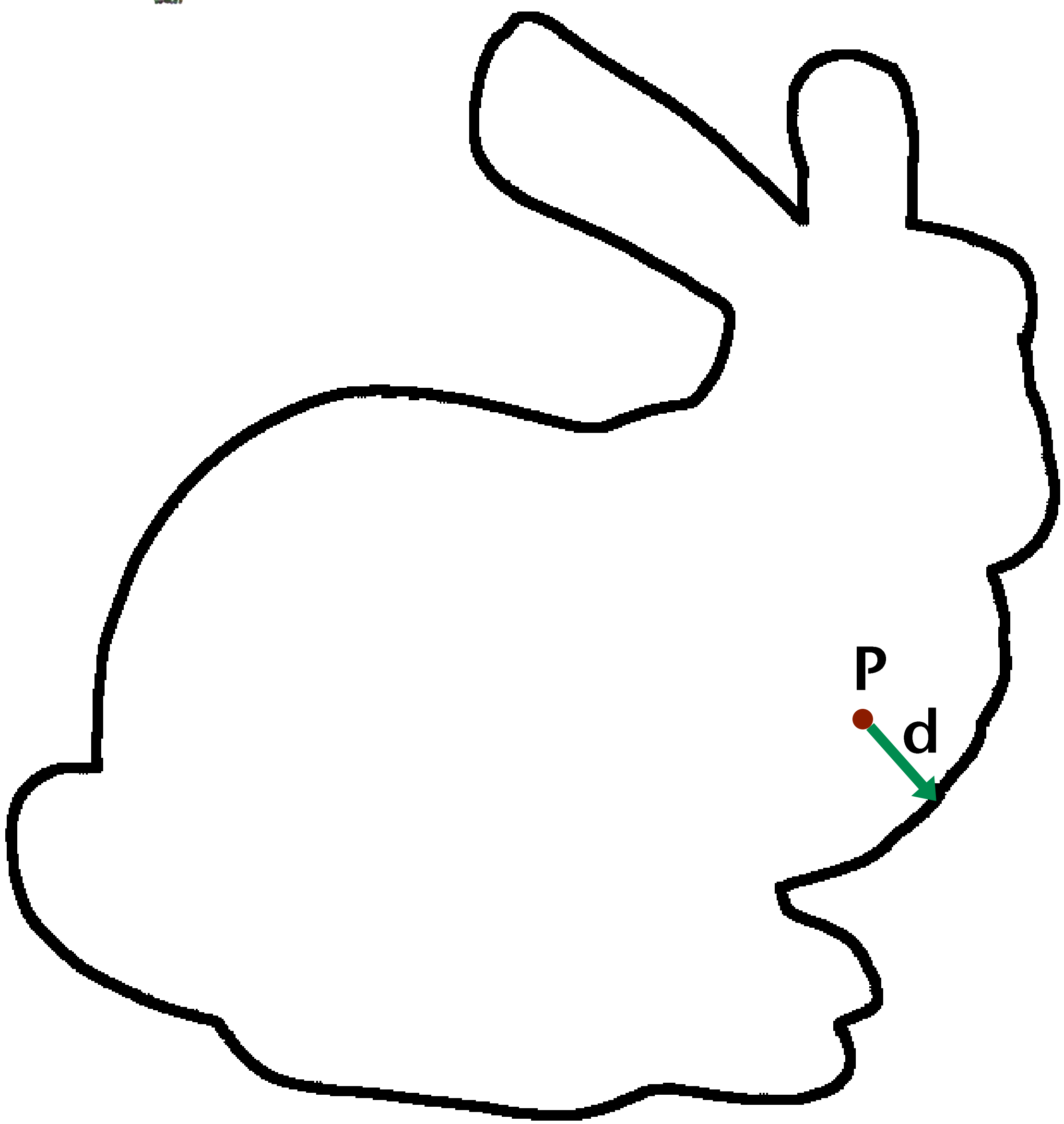
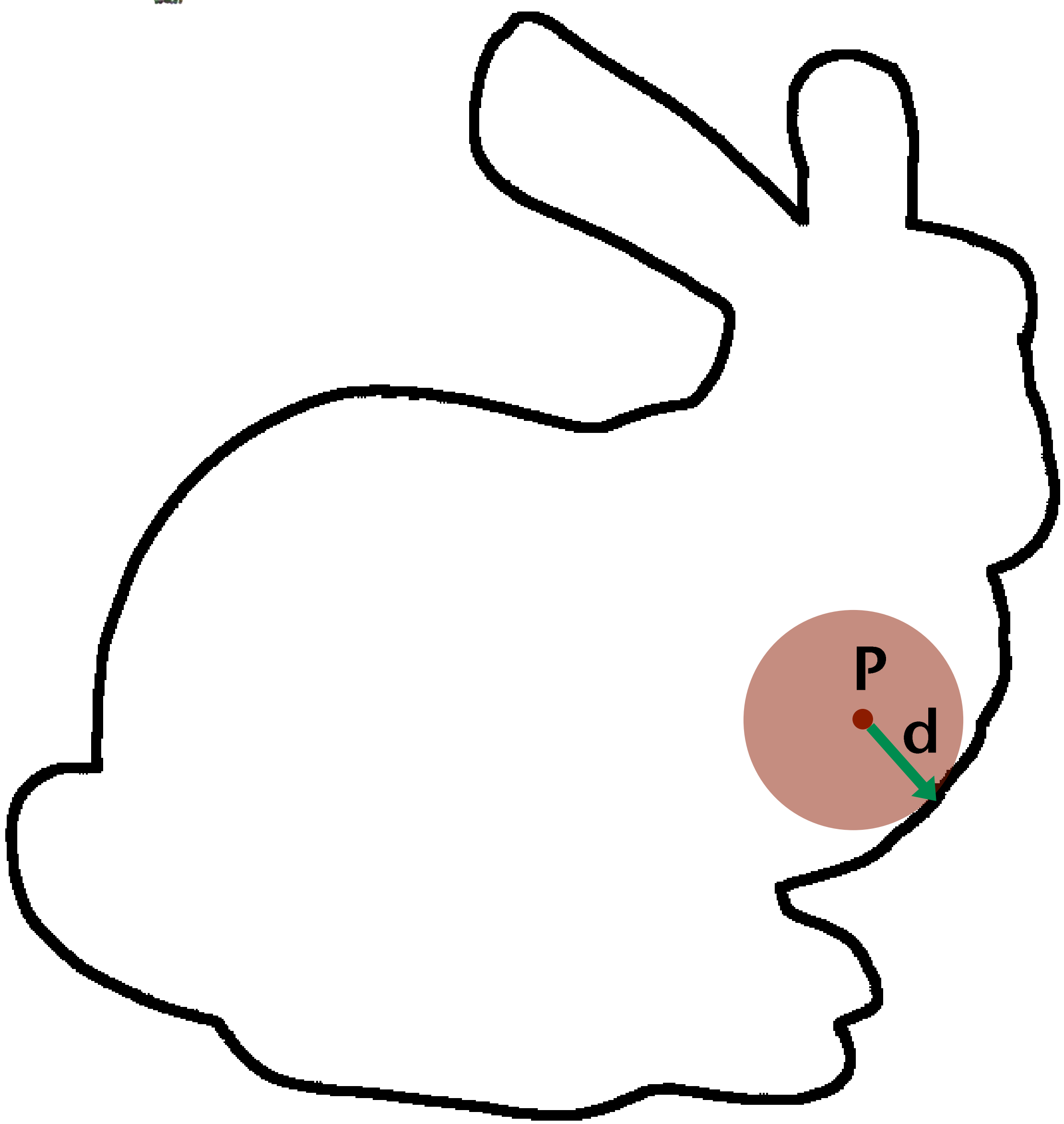
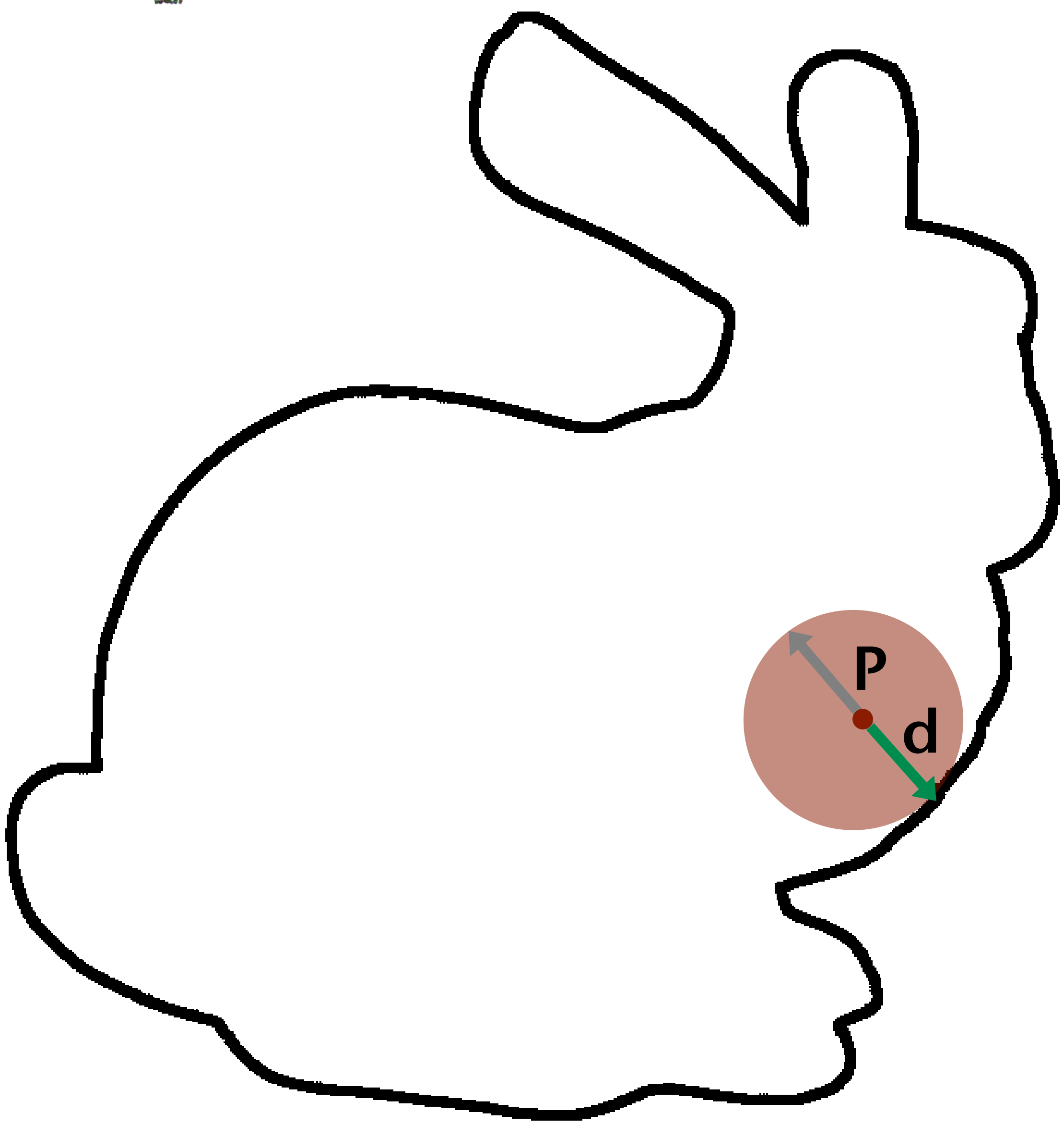
Protosphere - Basic Algorithm



Protosphere - Basic Algorithm

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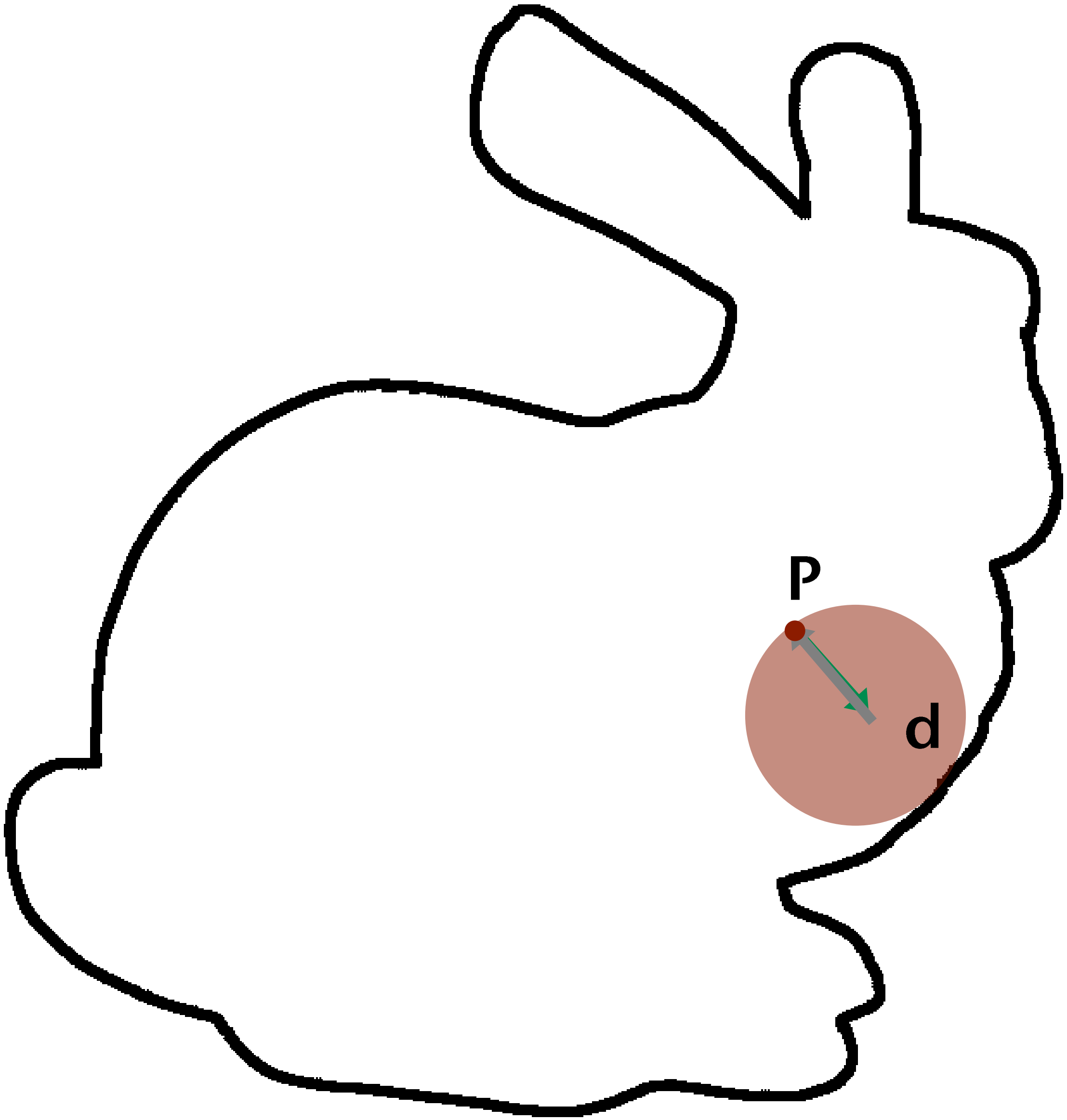
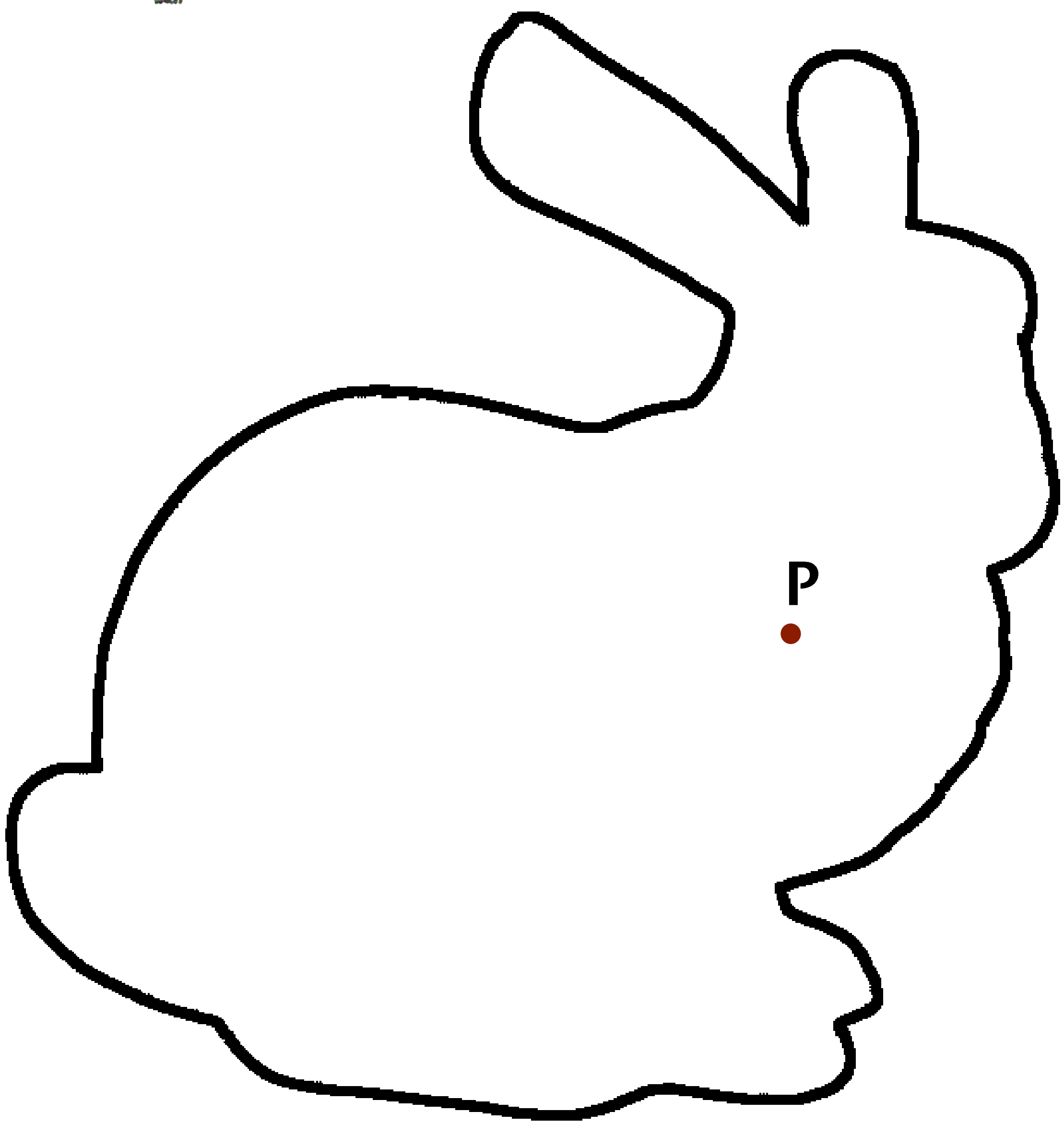
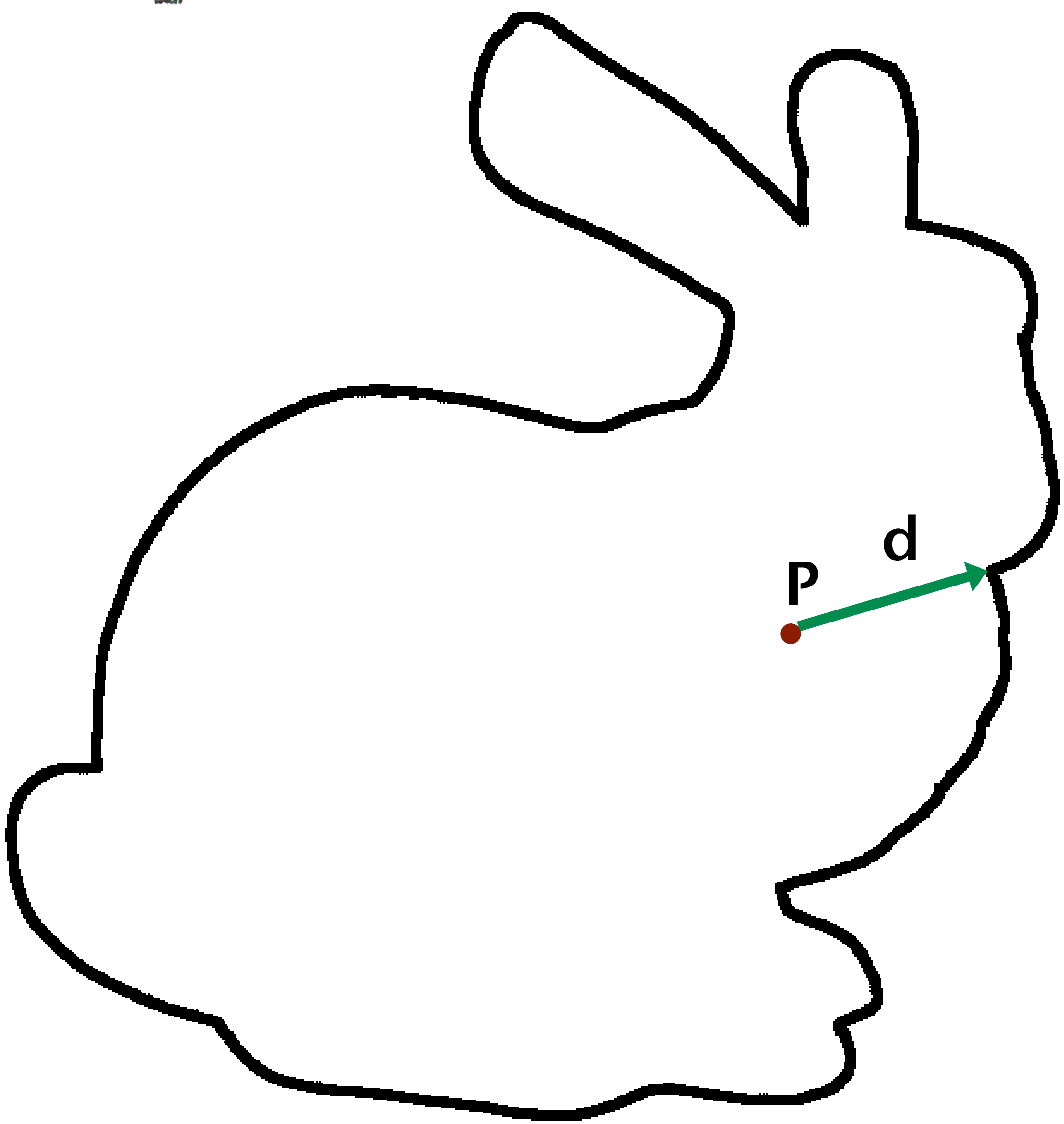
Protosphere - Basic Algorithm



Protosphere - Basic Algorithm



Protosphere - Basic Algorithm



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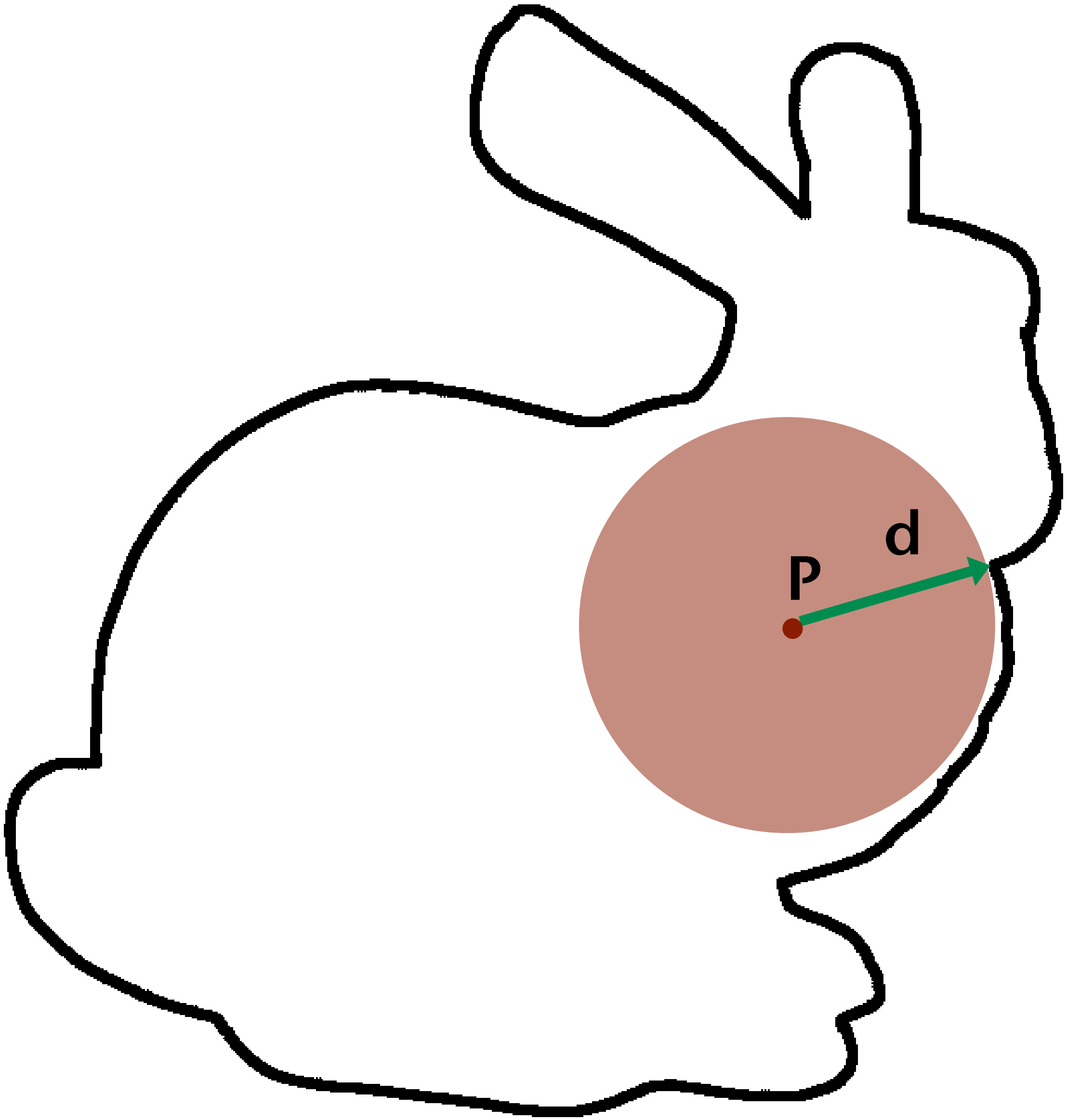
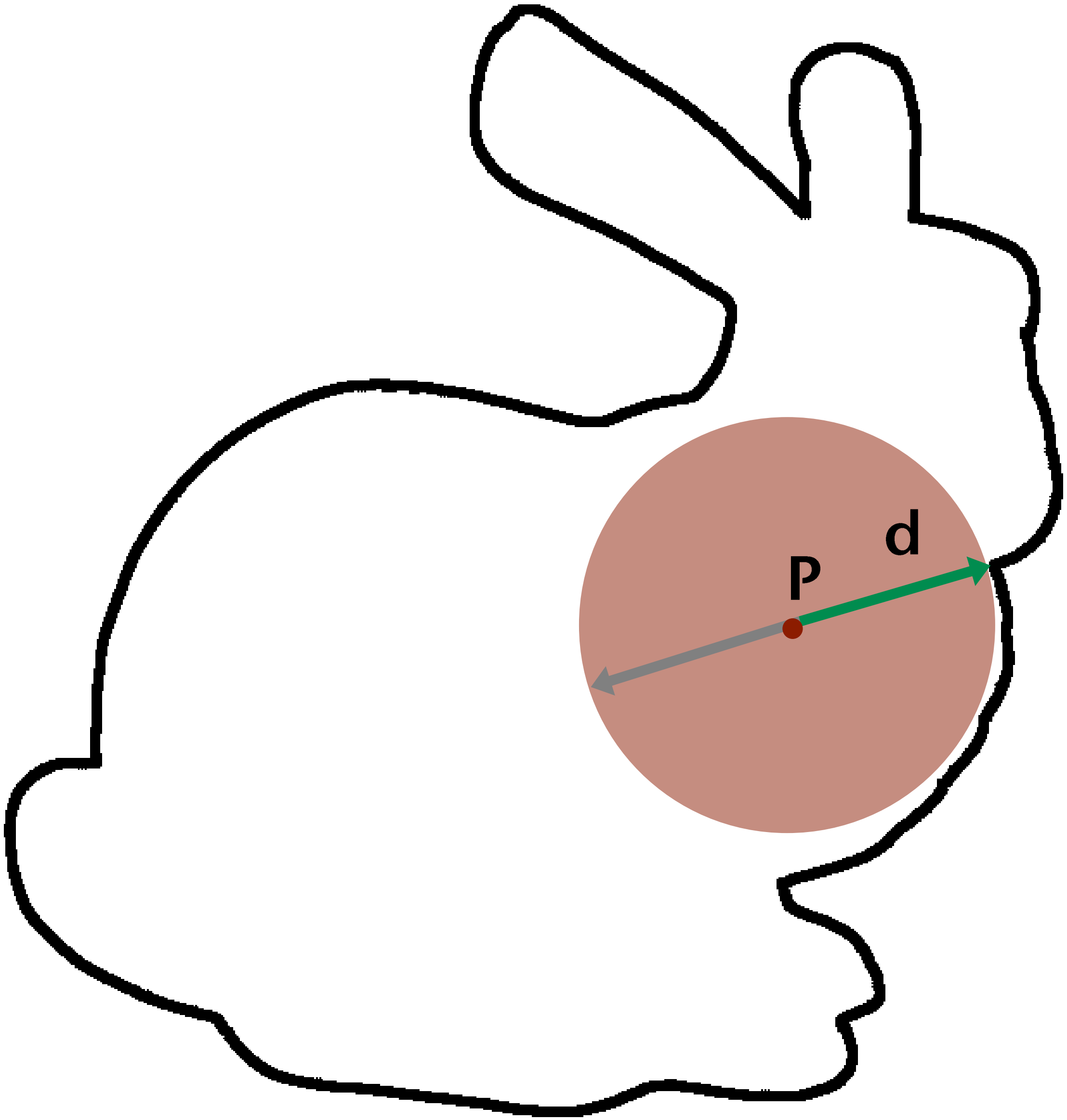
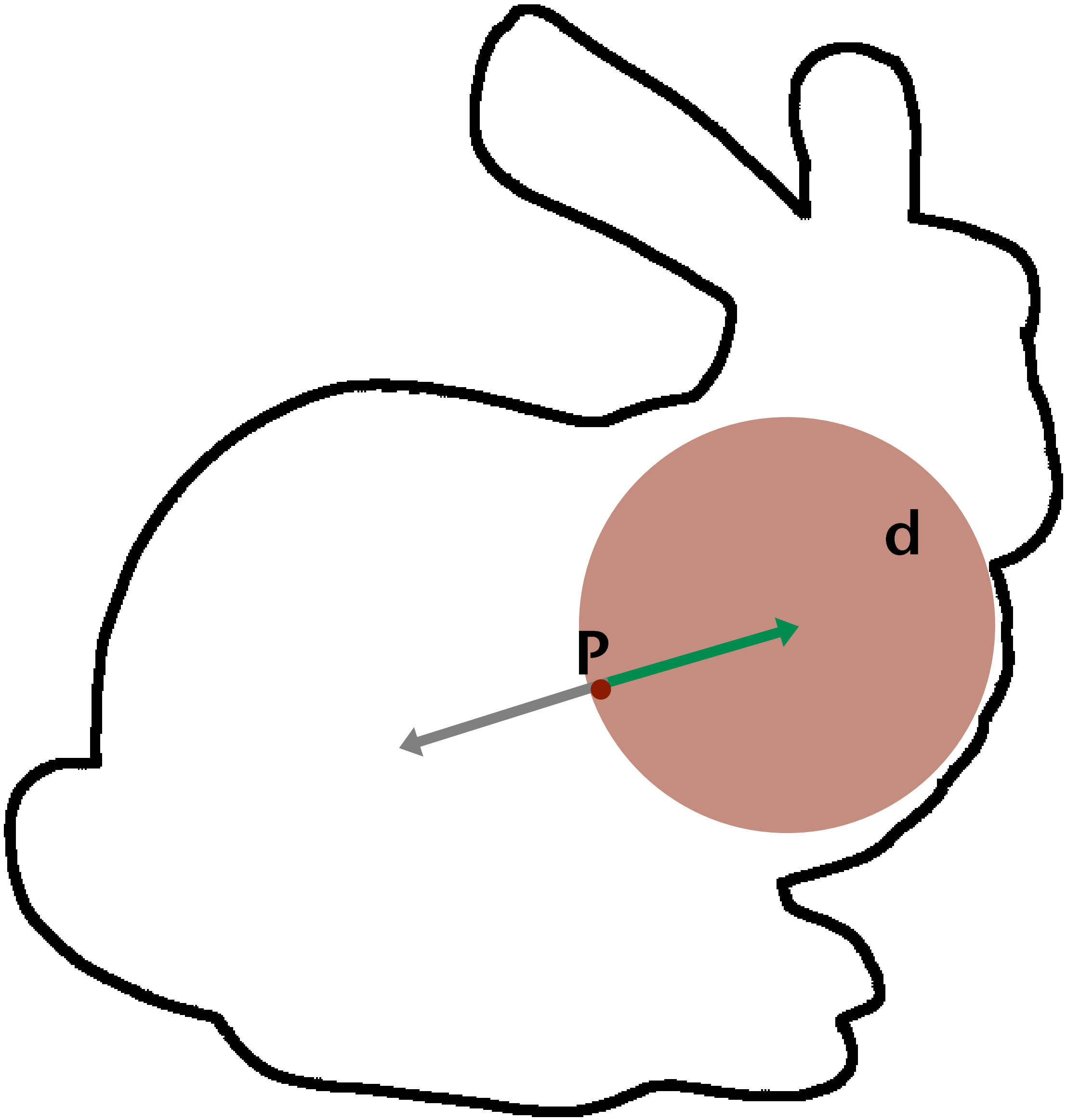
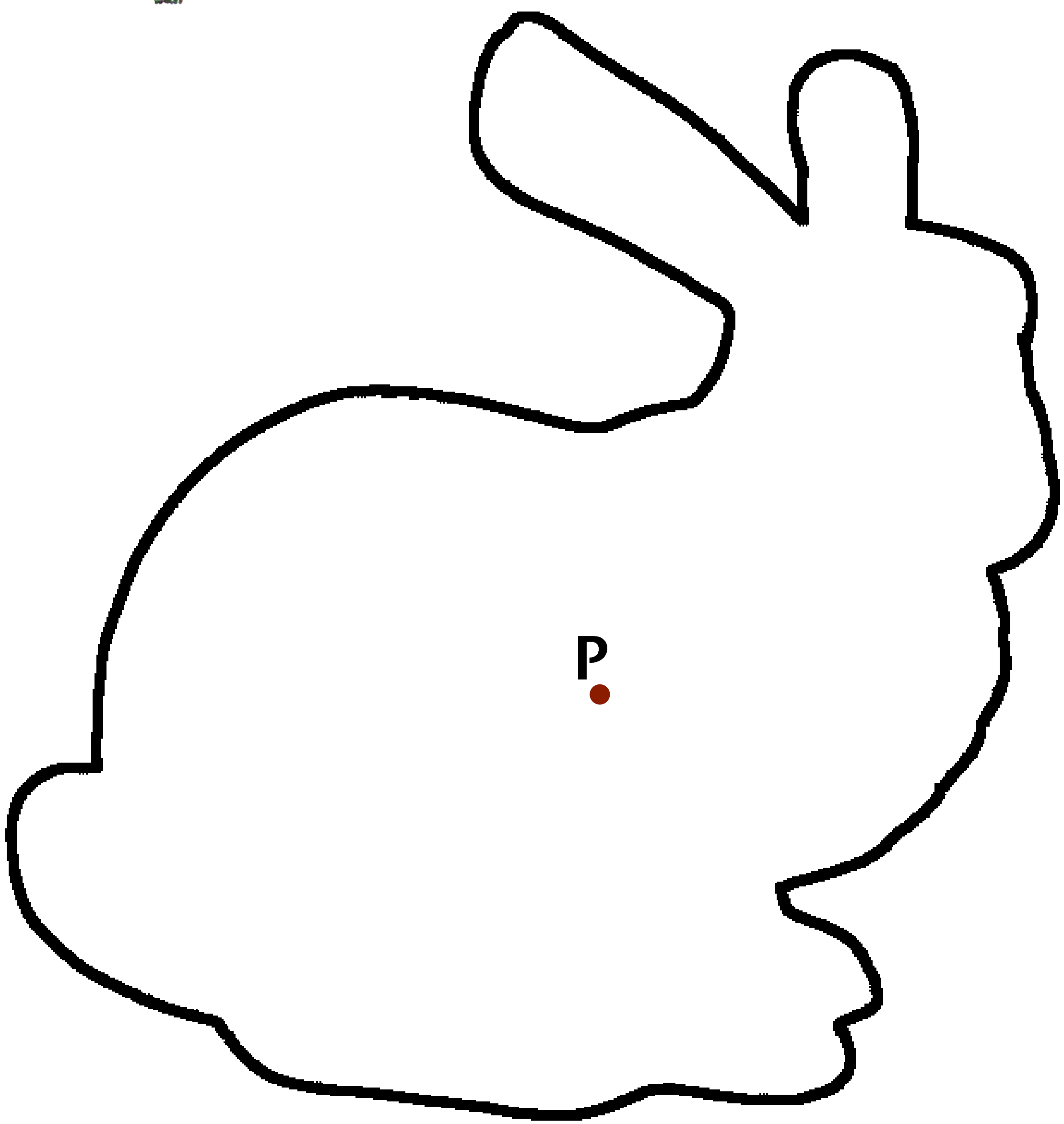


Protosphere - Basic Algorithm



Protosphere - Basic Algorithm

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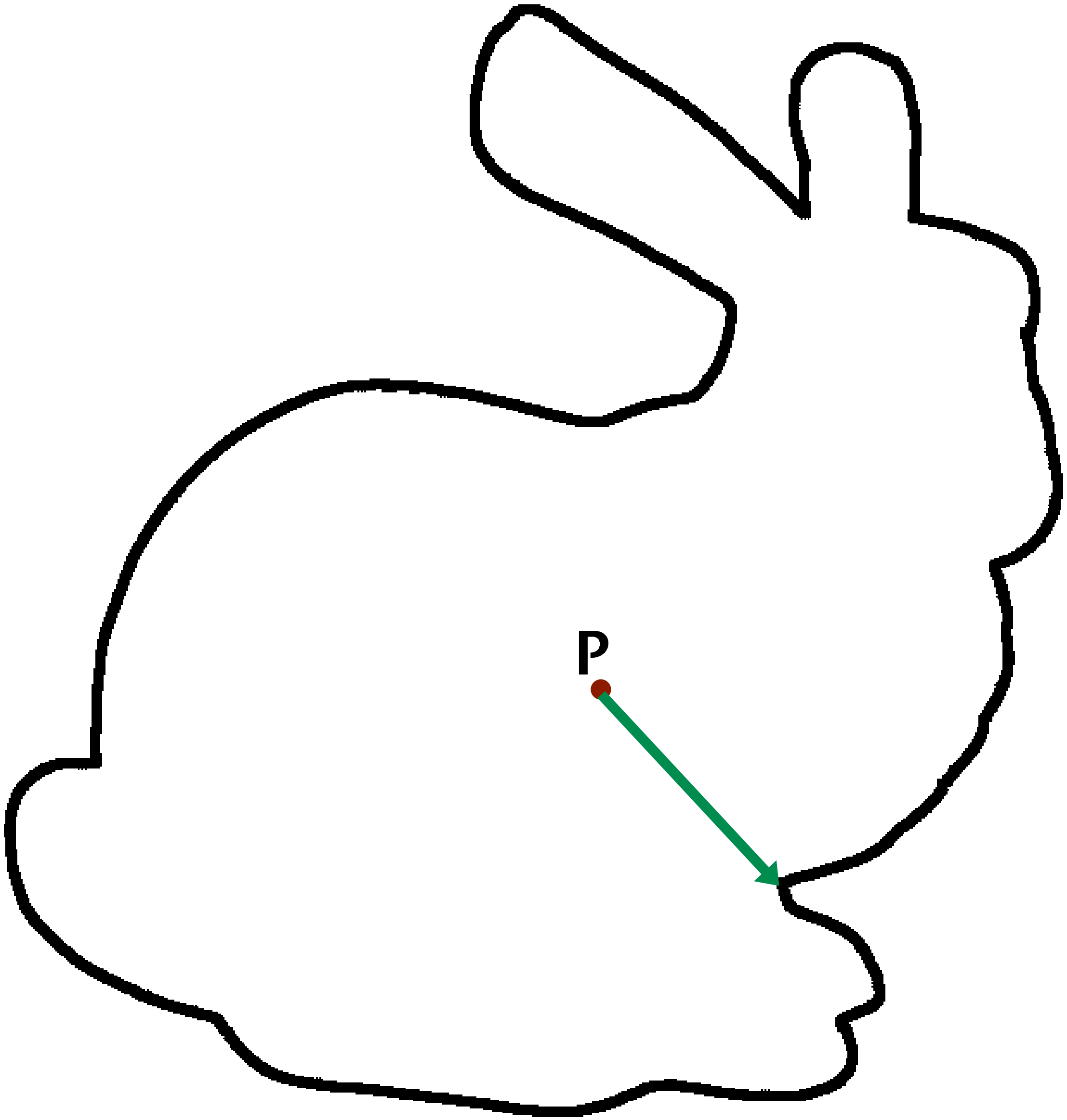
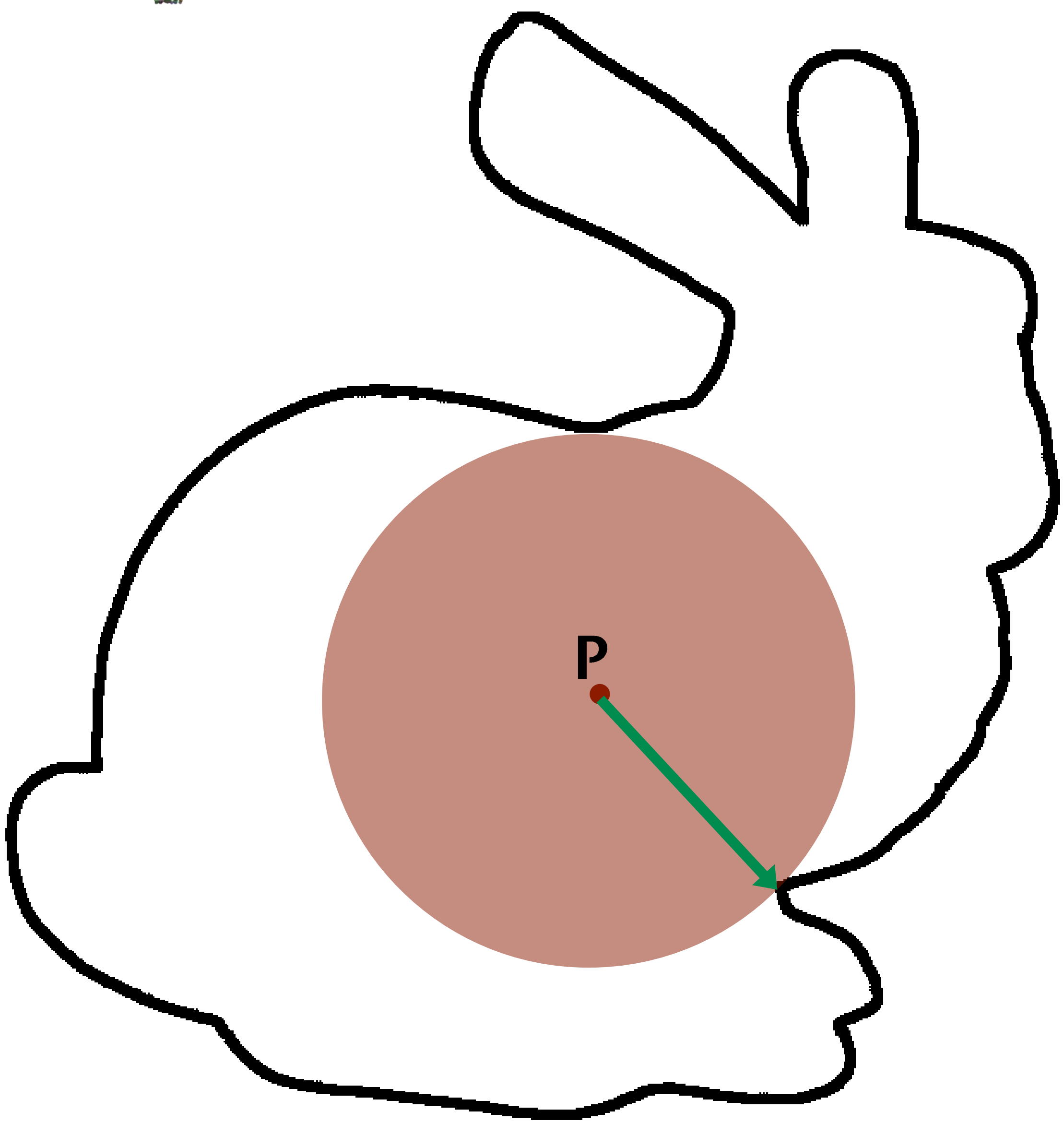
Protosphere - Basic Algorithm

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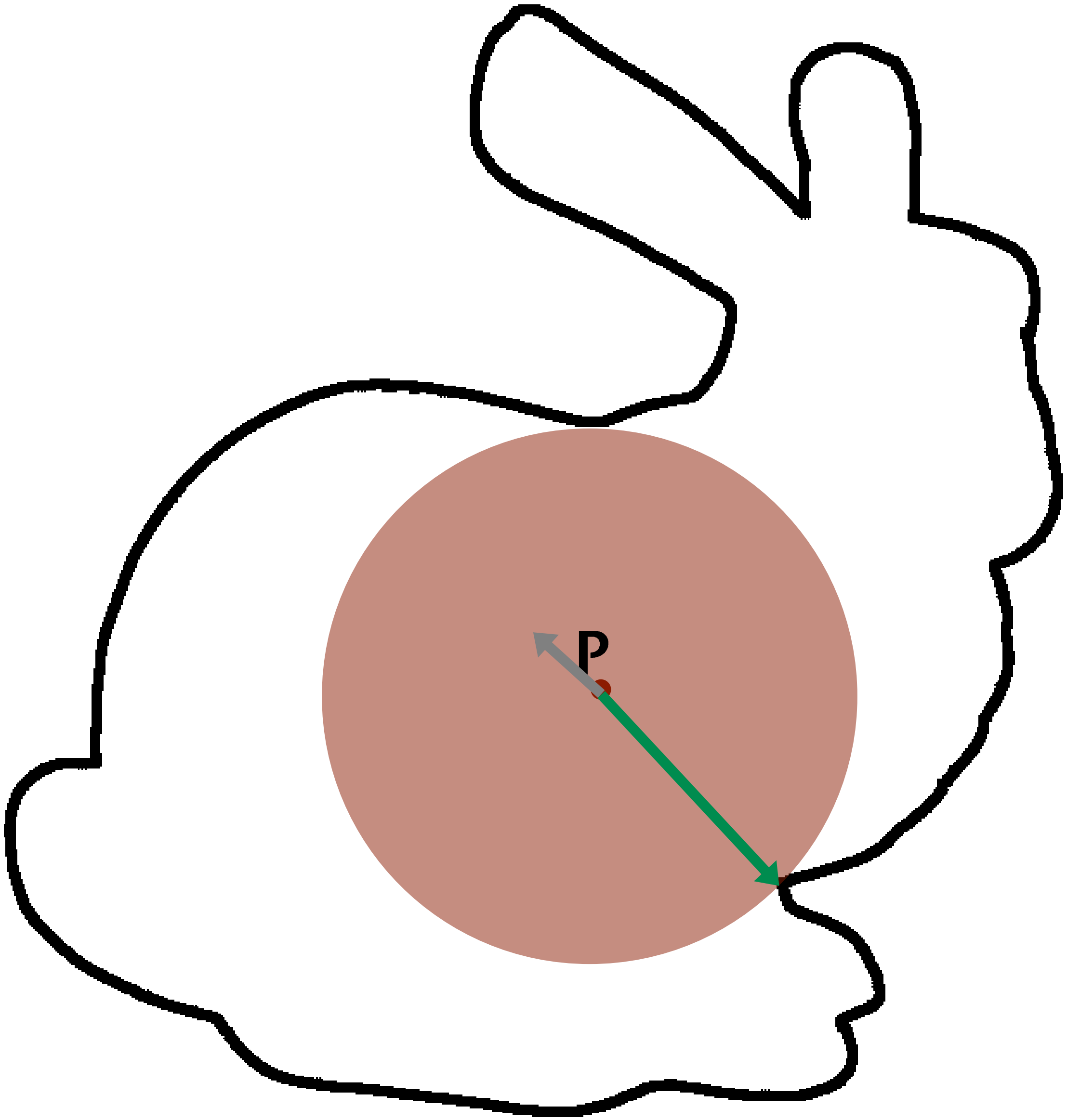

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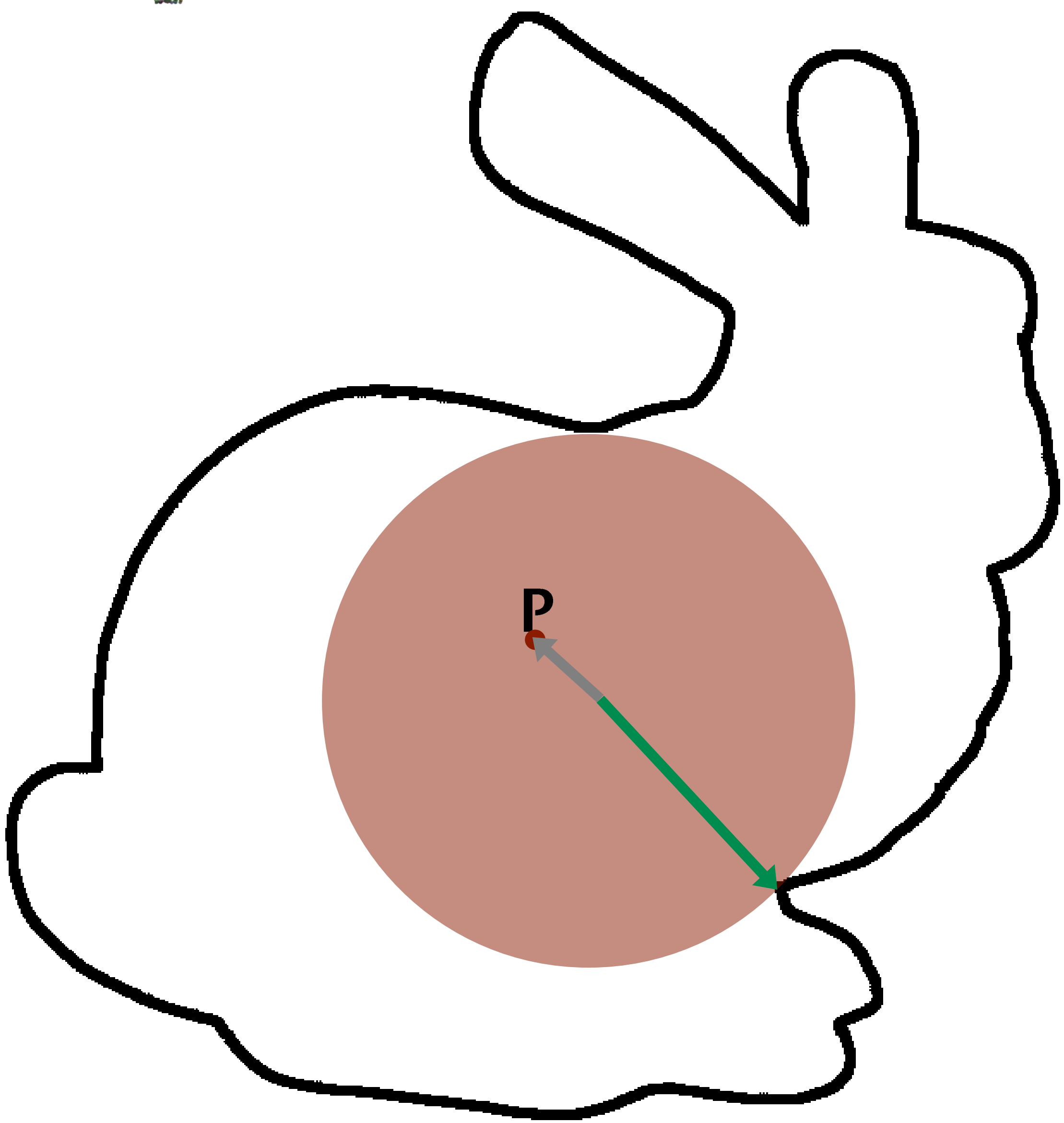

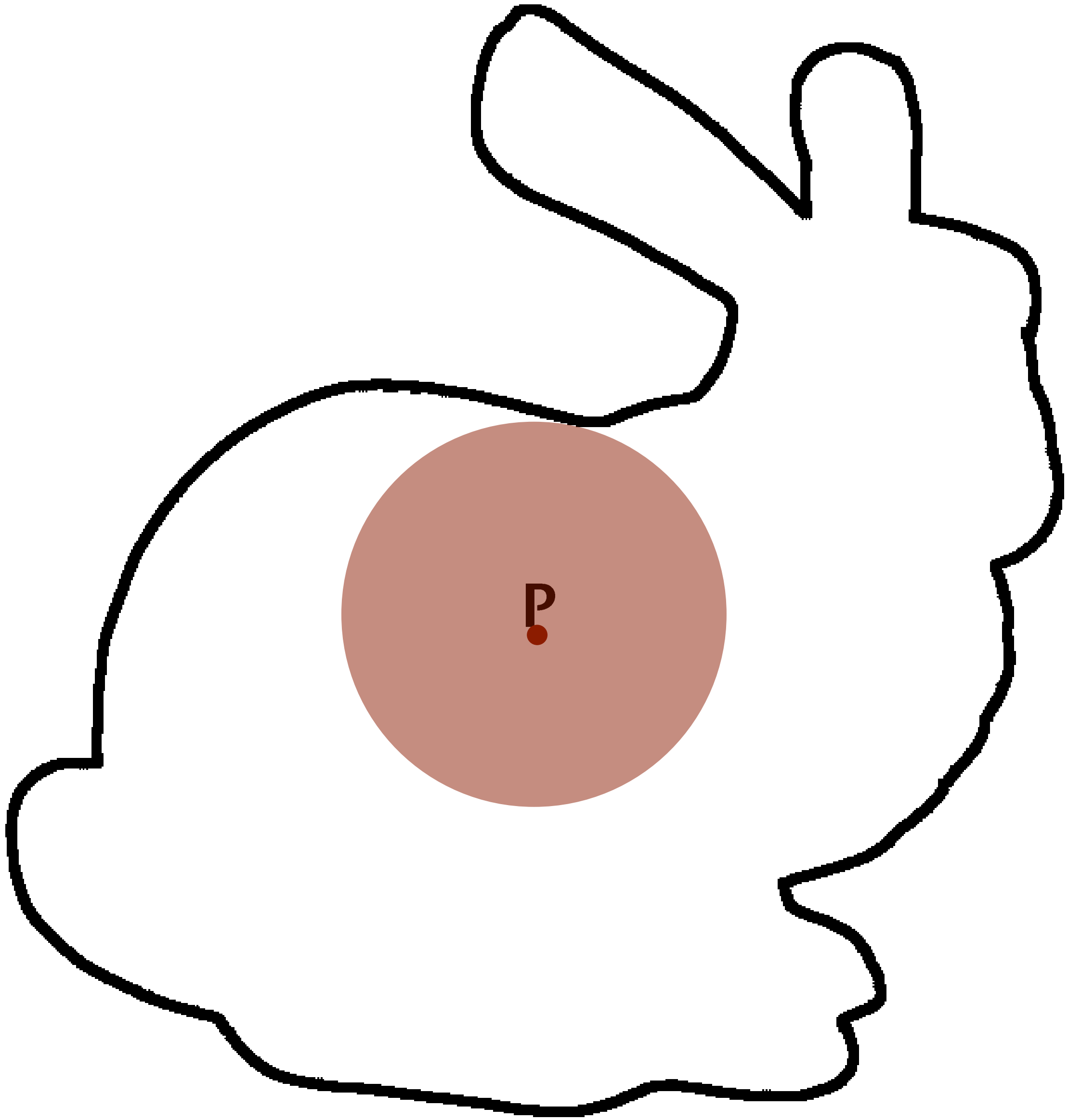
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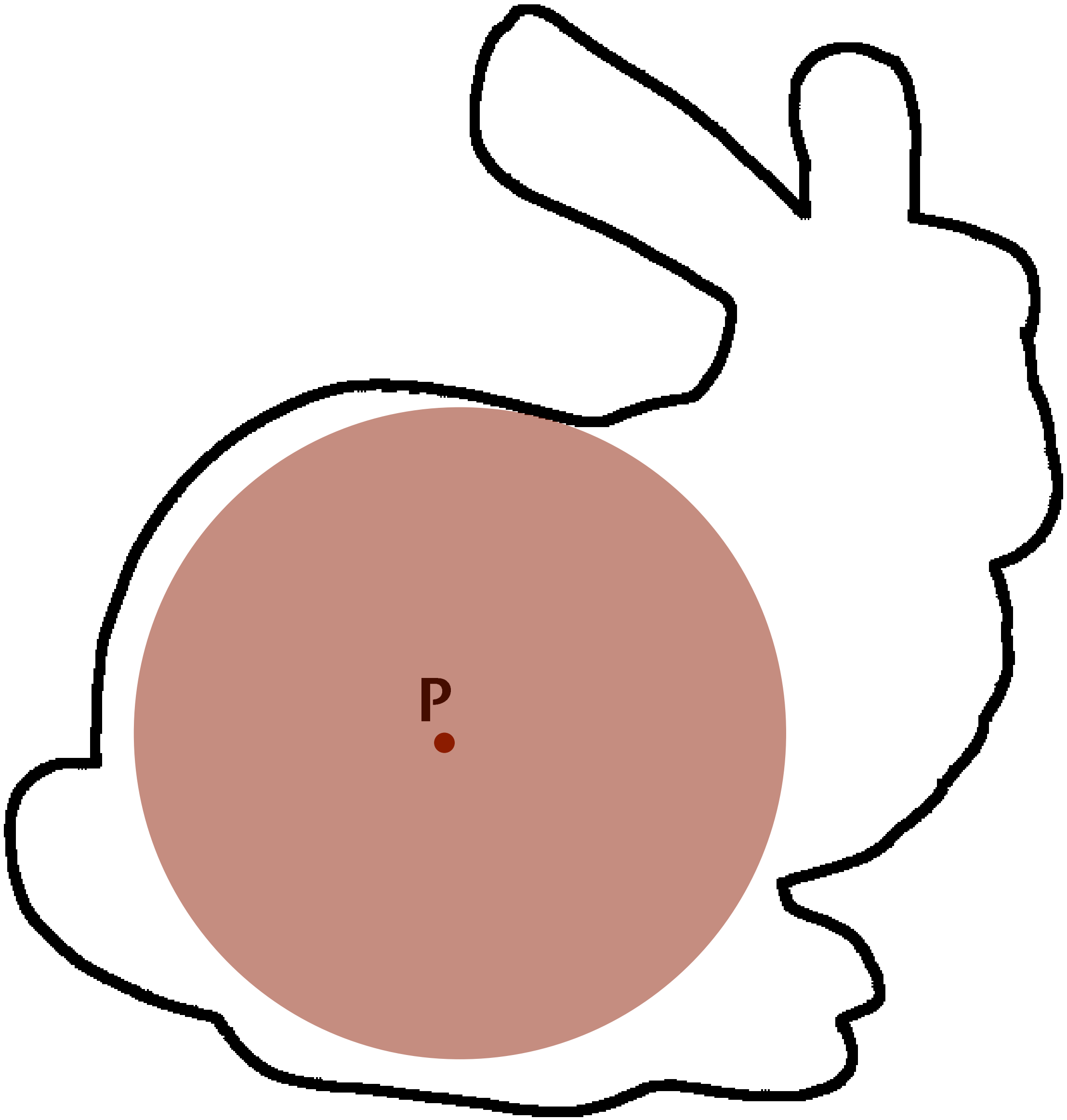
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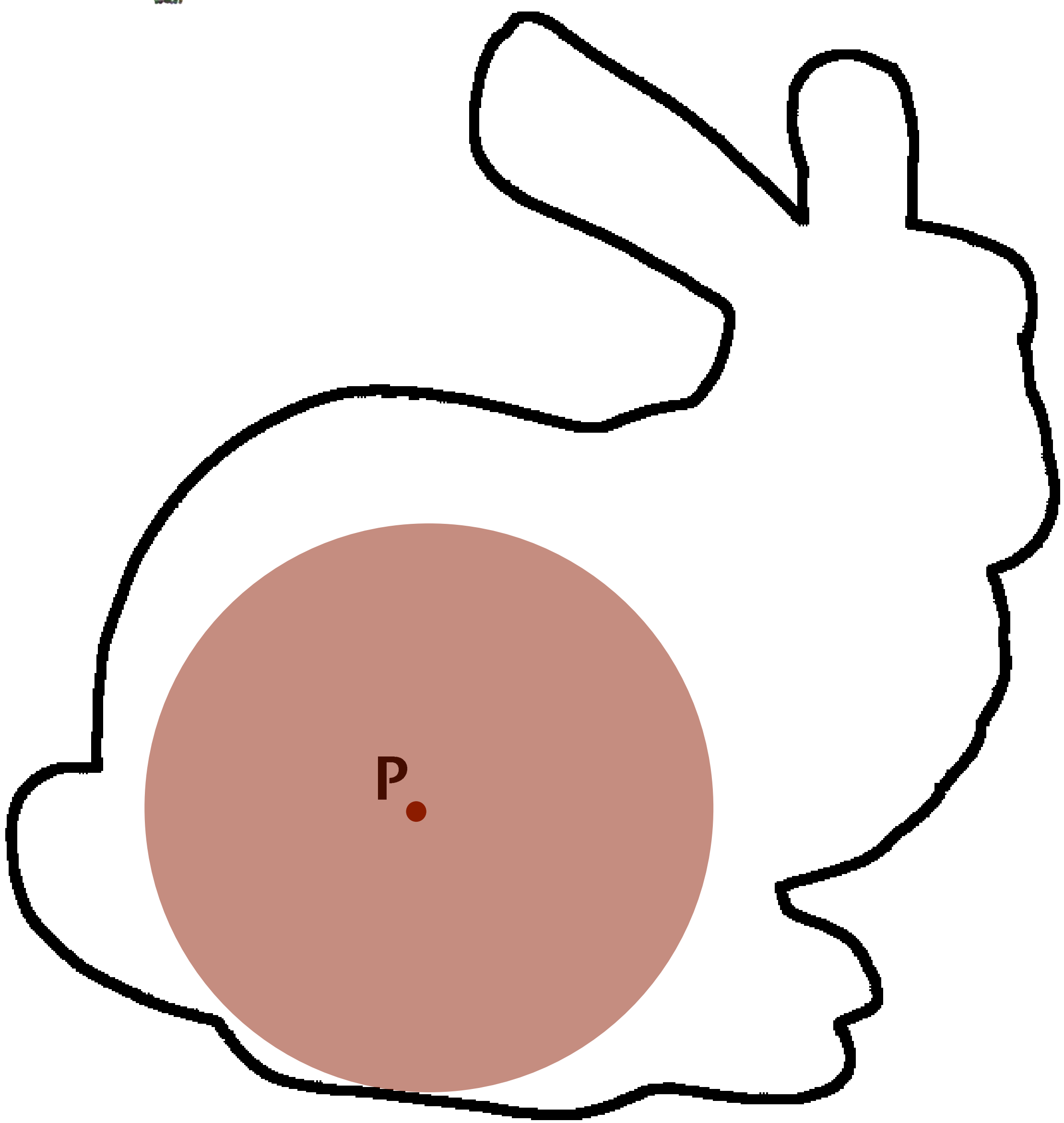
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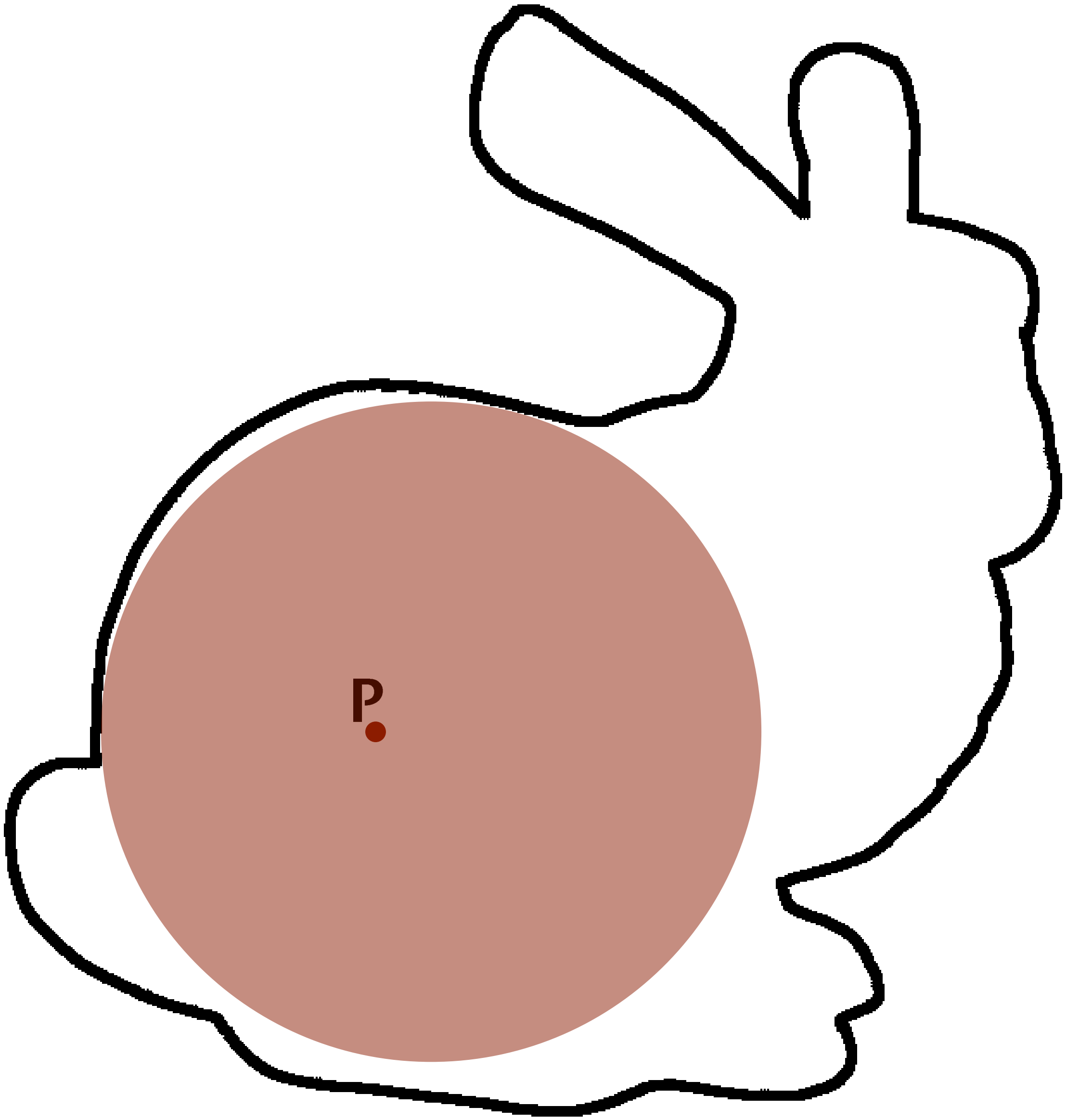
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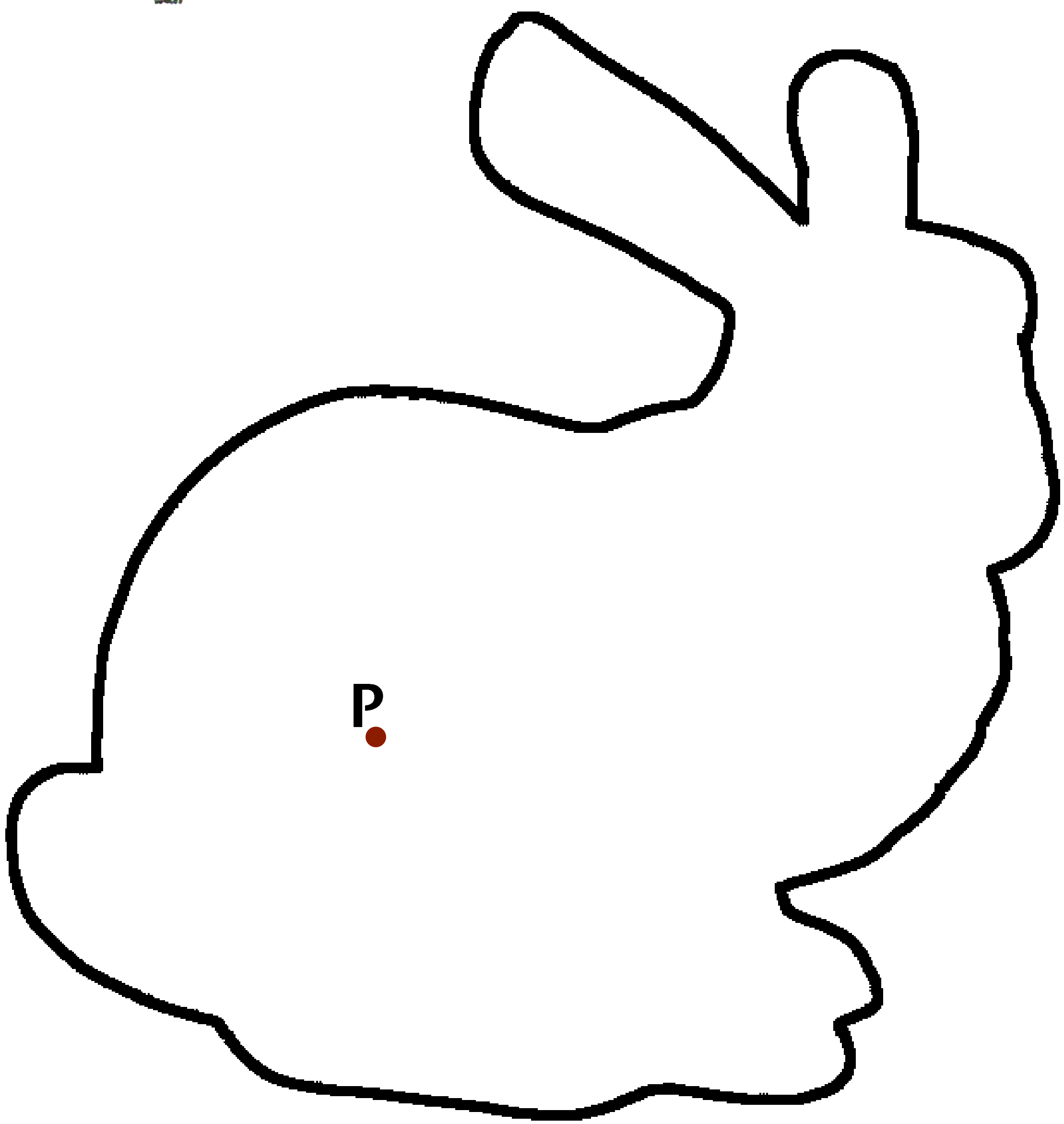
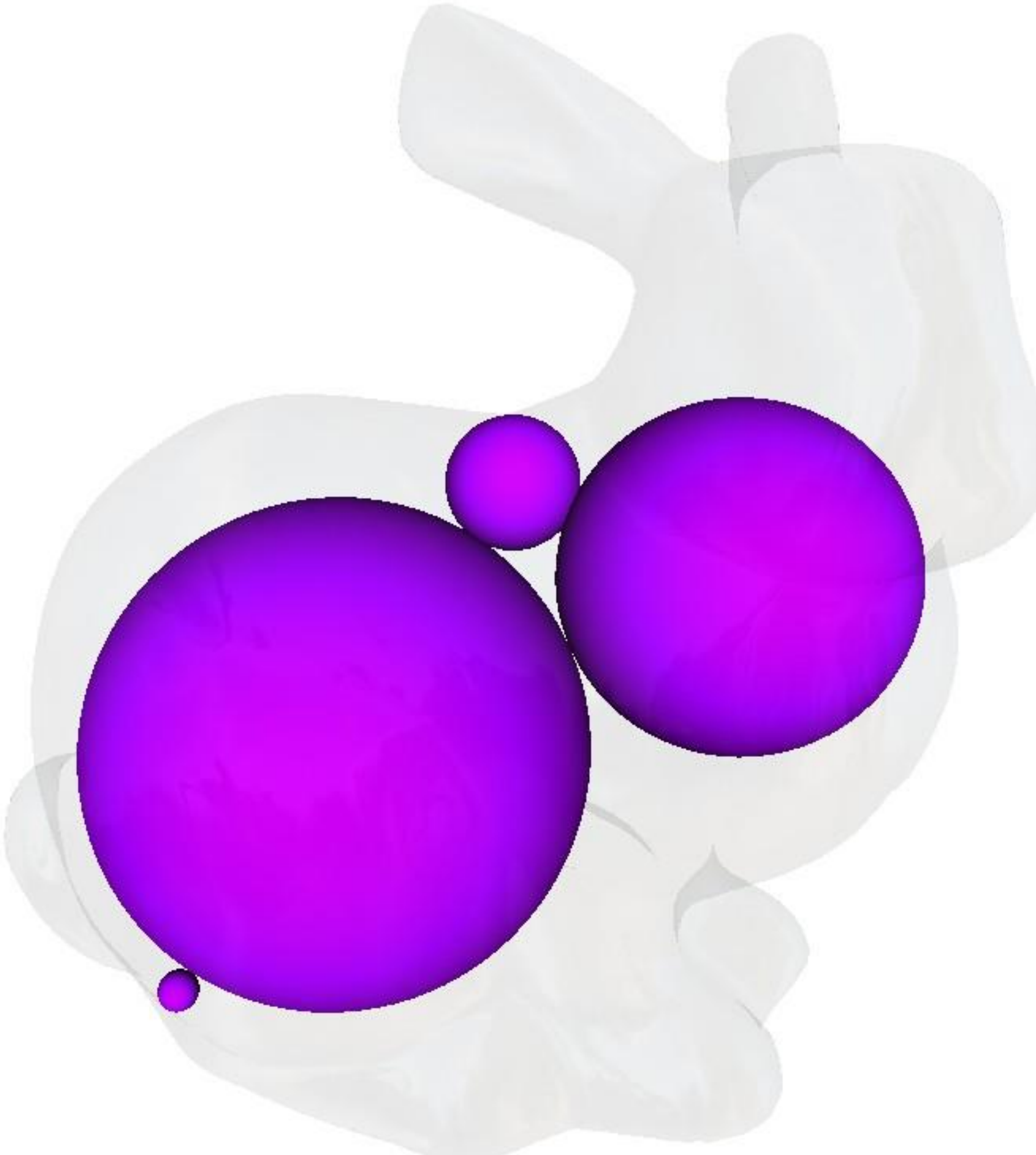
Protosphere - Basic Algorithm



Protosphere - Basic Algorithm

Protosphere - Basic Algorithm



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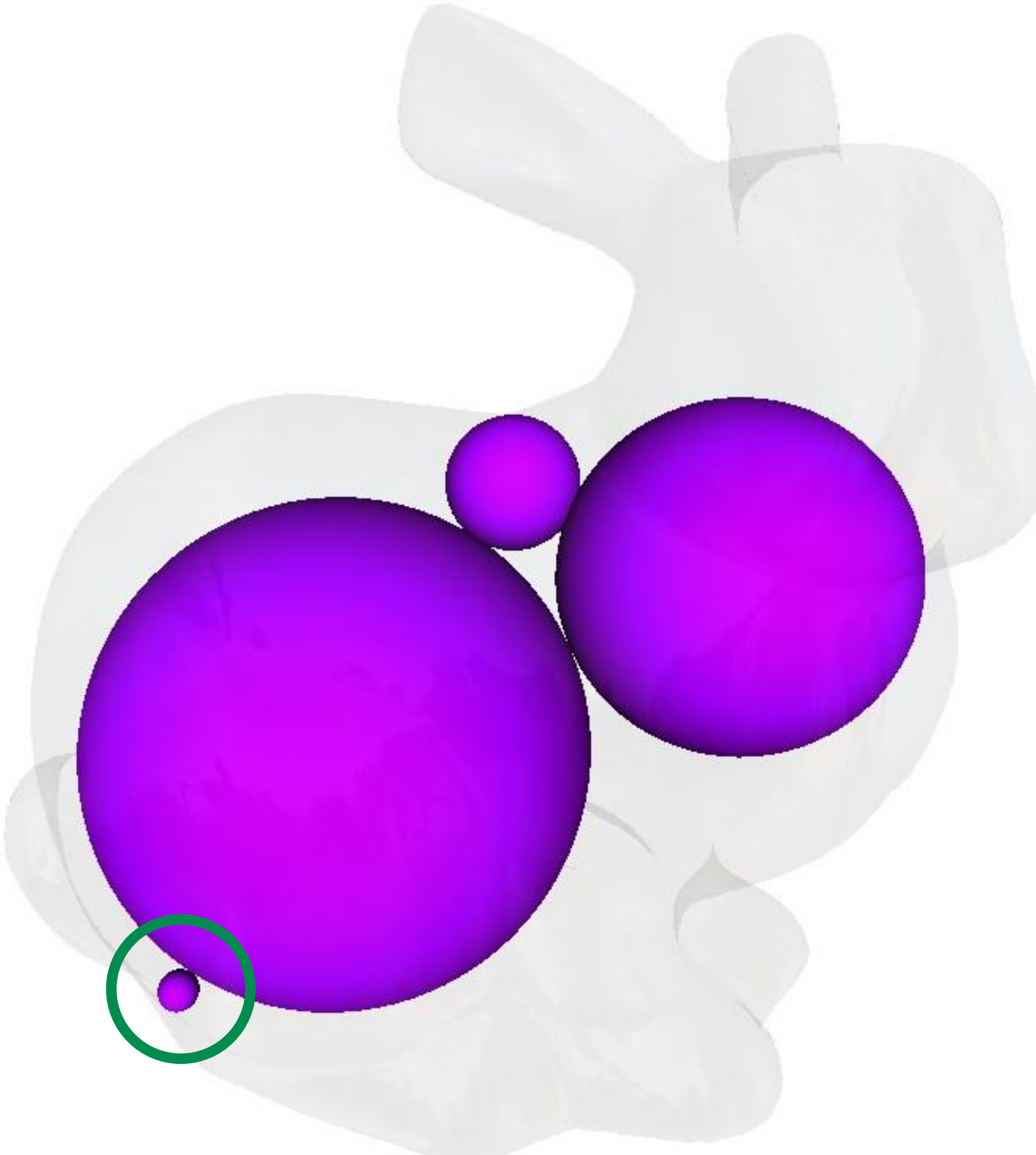


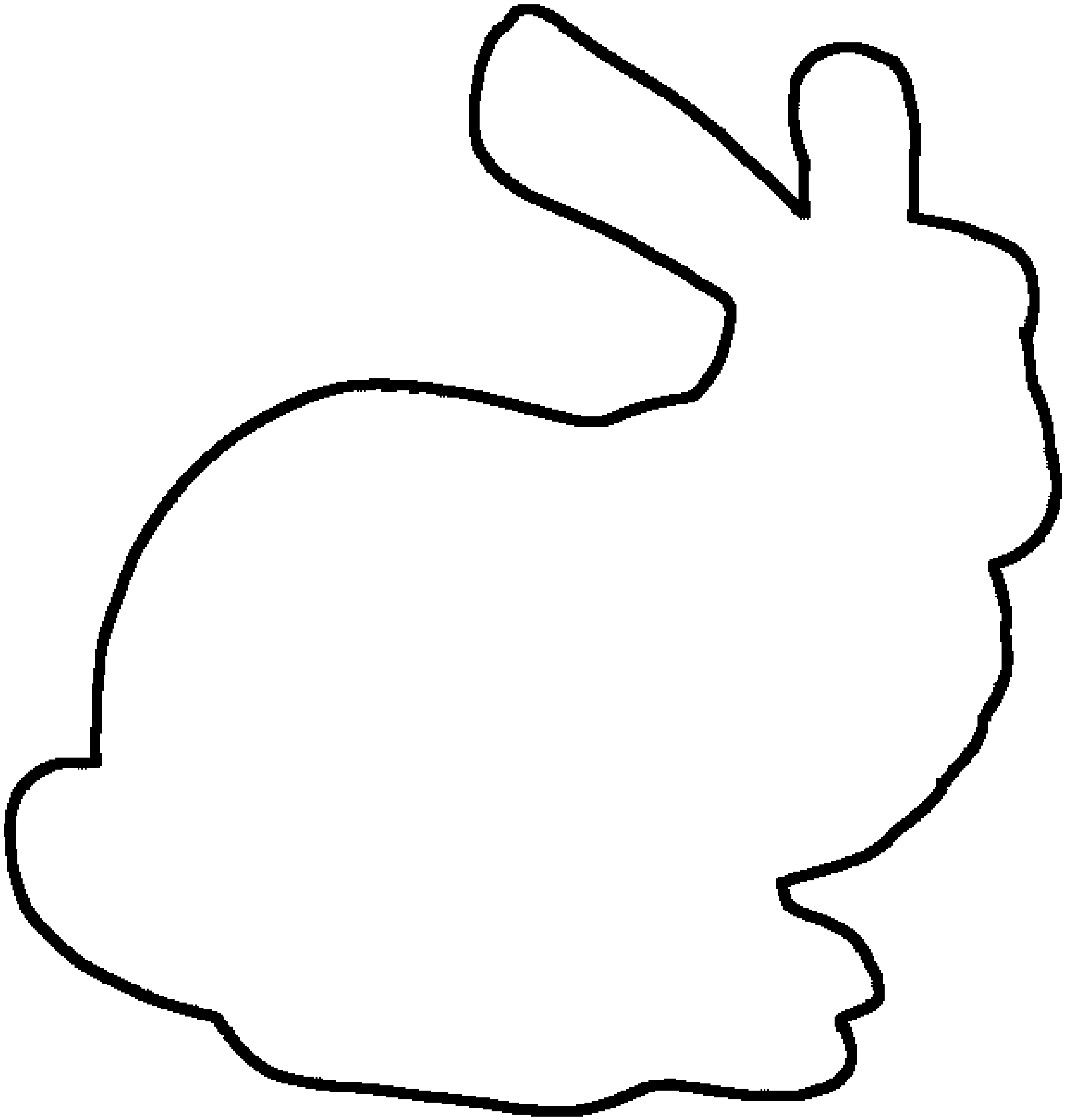
Protosphere - Basic Algorithm

Protosphere - Basic Algorithm

Protosphere - Basic Algorithm

Protosphere - Basic Algorithm

Parallelization



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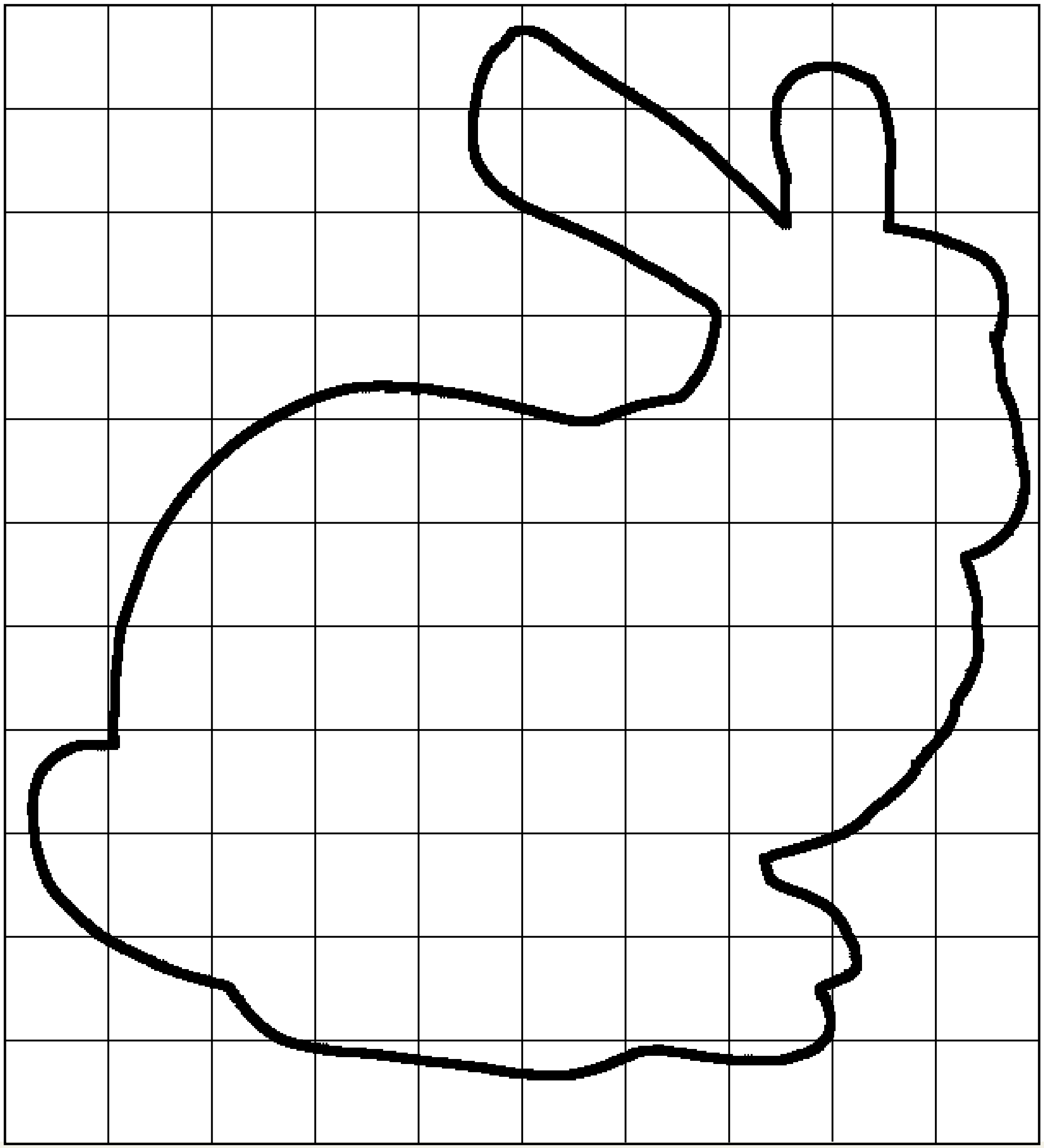

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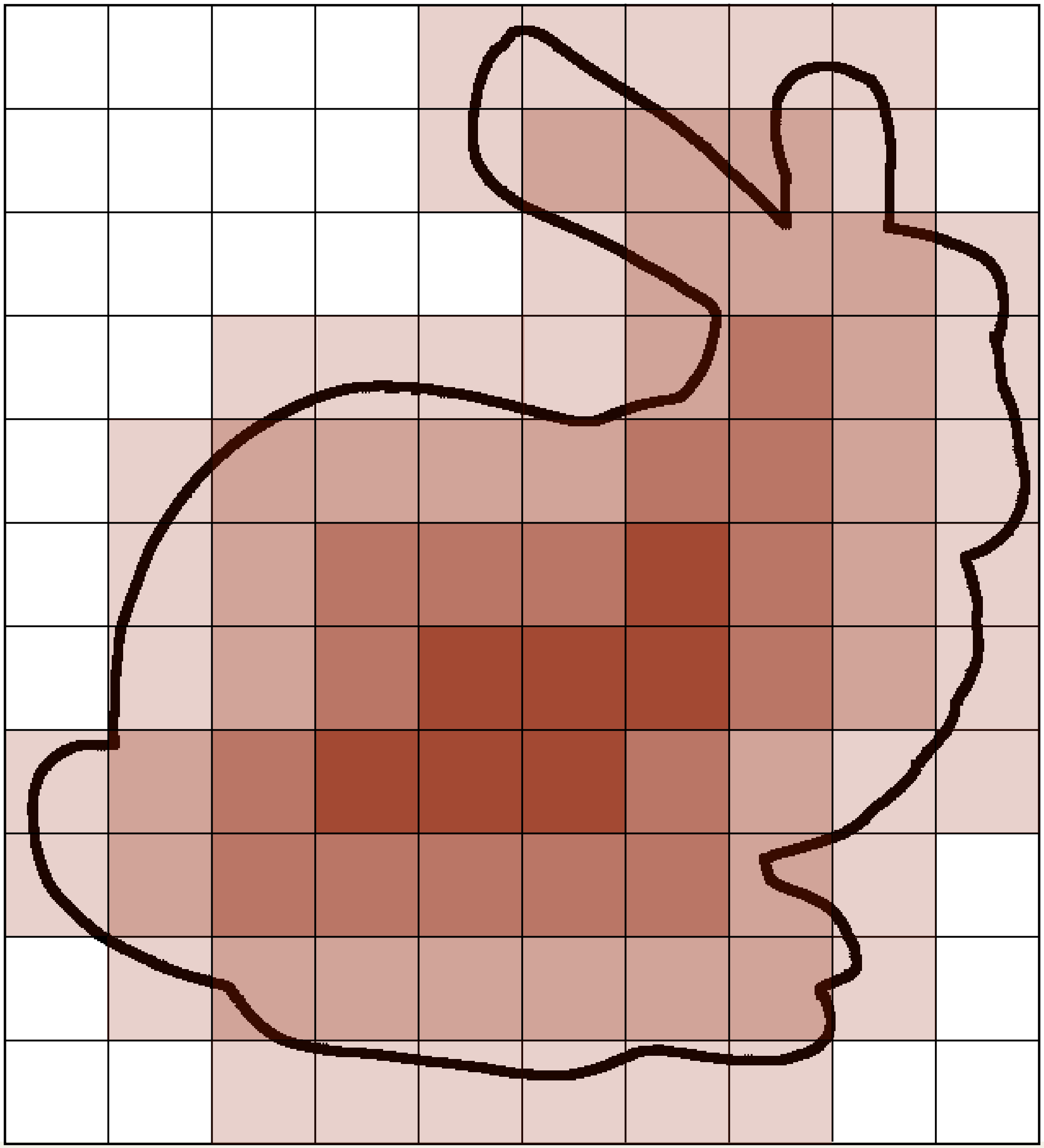

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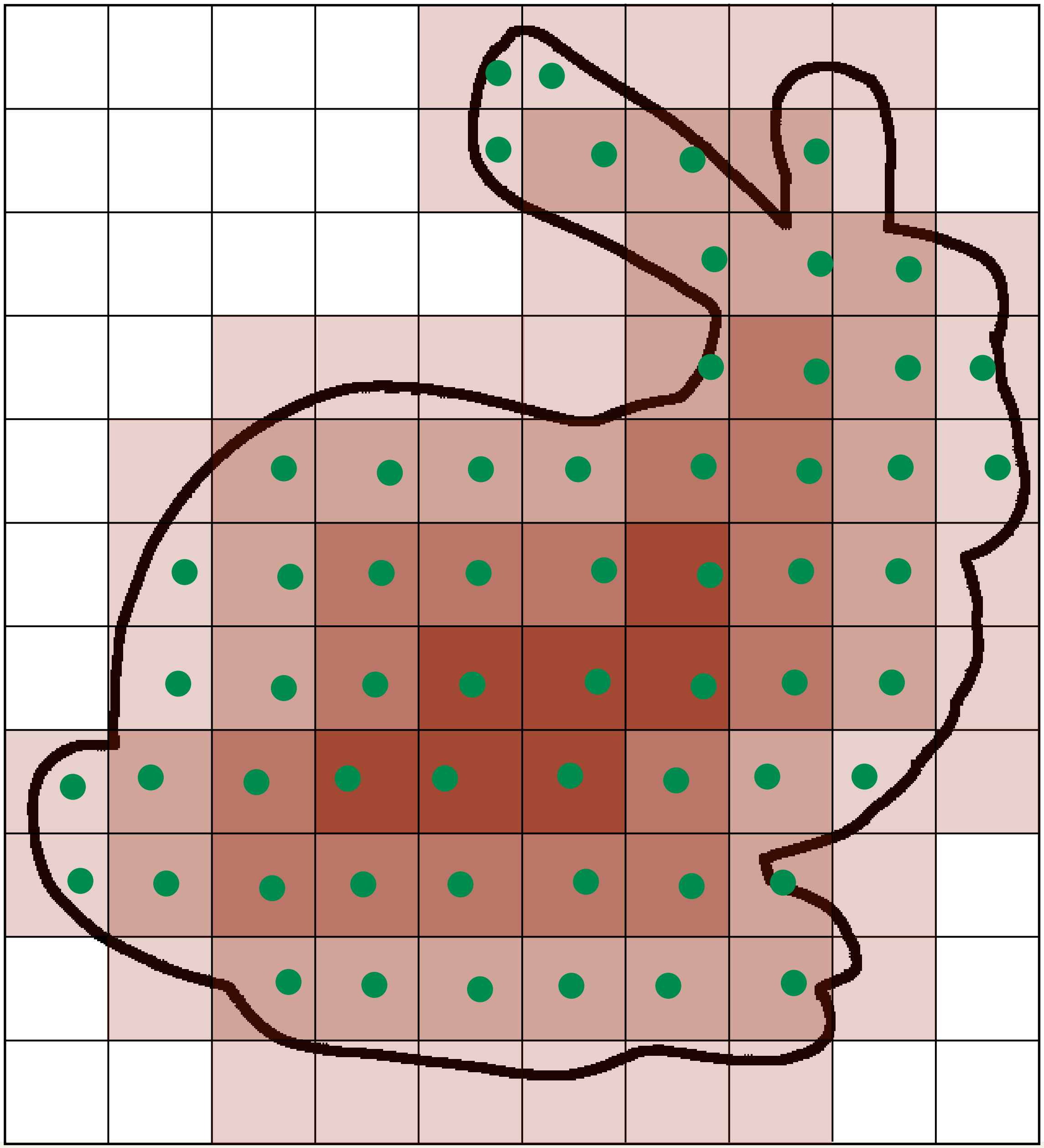

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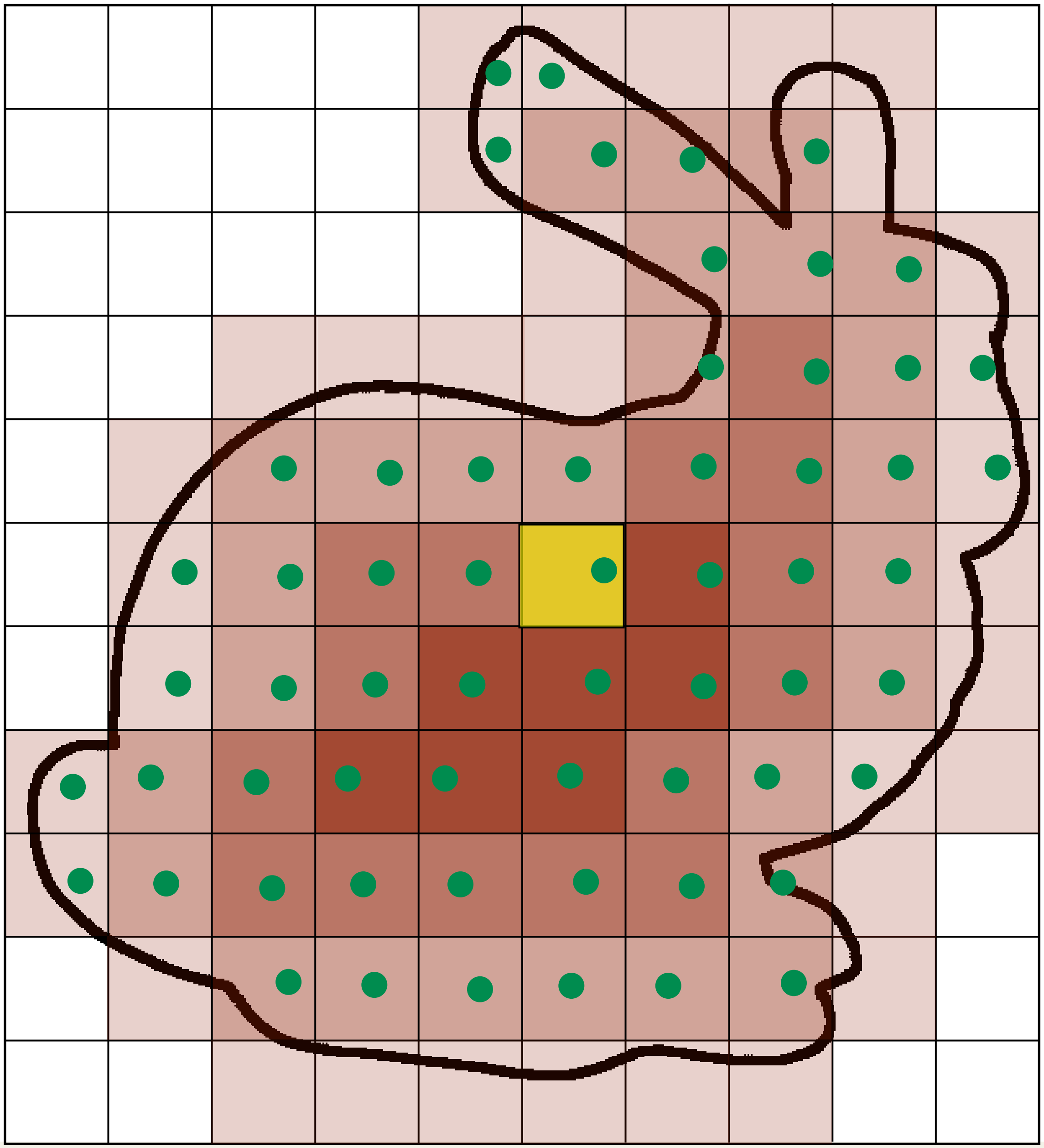

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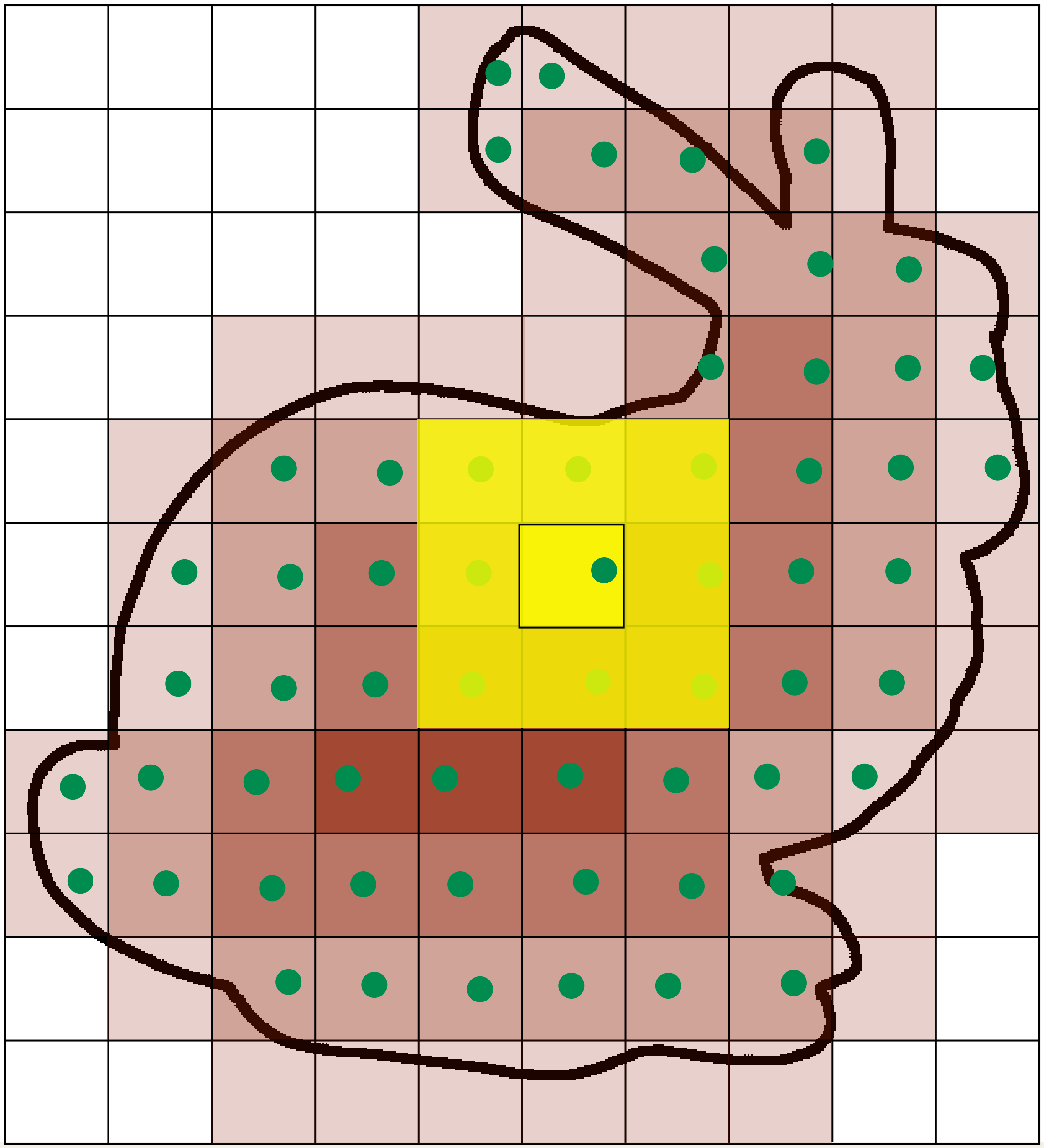

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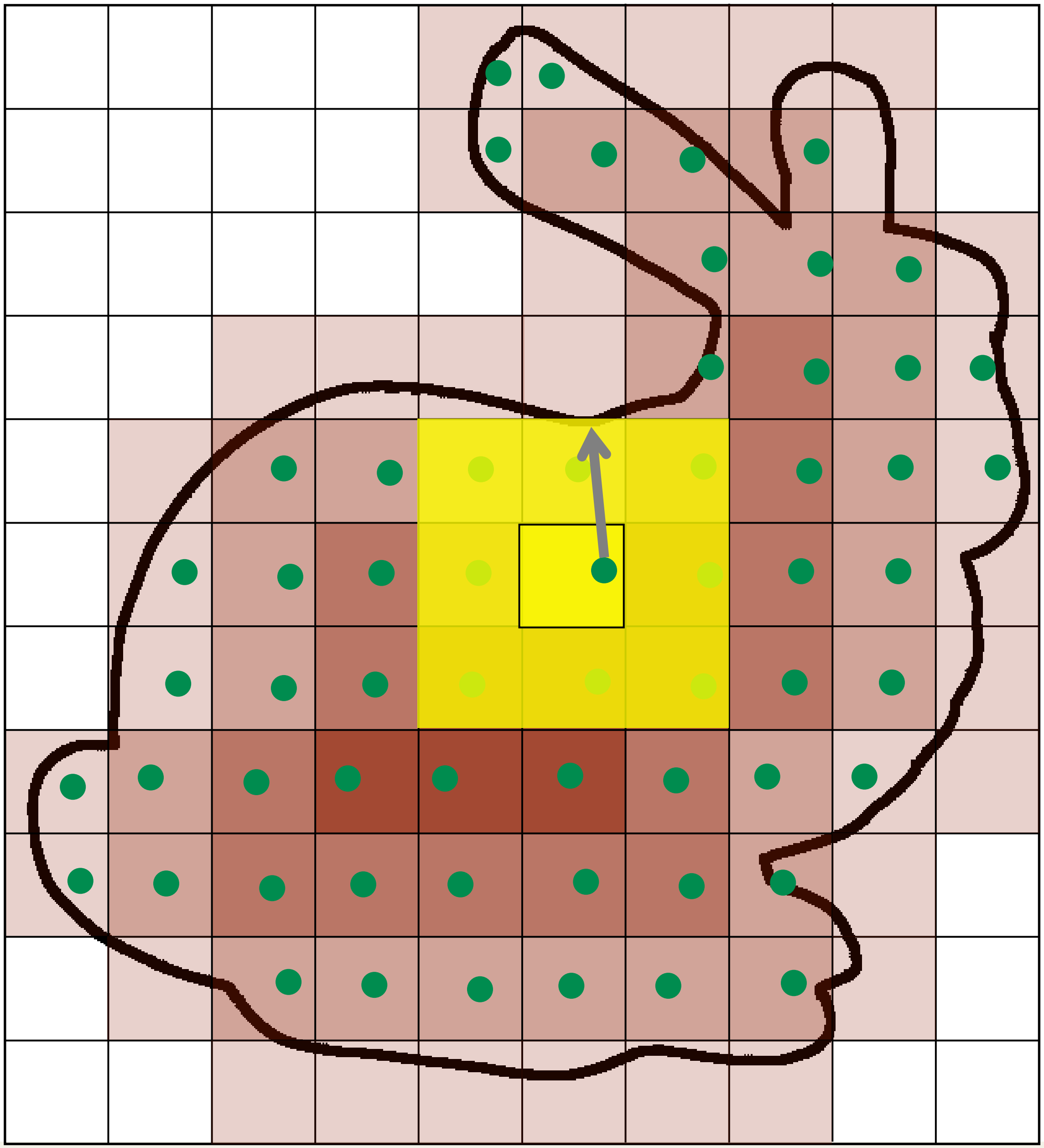

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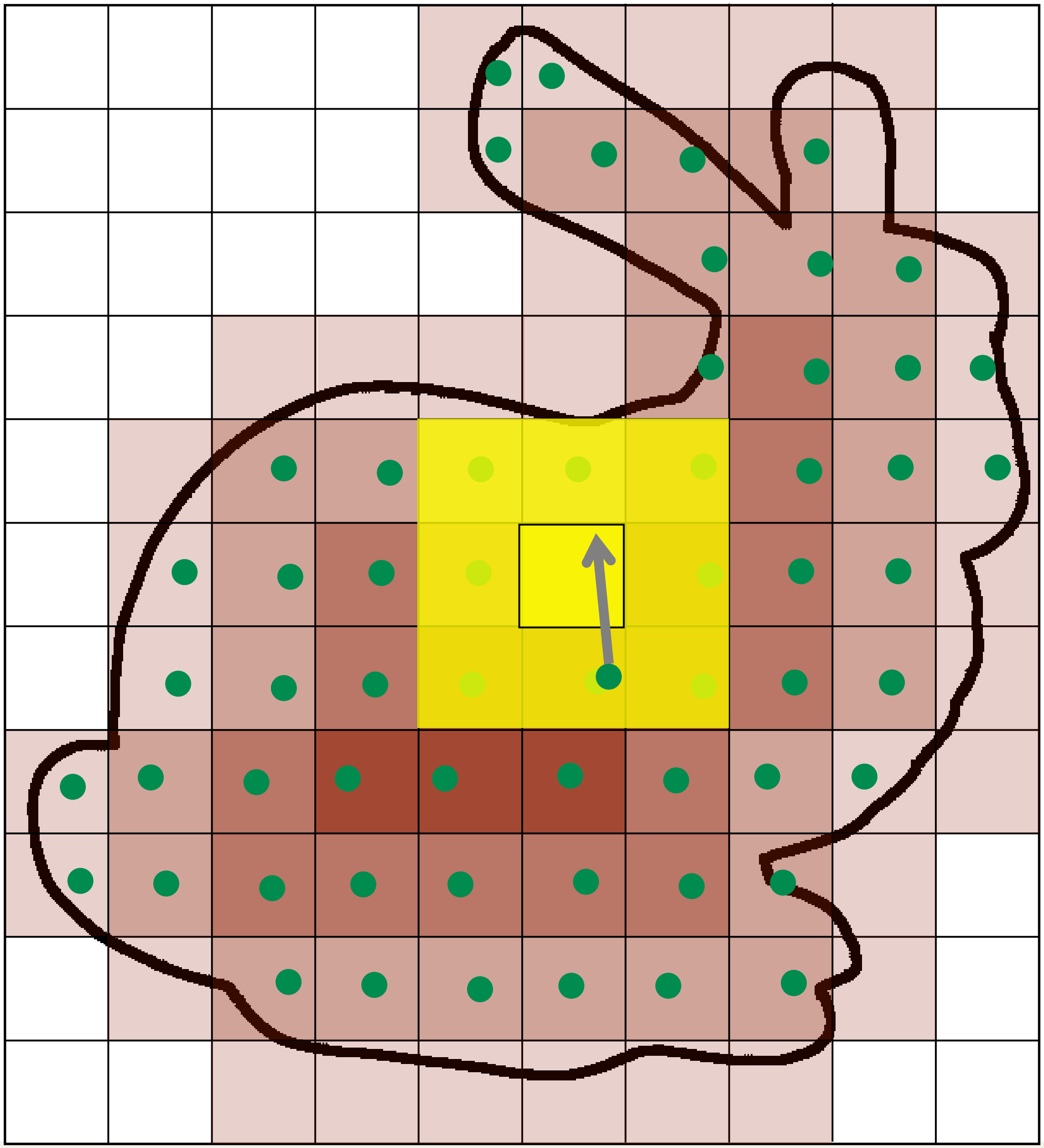

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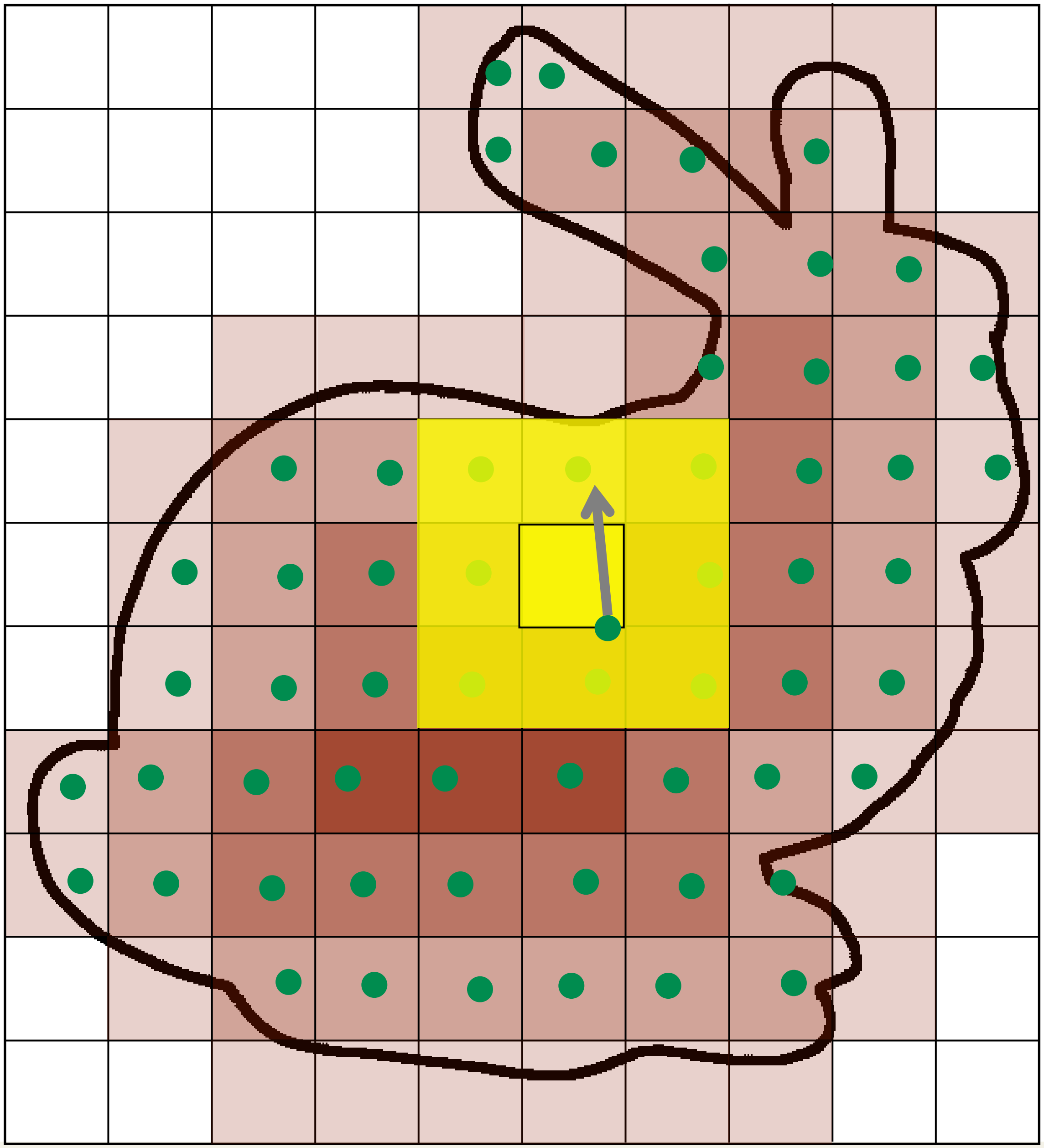

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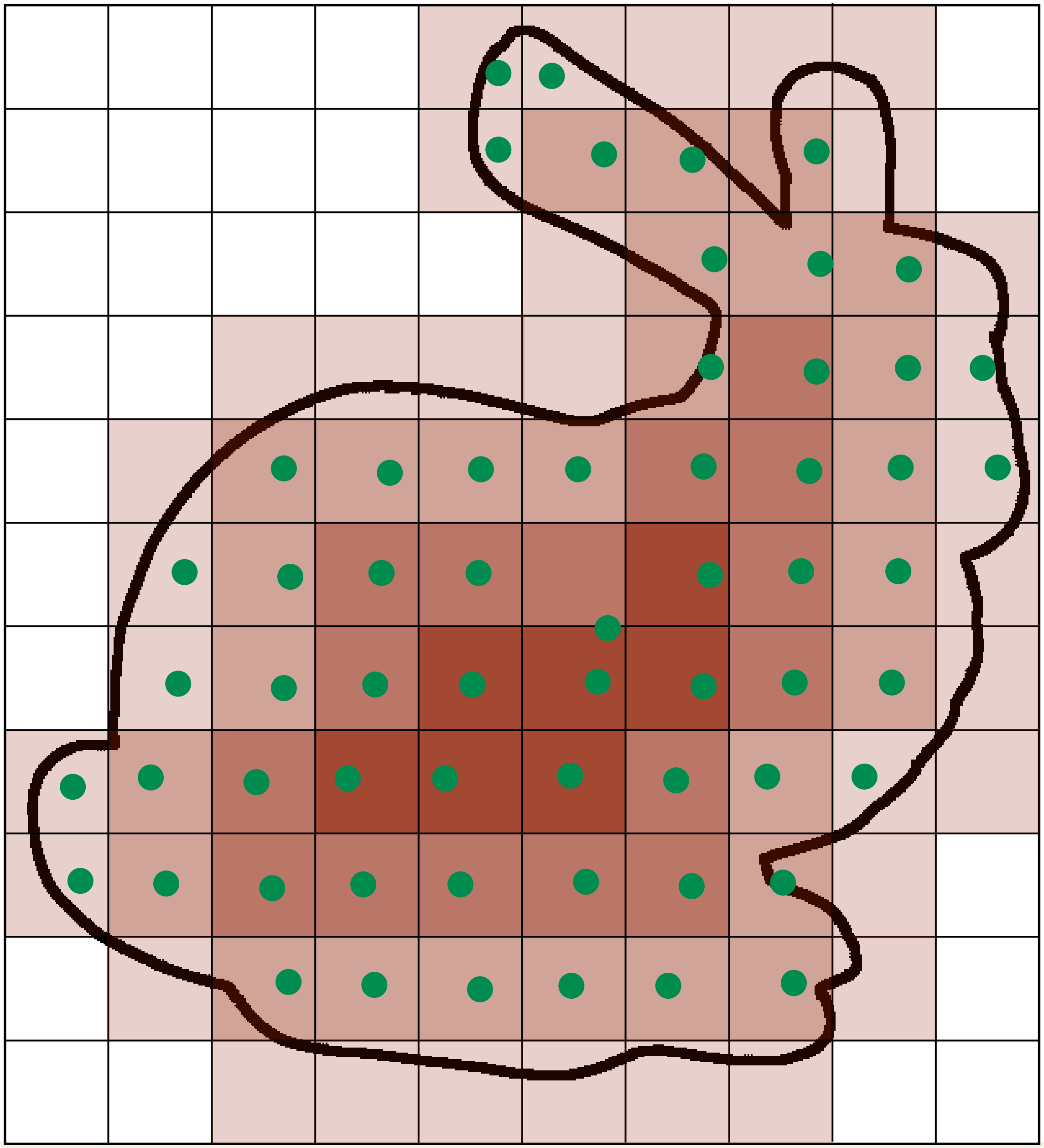

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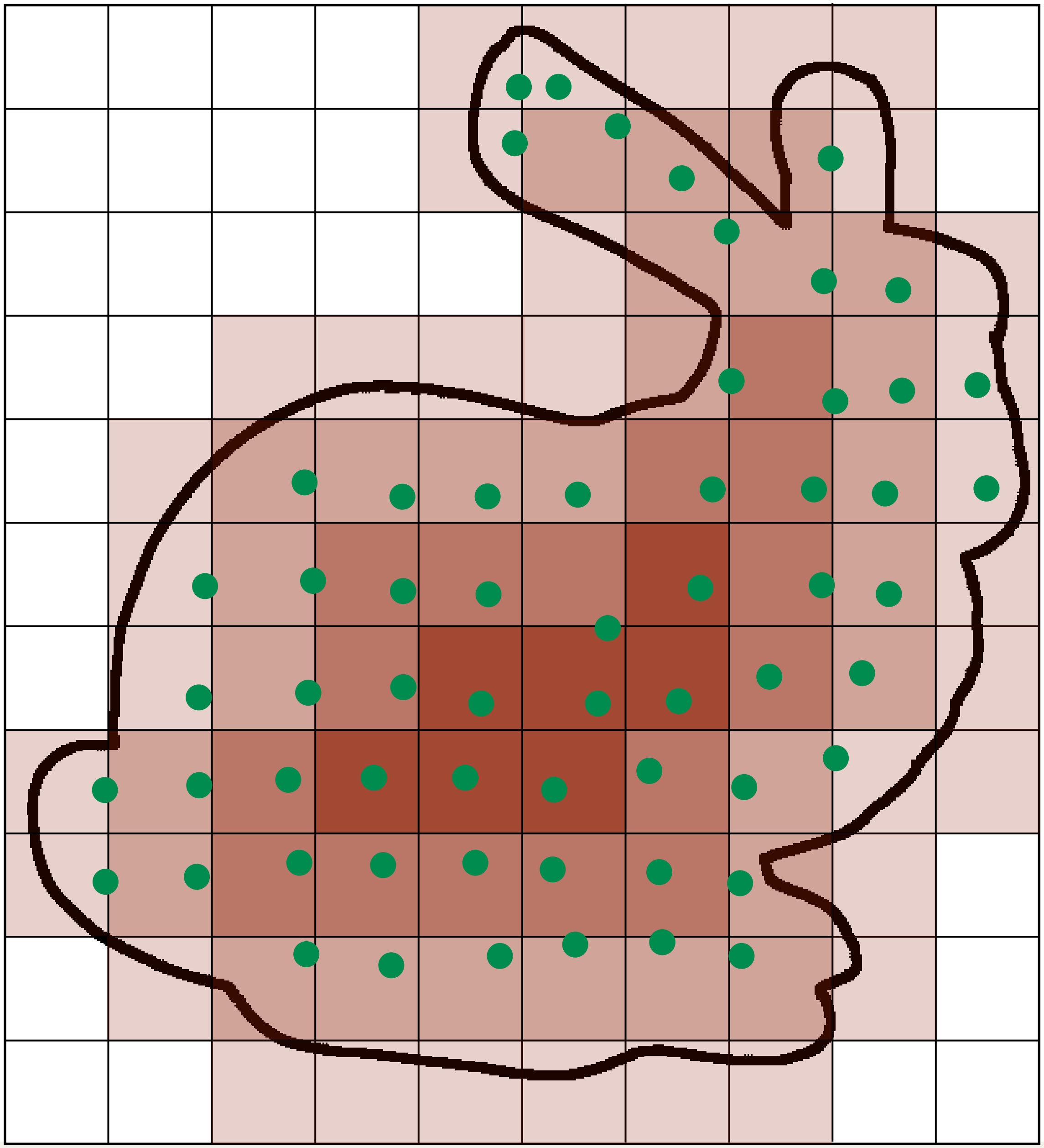
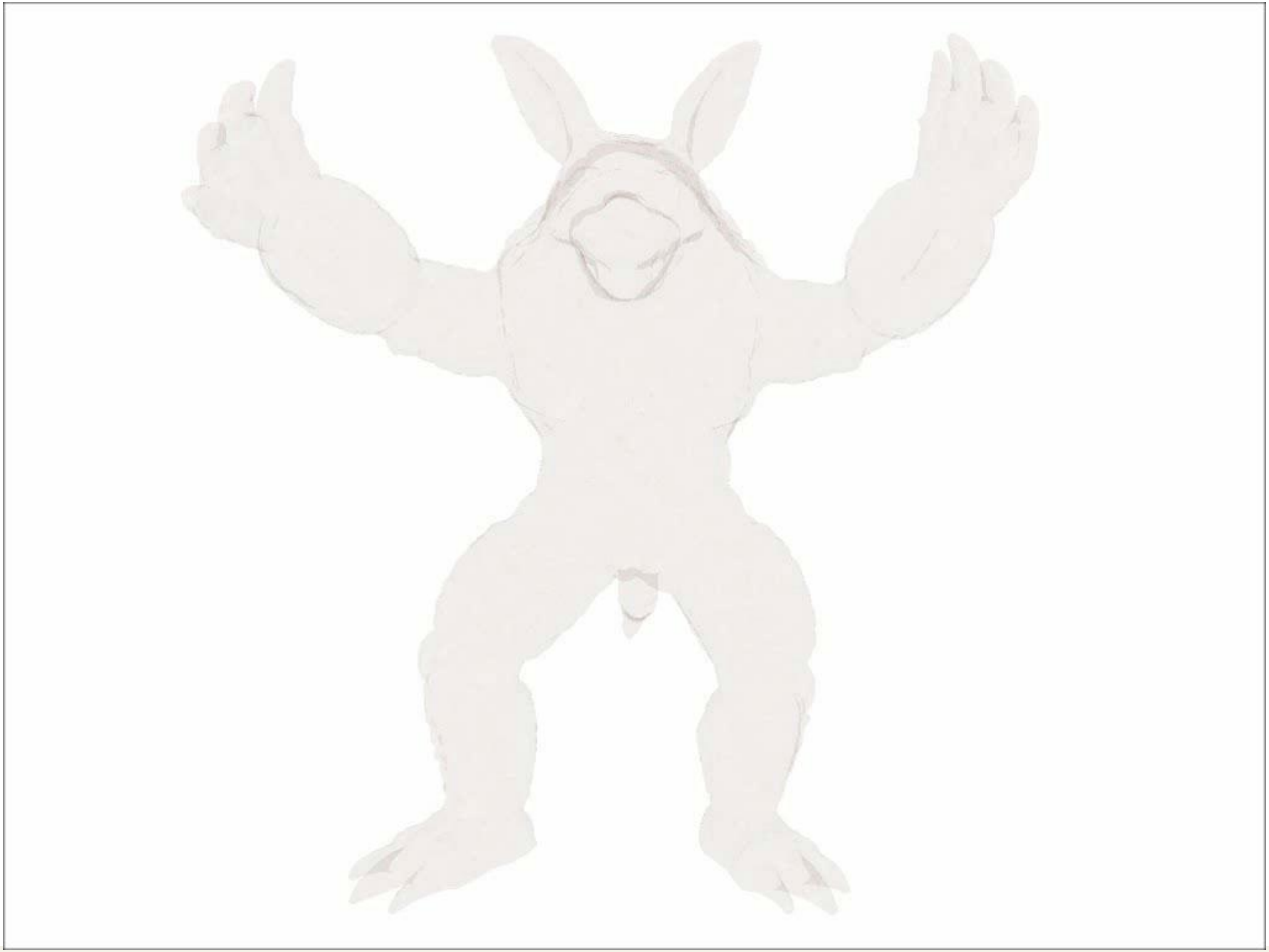


Parallel Protosphere



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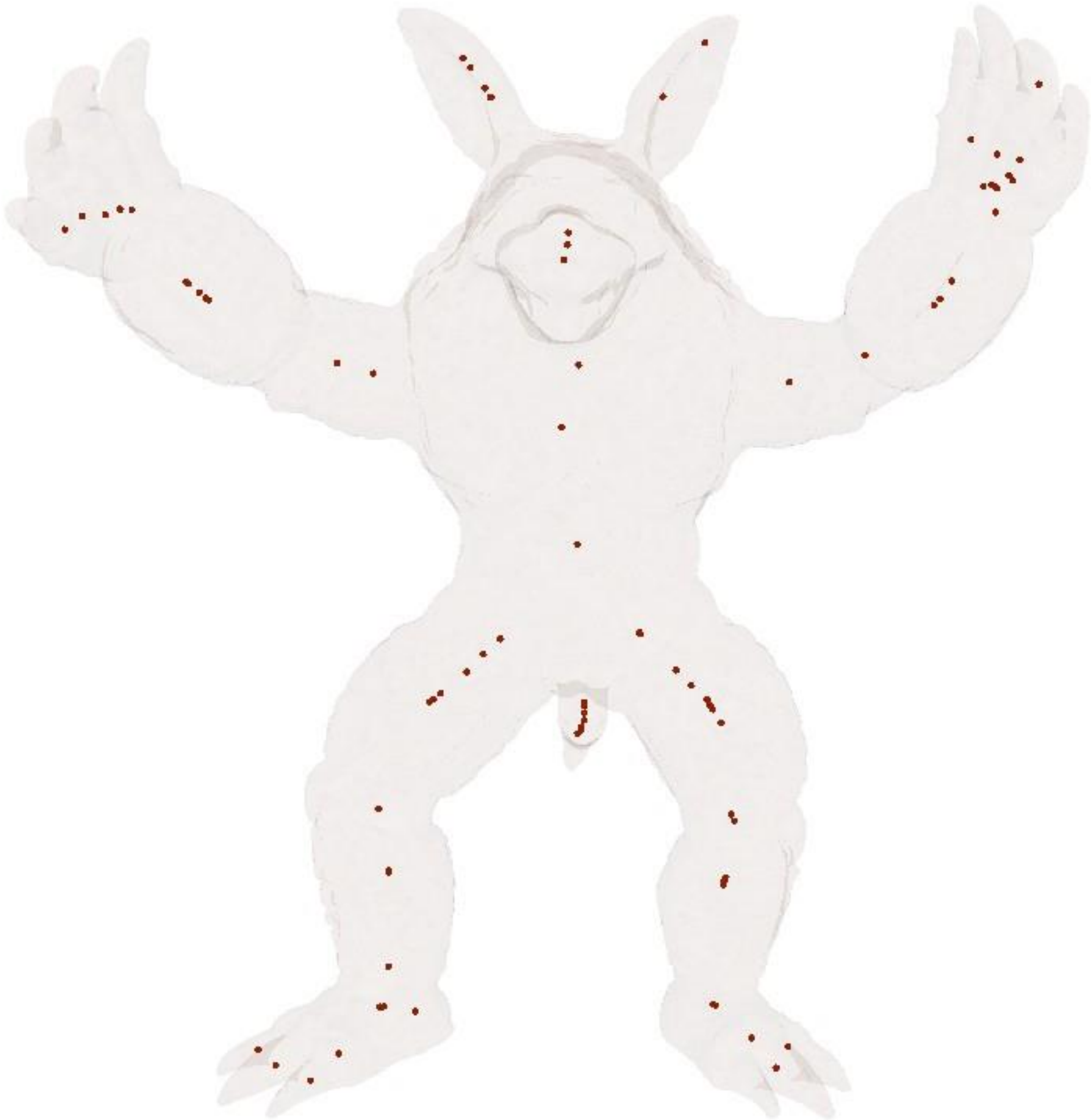

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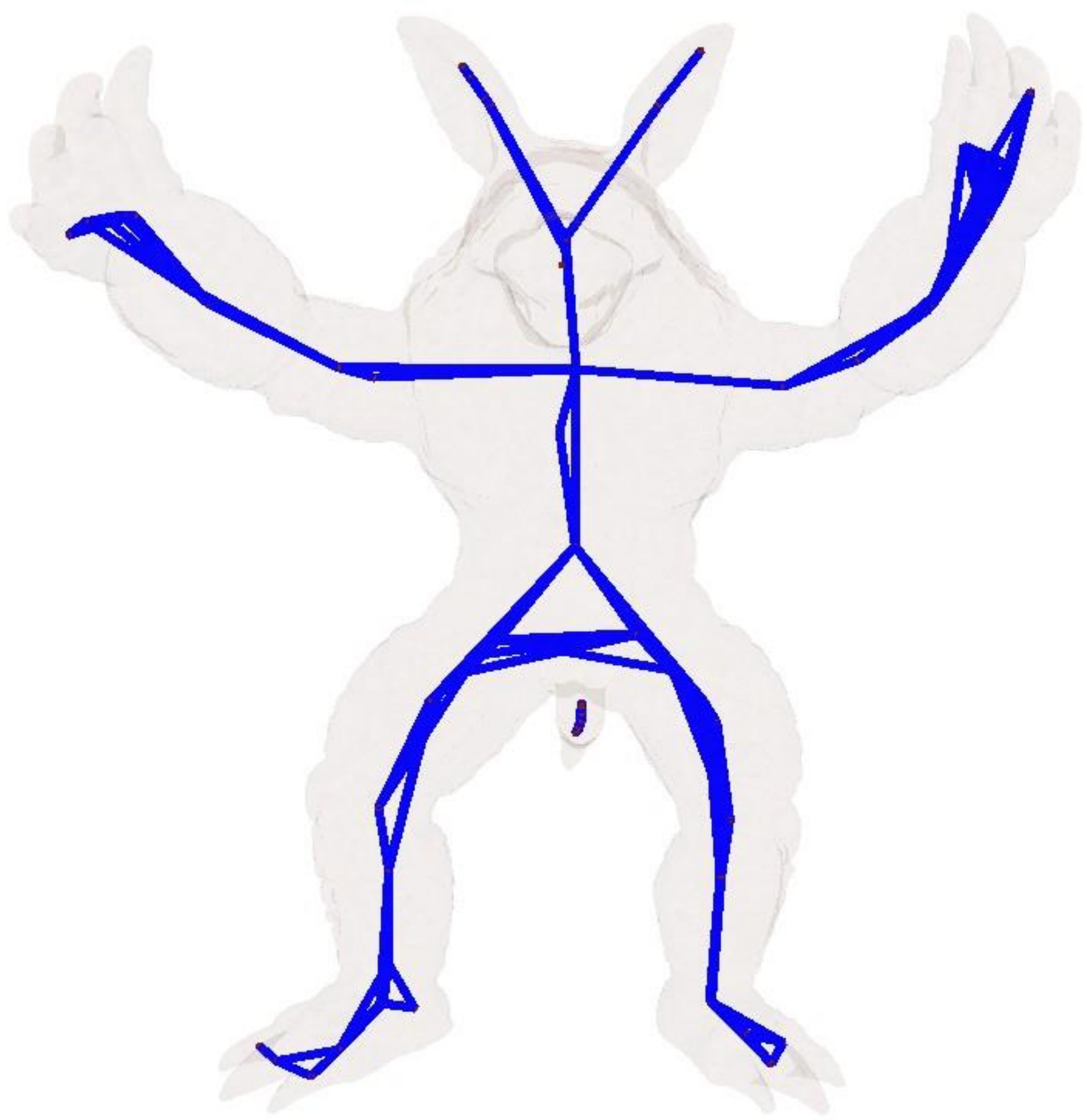
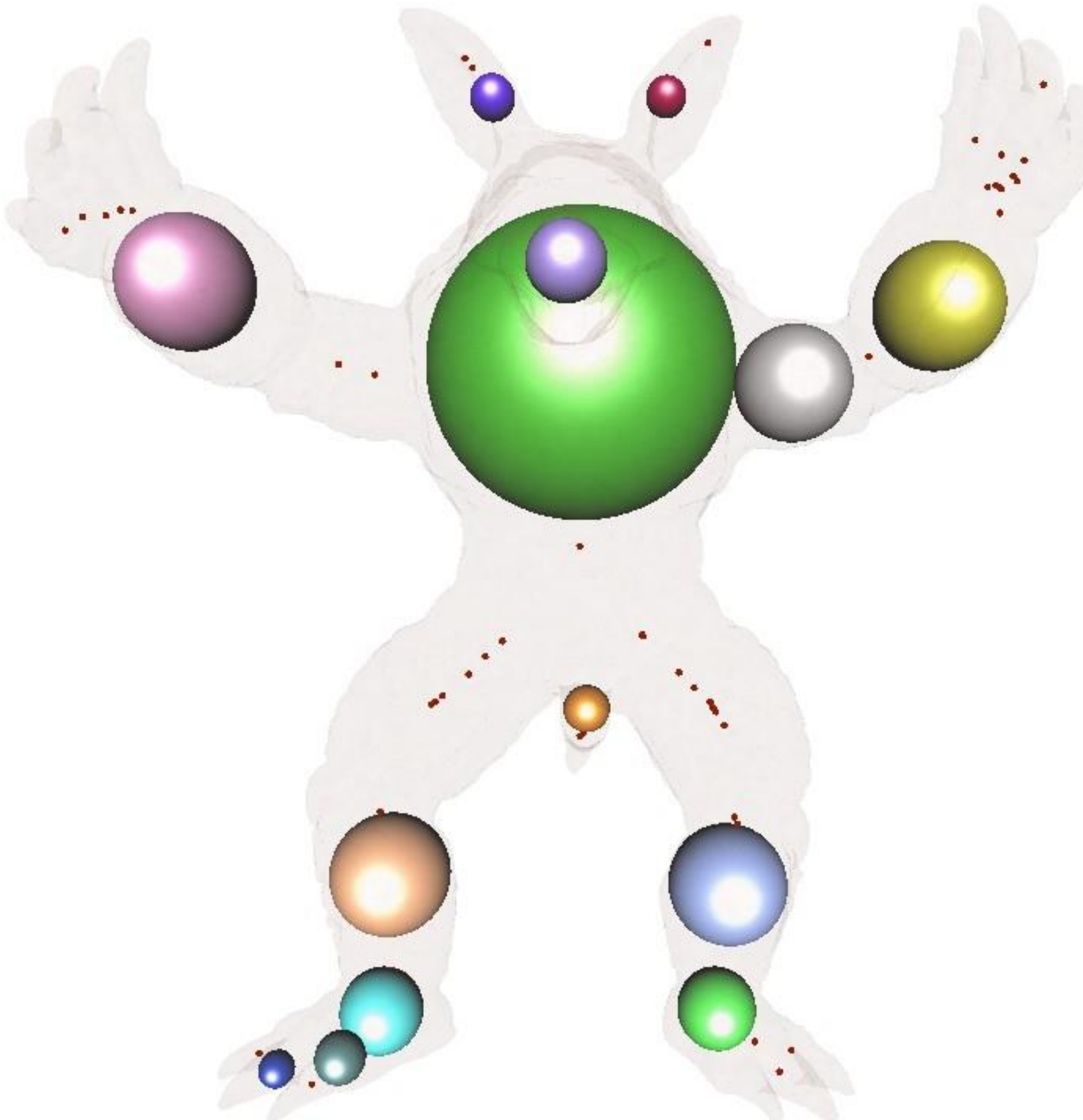


Parallel Protosphere

Parallel Protosphere



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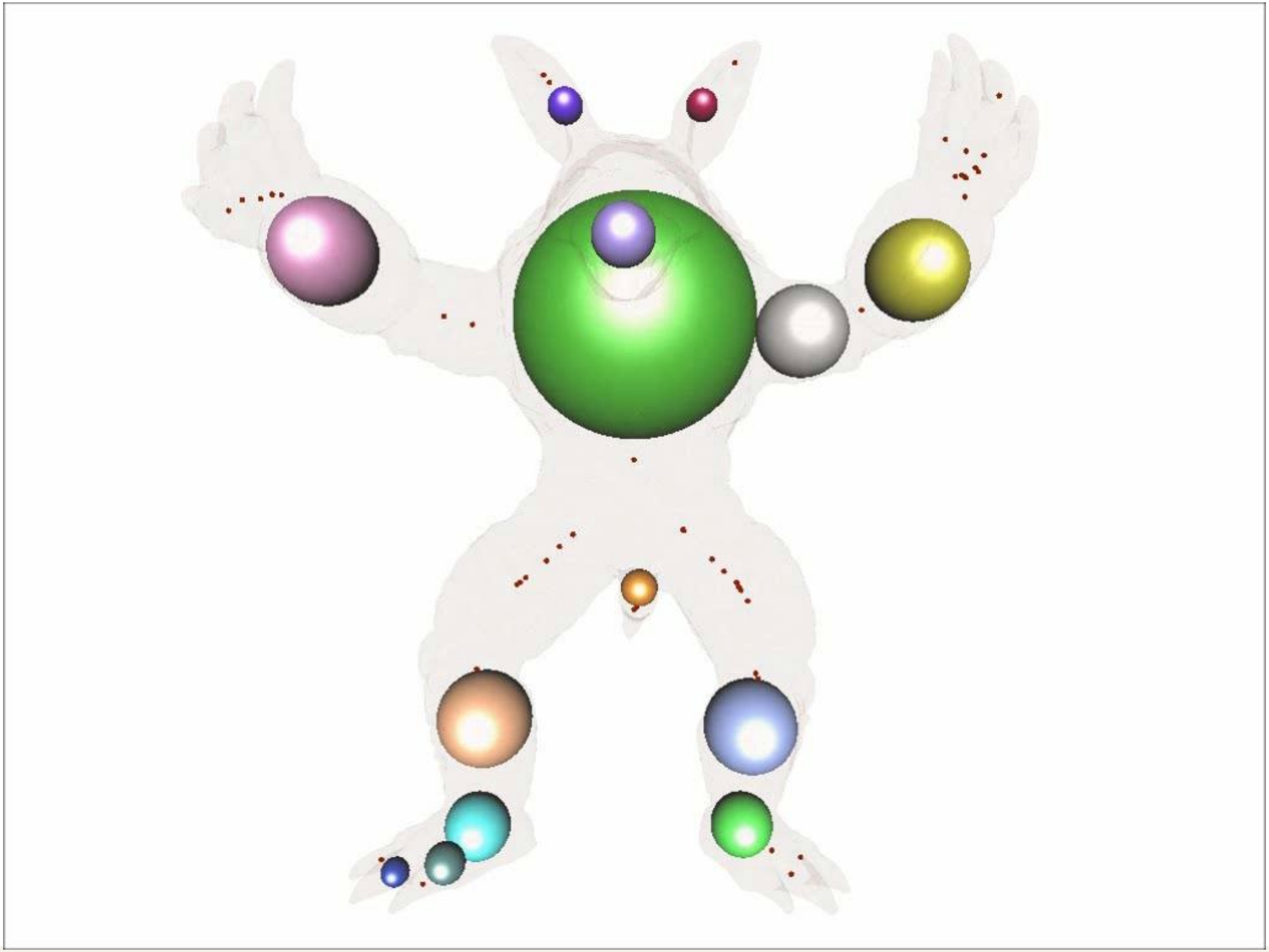
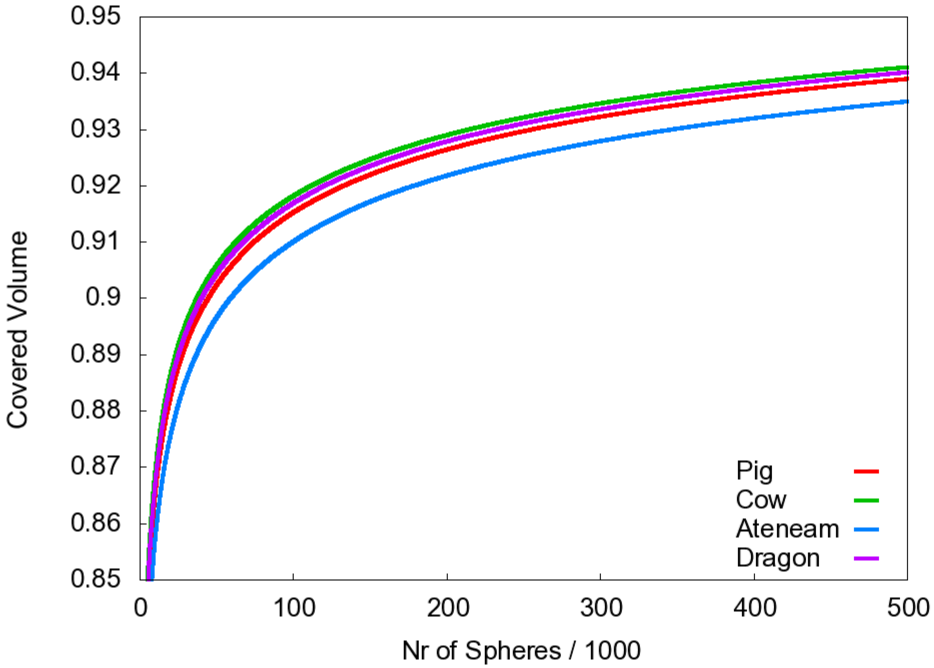
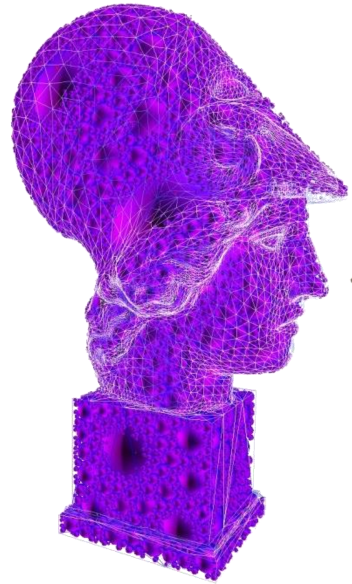
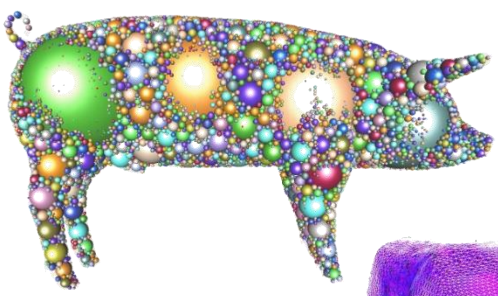
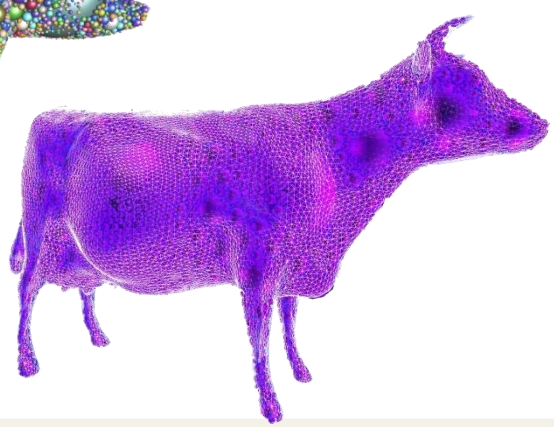


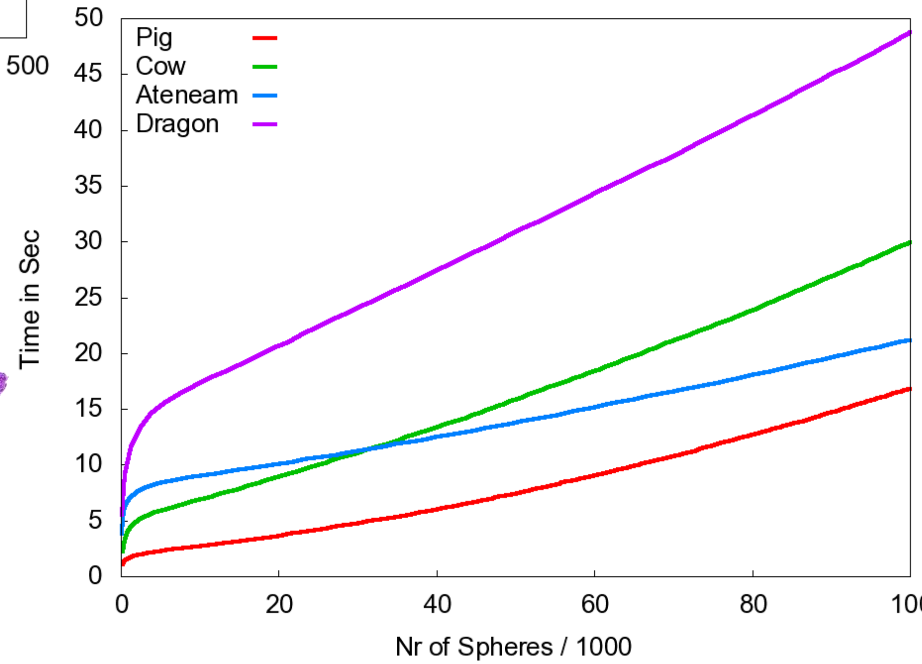
Results





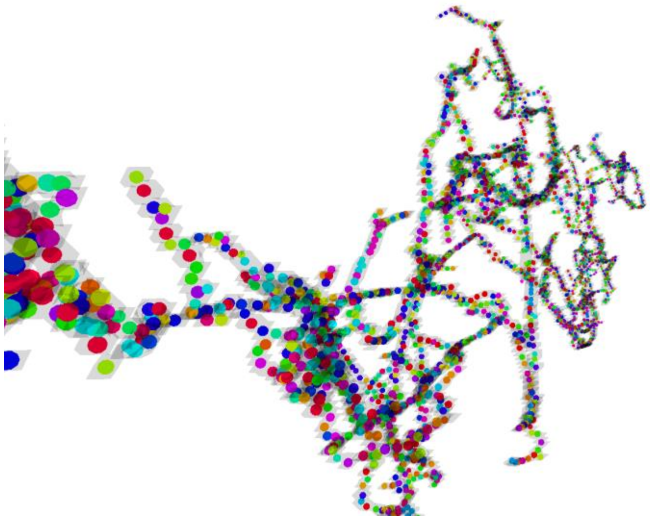






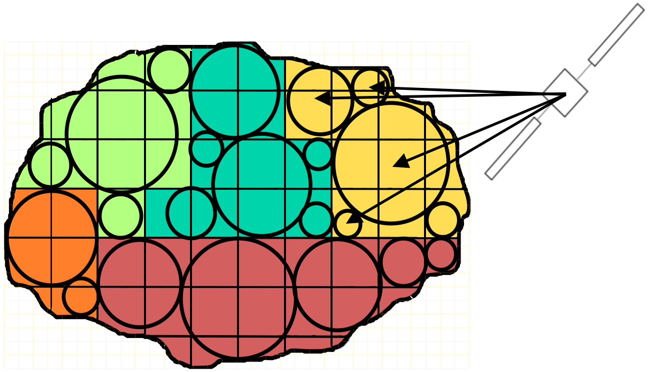
Nvidia Geforce GTX 480

Further Applications of our Sphere Packings

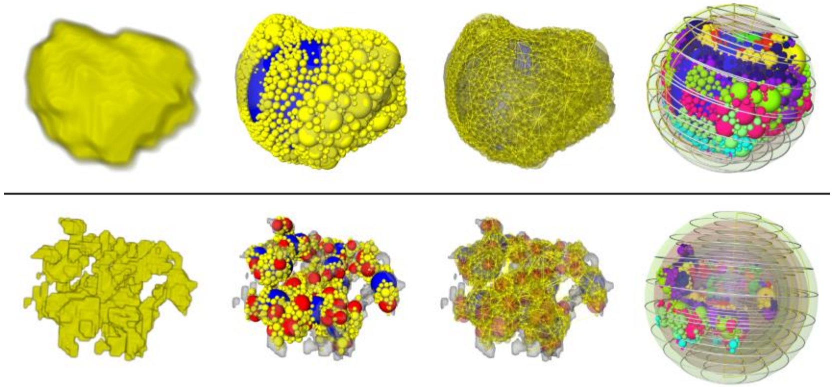


Material Science

3D Printing

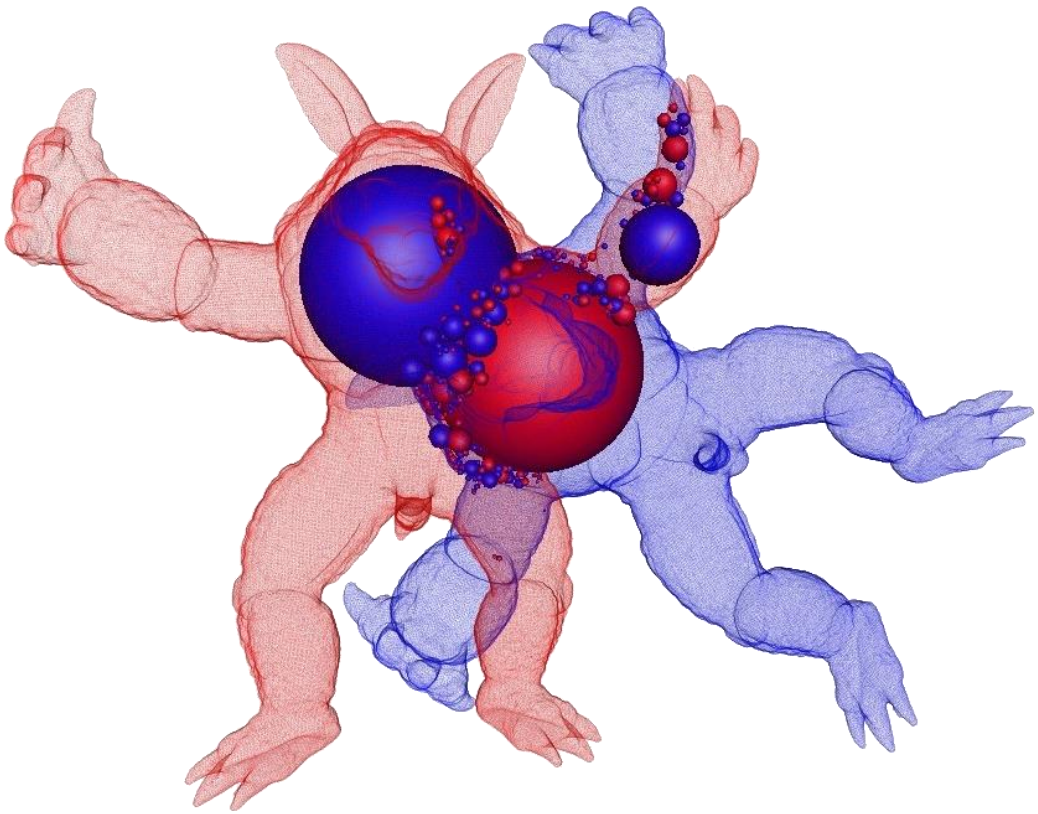


Gravitational Field of Asteroids

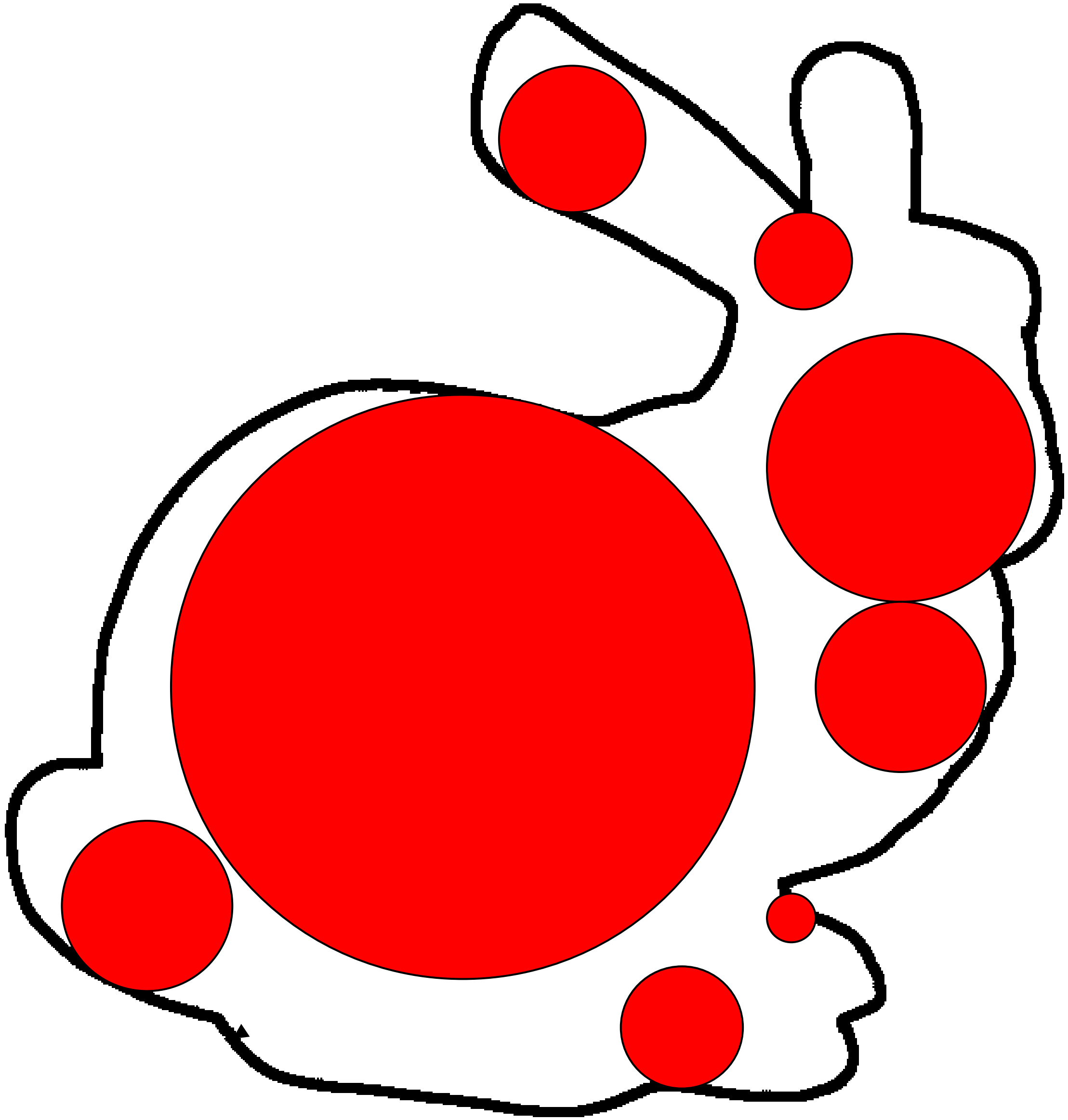


Classification of Carcinoms

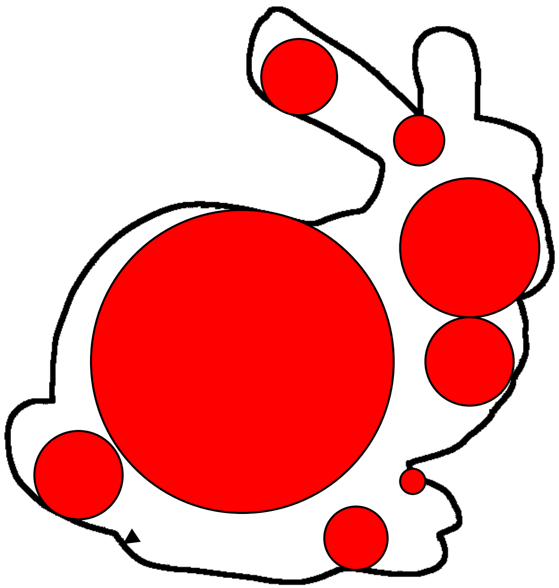
Back to Collision Detection



Hierachy Creation



Hierachy Creation



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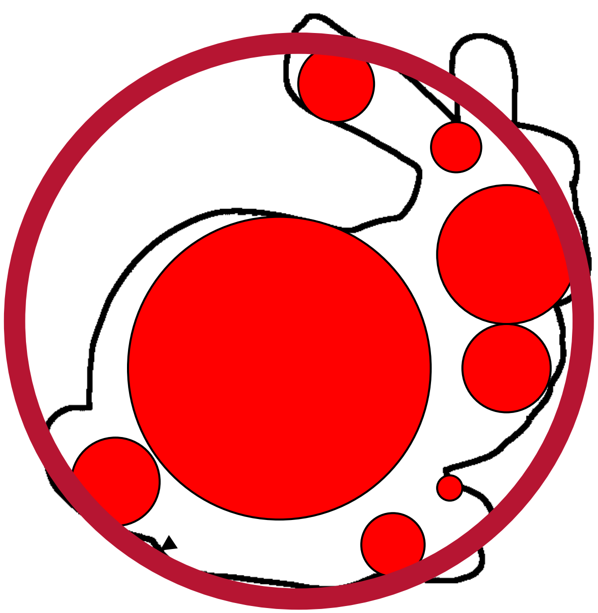

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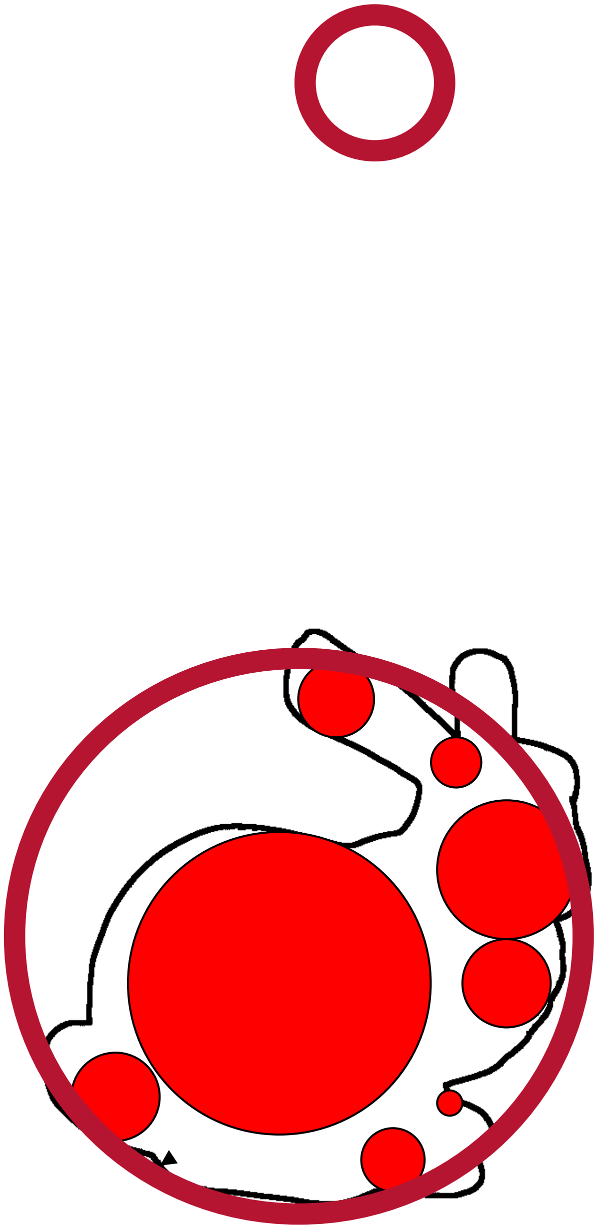

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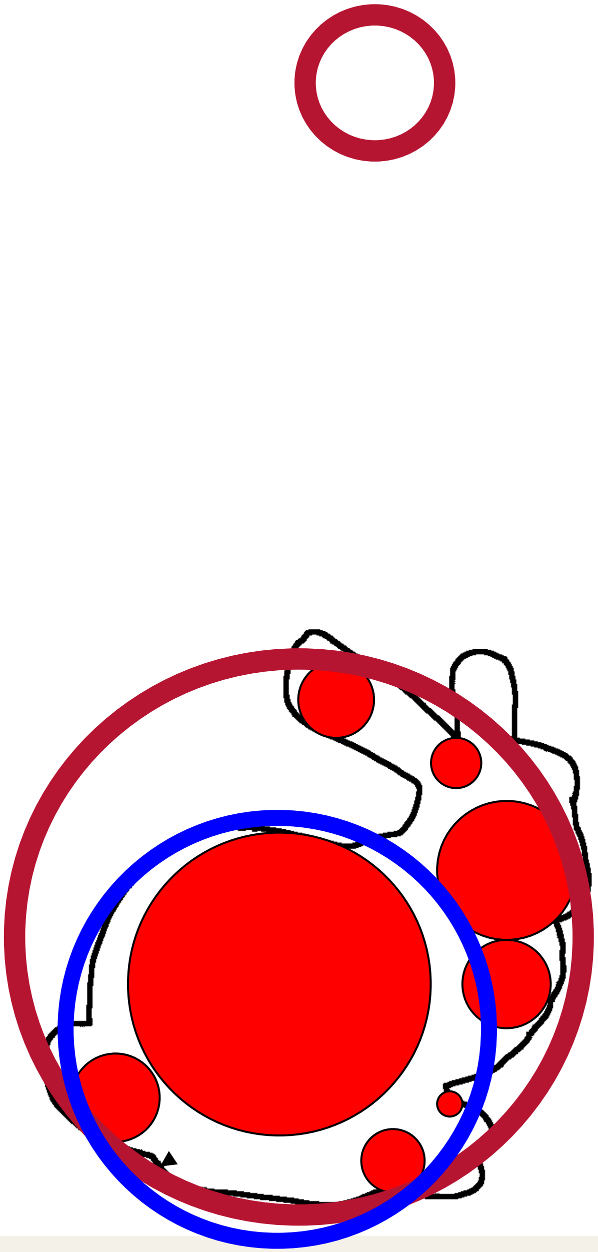

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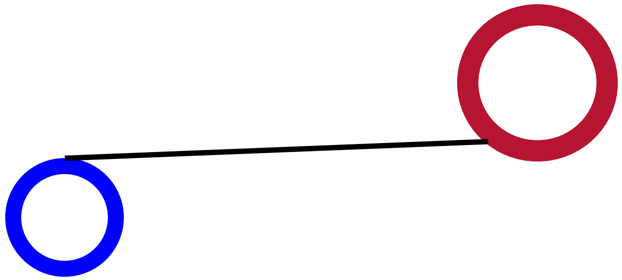

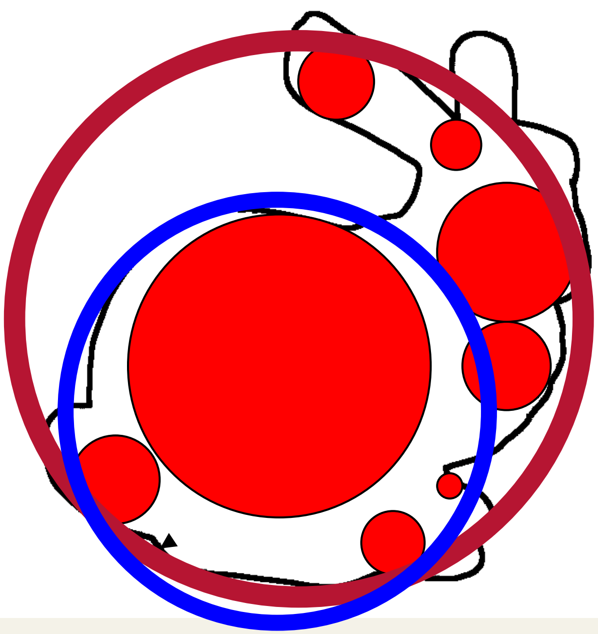

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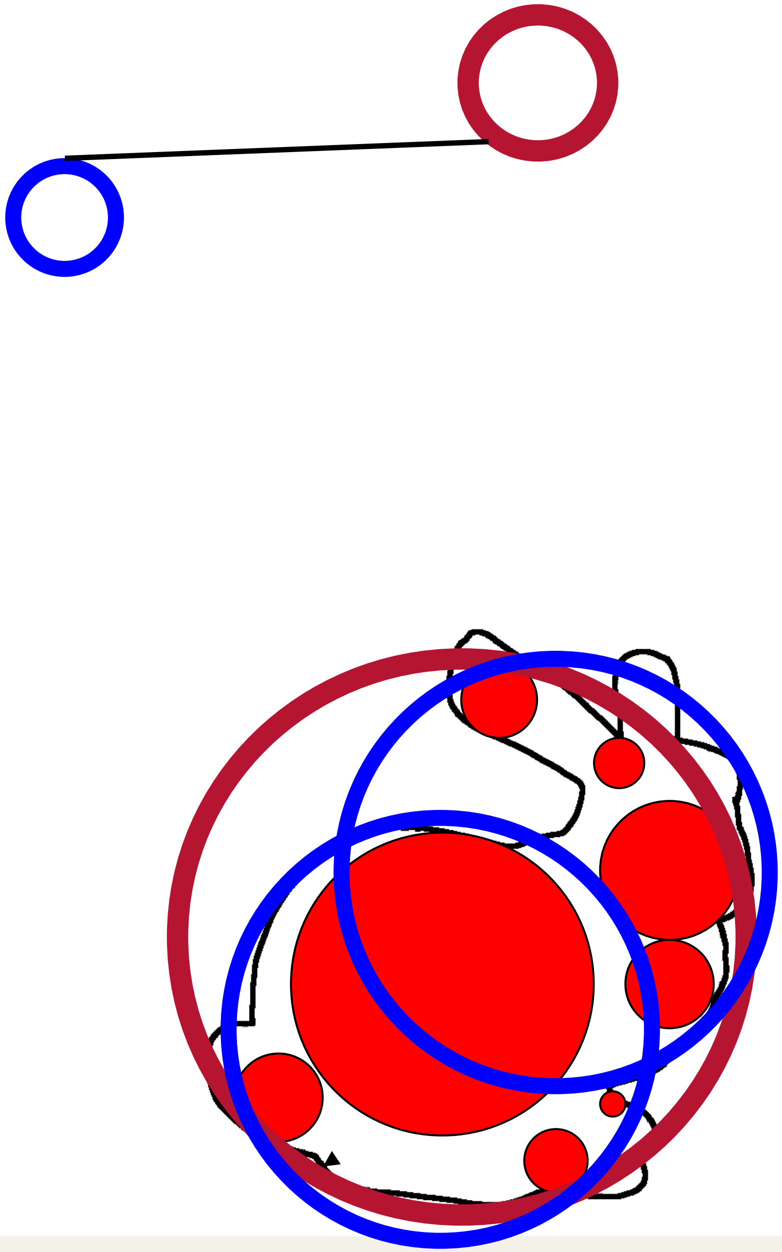

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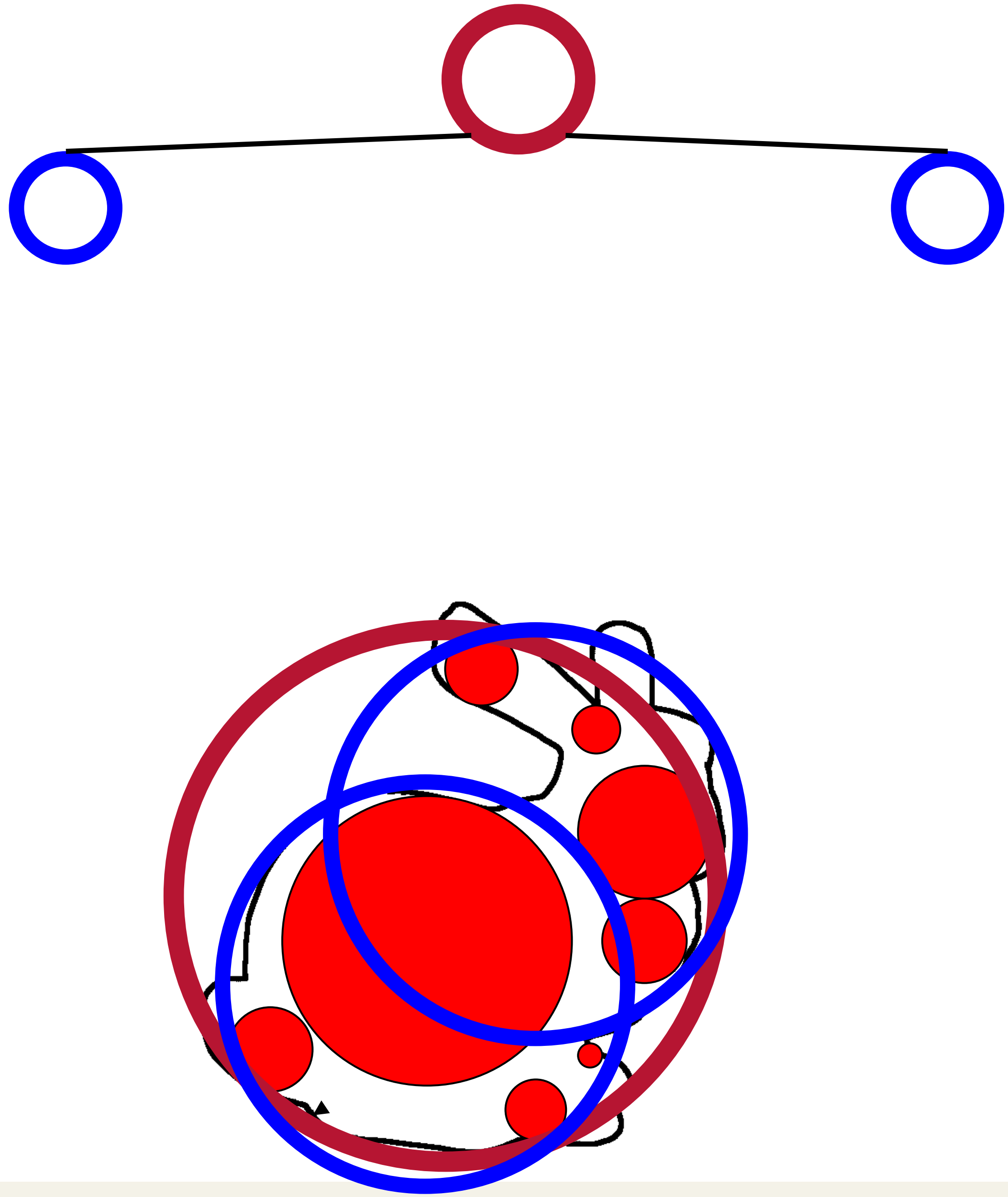

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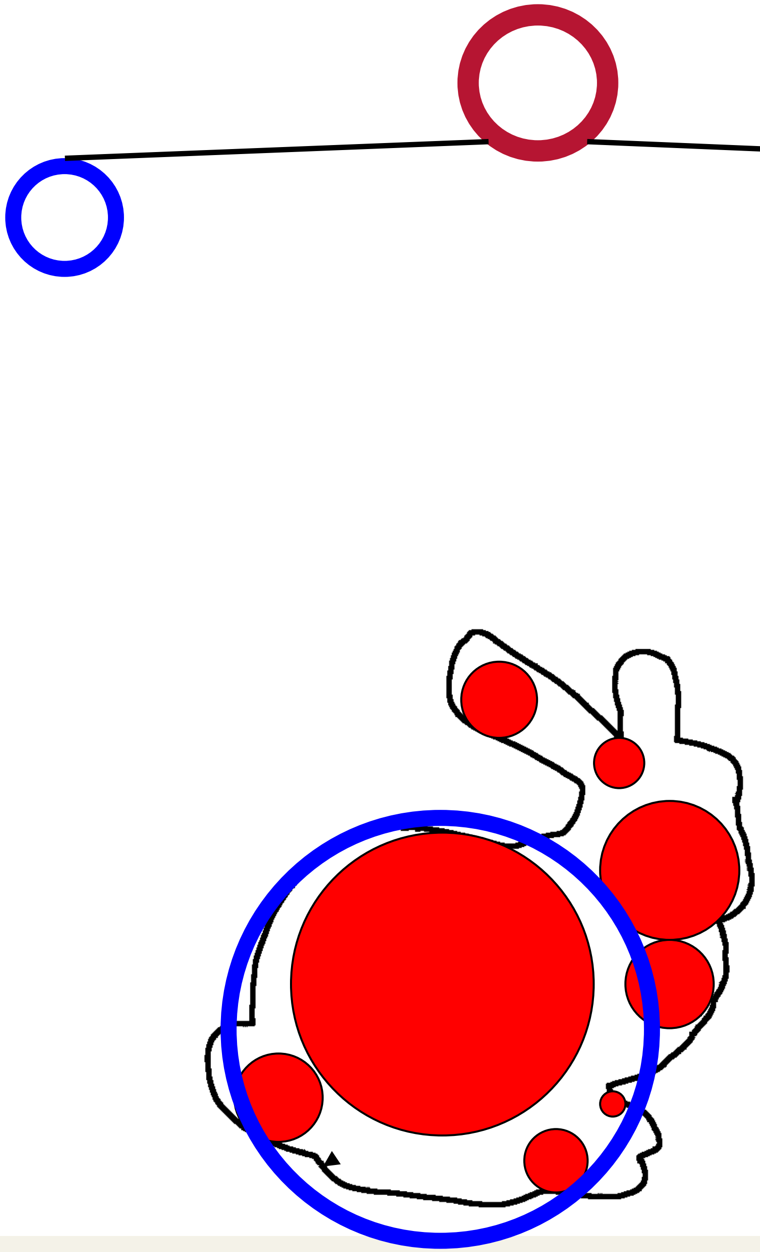

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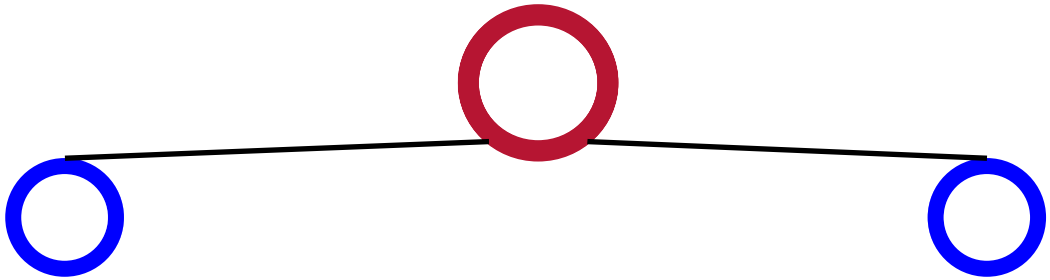

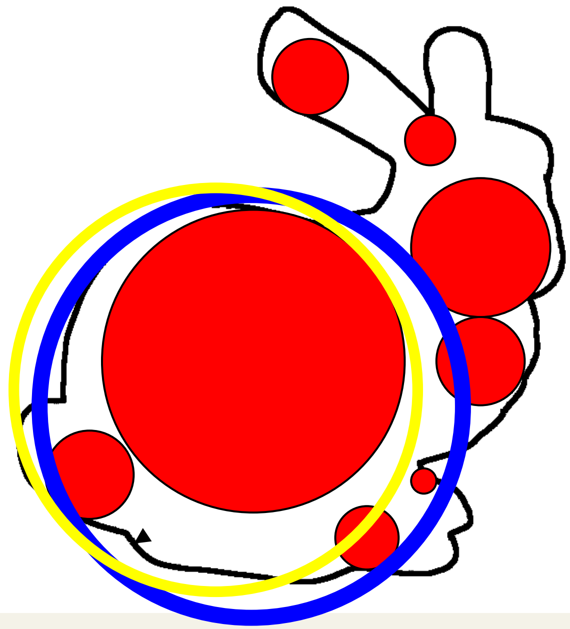

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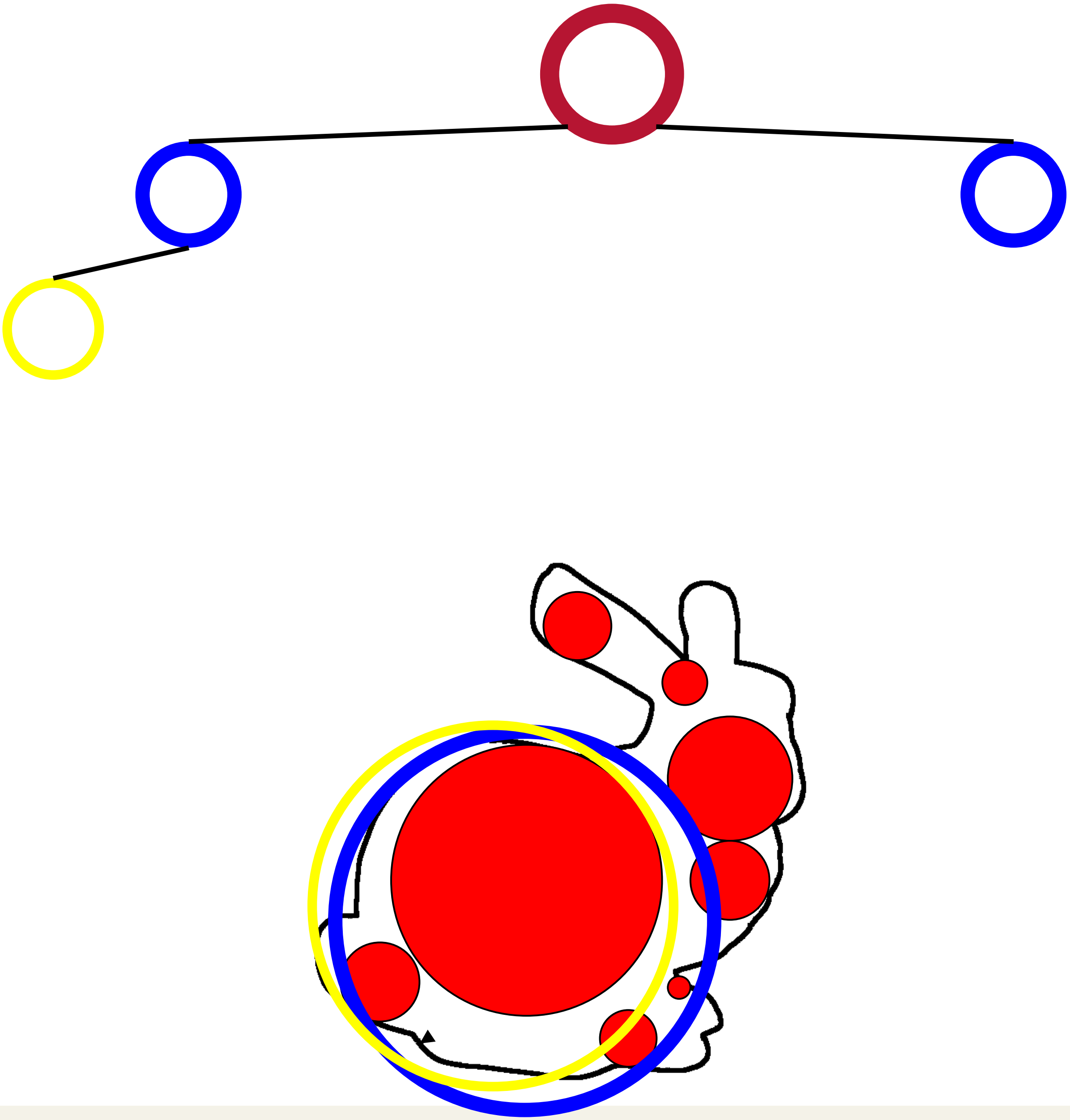

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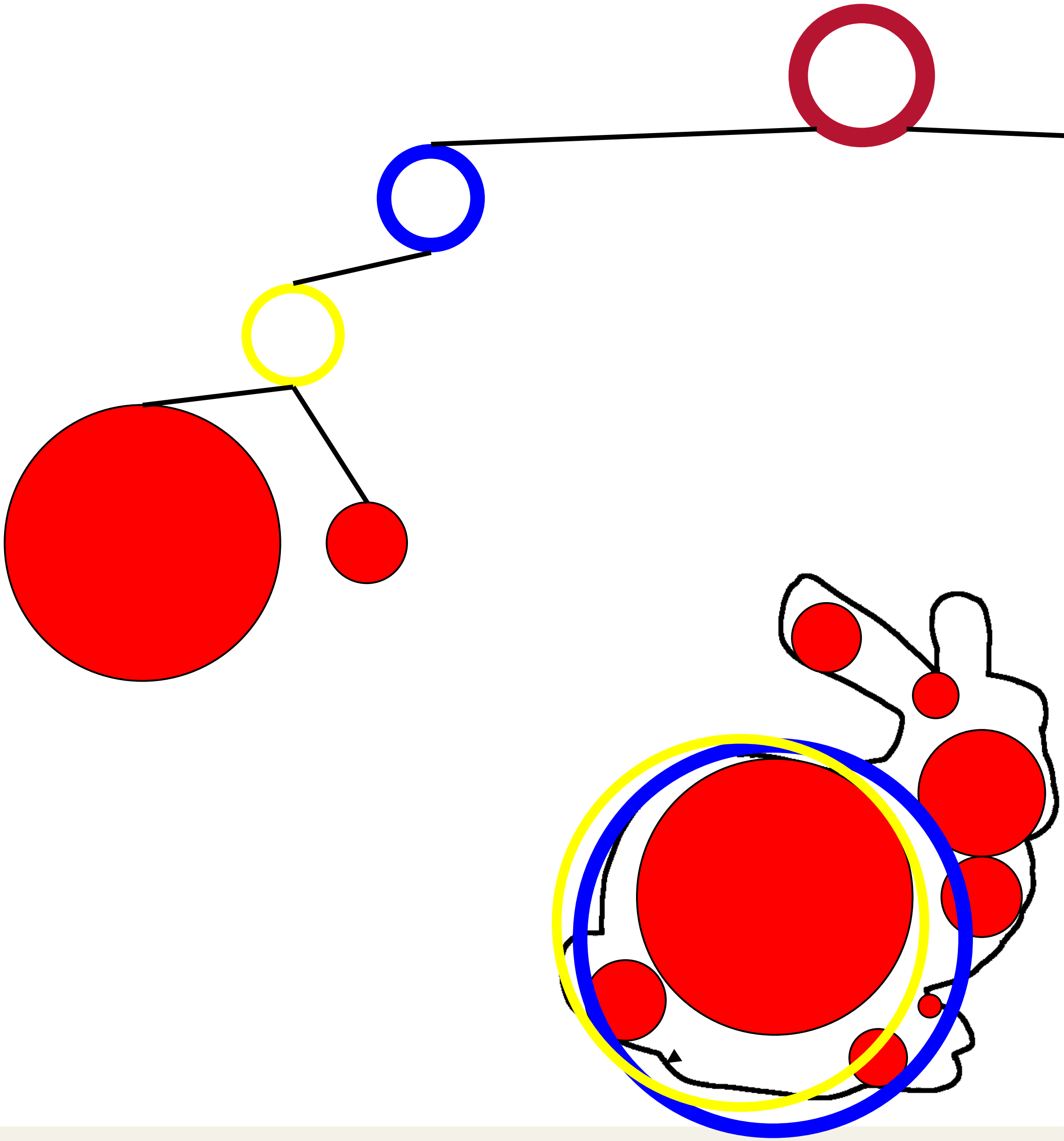

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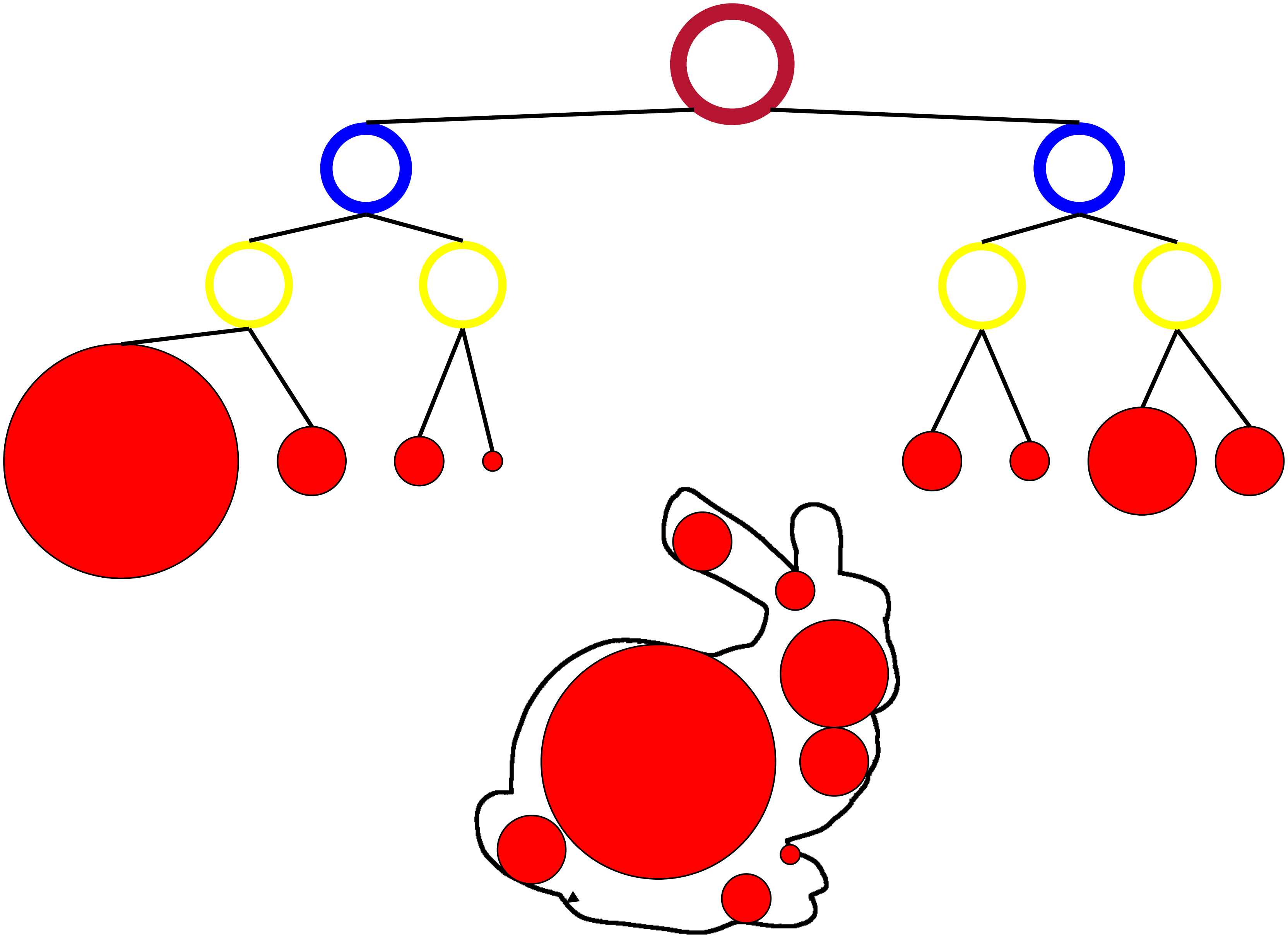
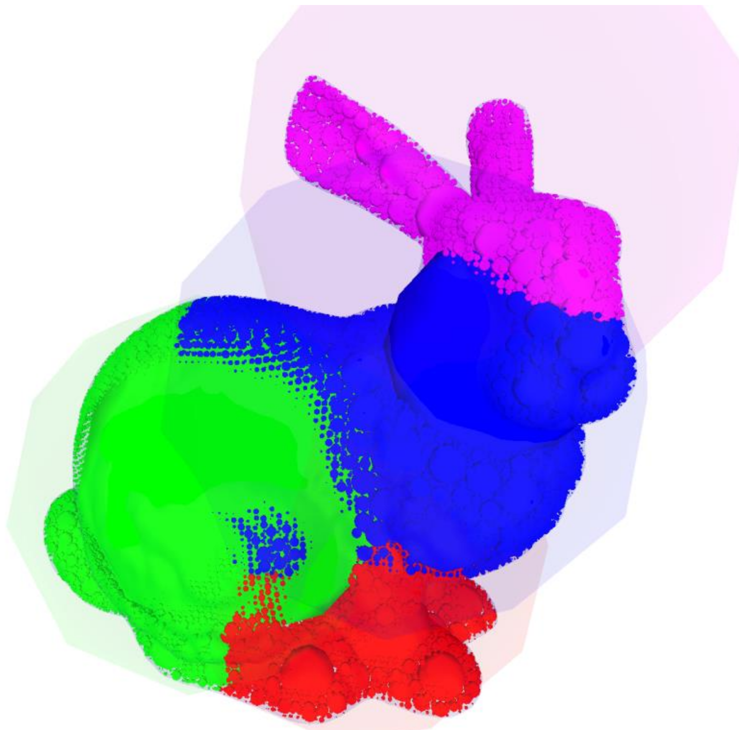
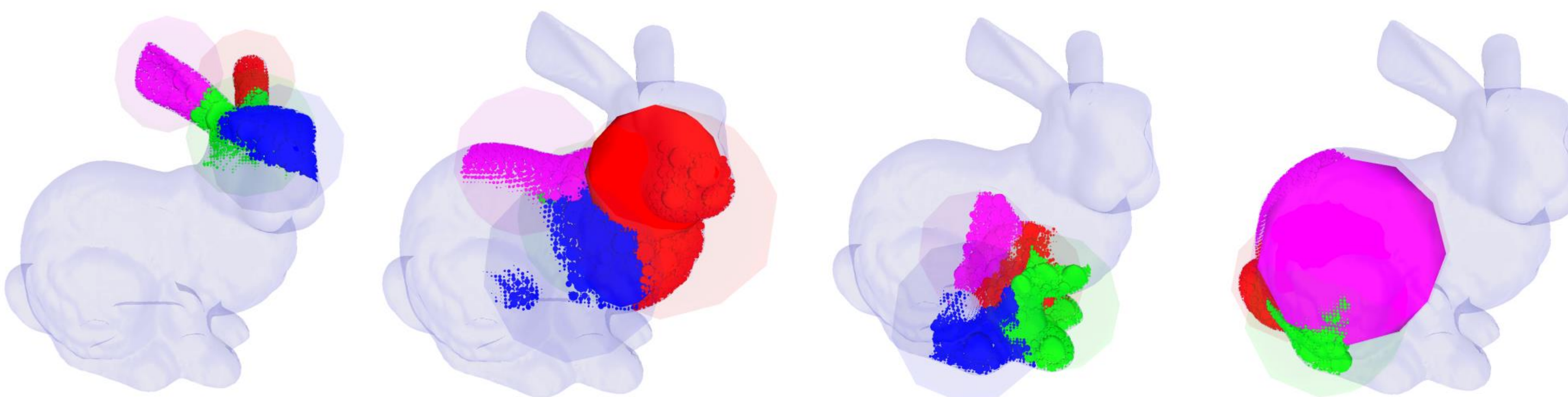
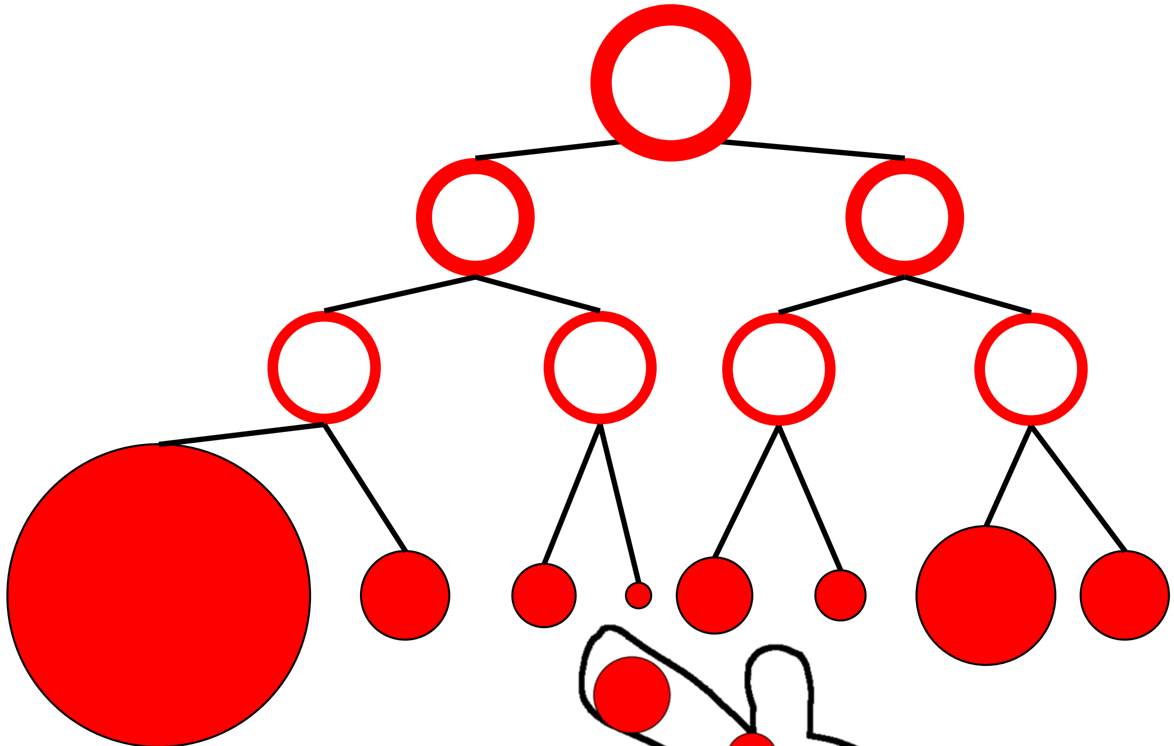


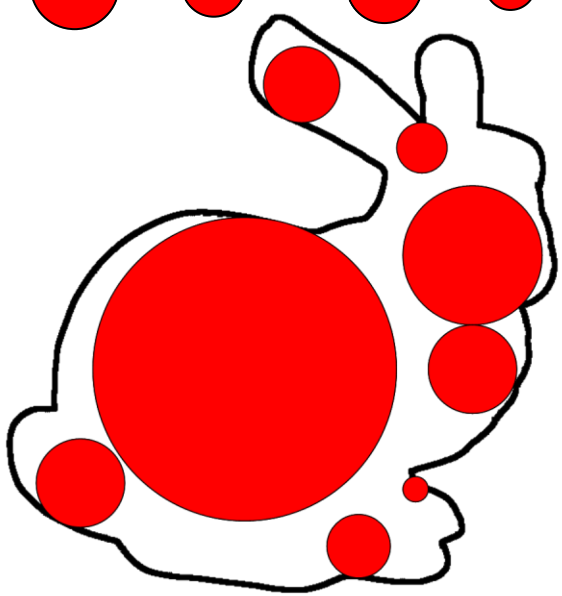
Bounding Volume Hierarchy for Inner Spheres

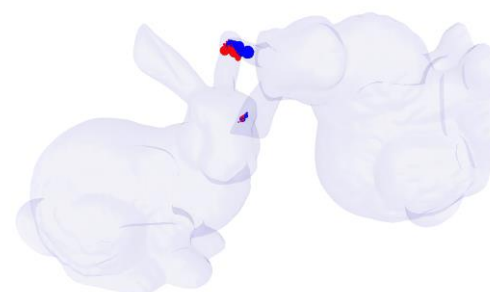




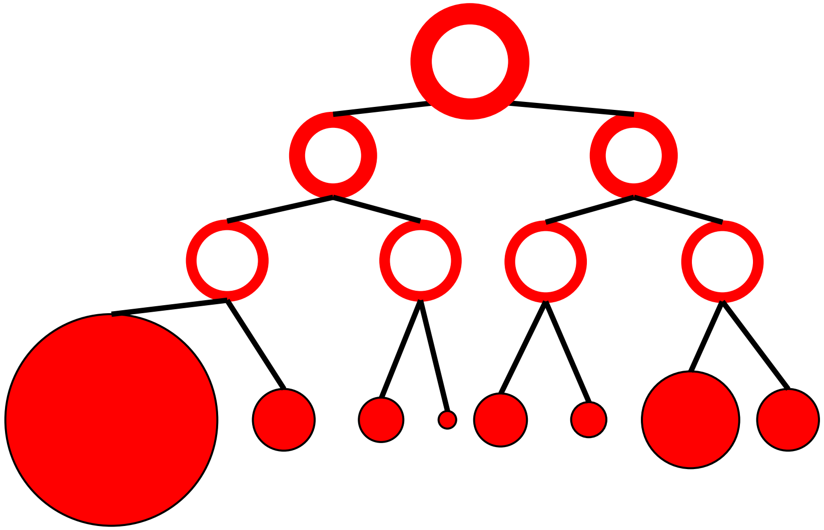
BVH Traversal: Penetration Volume Queries

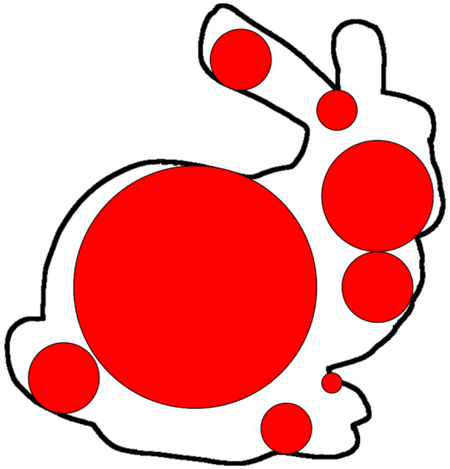


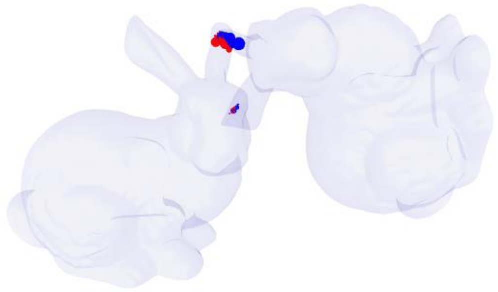




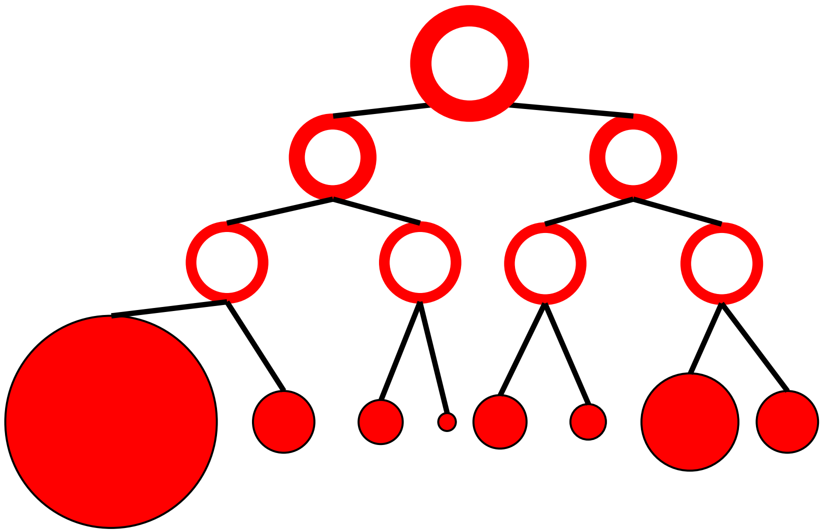
BVH Traversal: Penetration Volume Queries

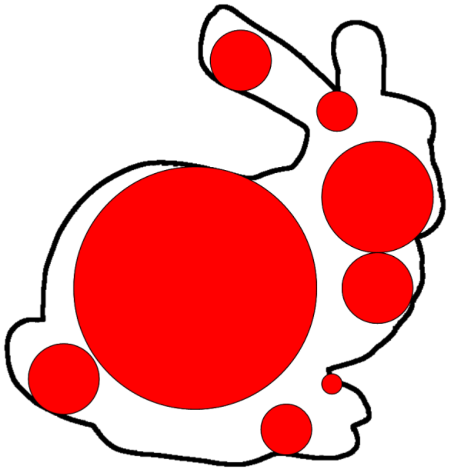


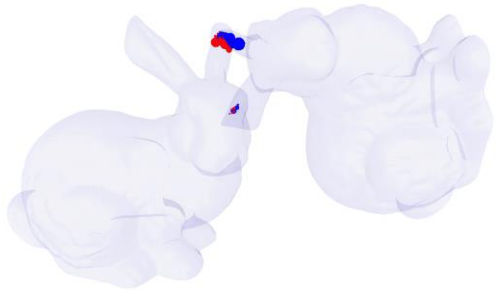




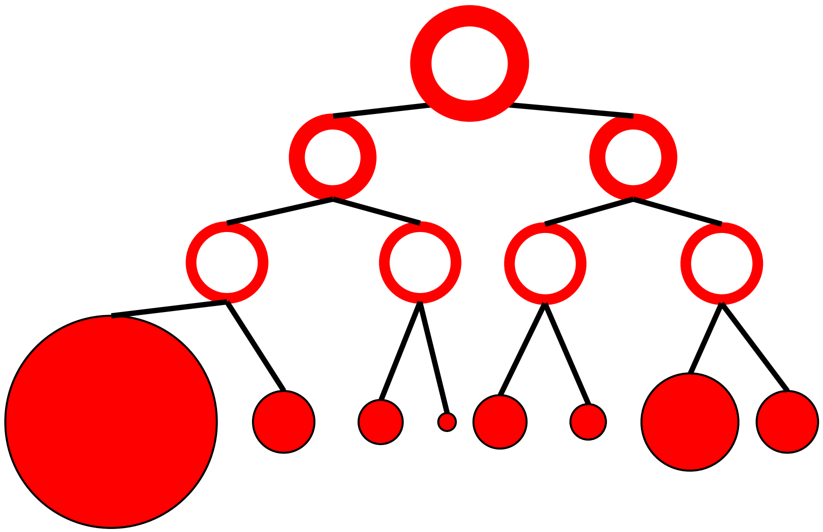
BVH Traversal: Penetration Volume Queries

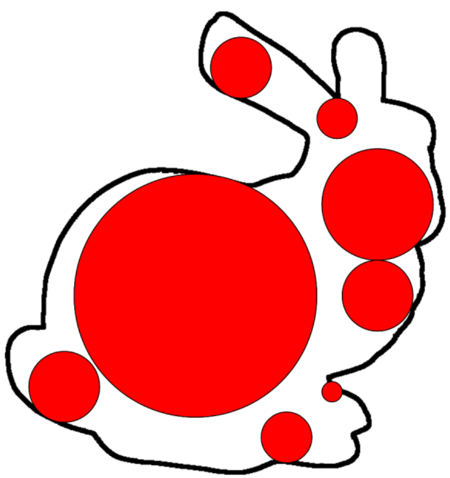


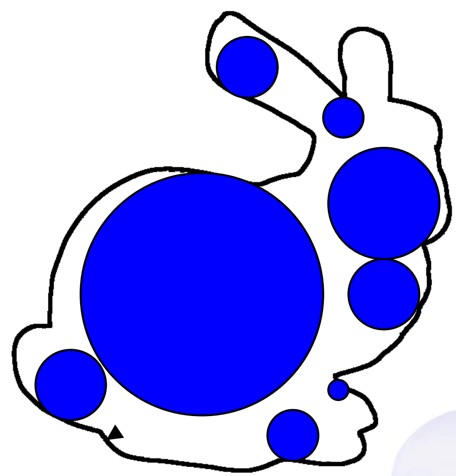


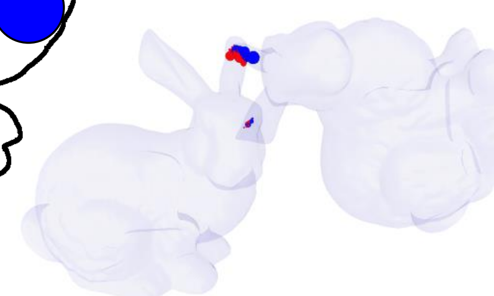


BVH Traversal: Penetration Volume Queries

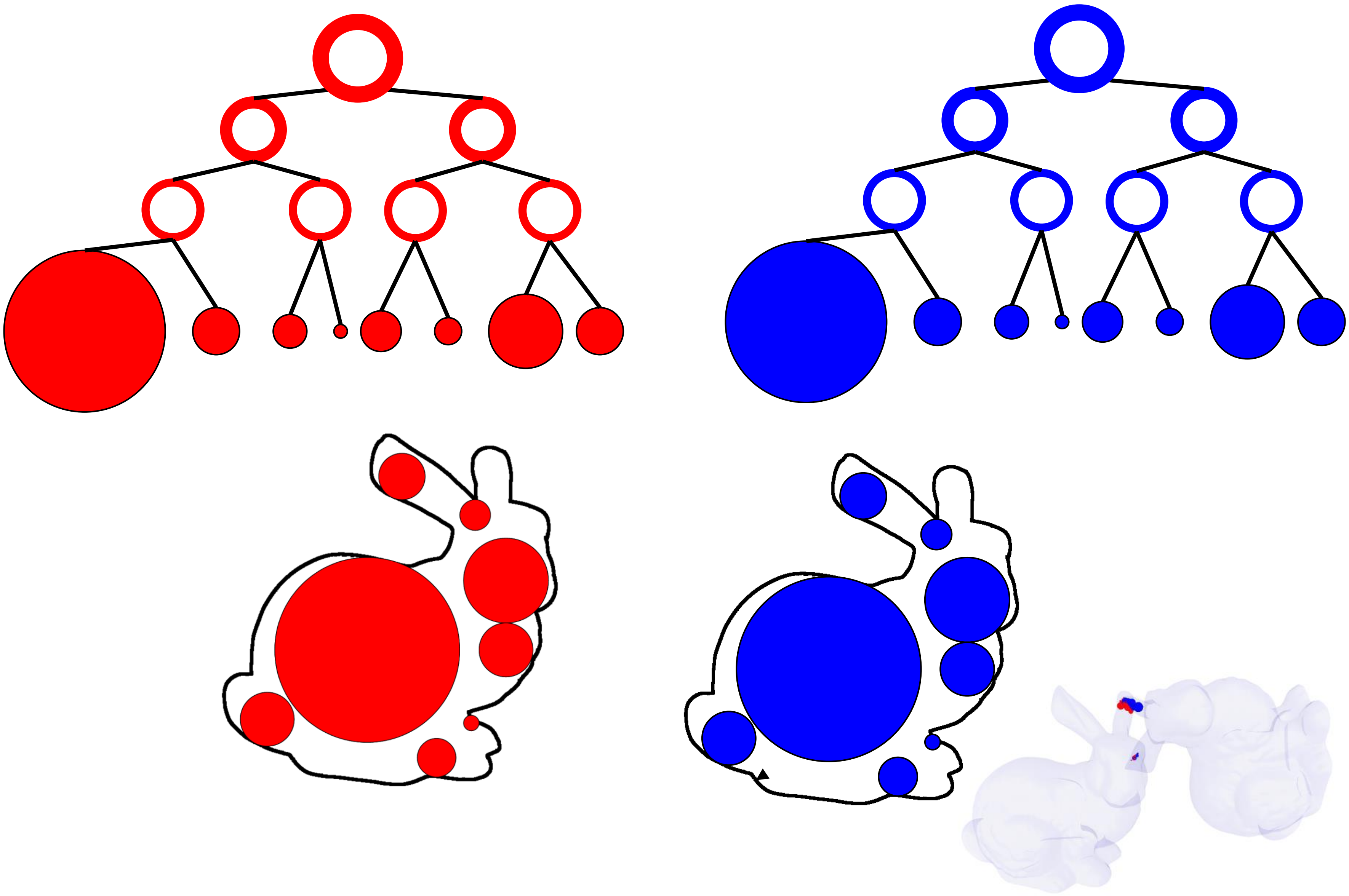




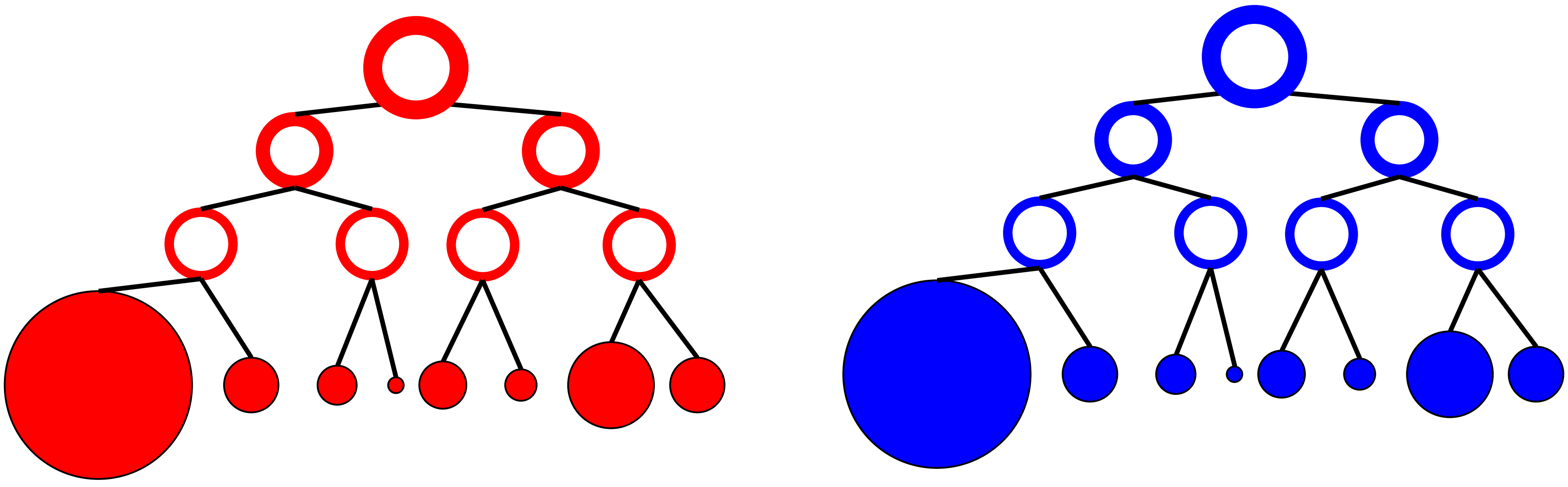


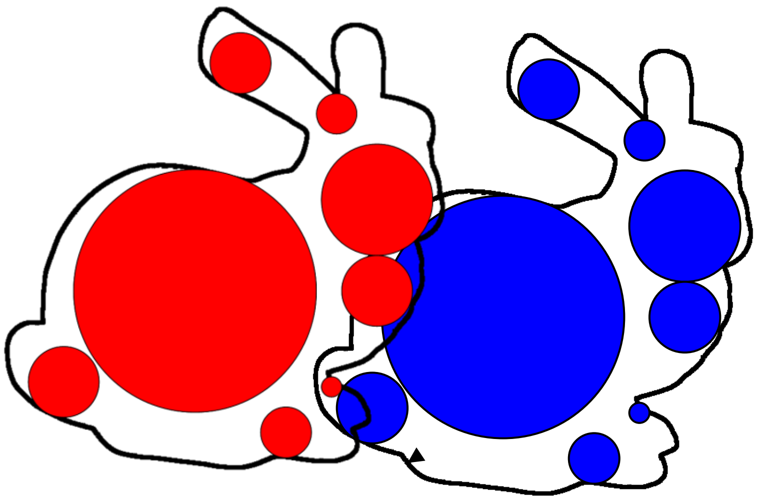


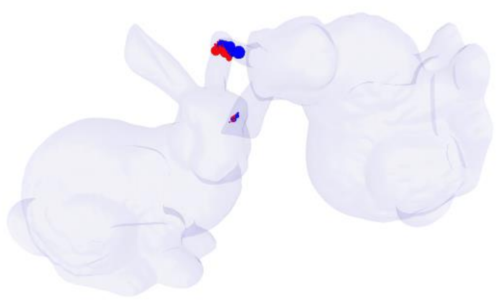
BVH Traversal: Penetration Volume Queries



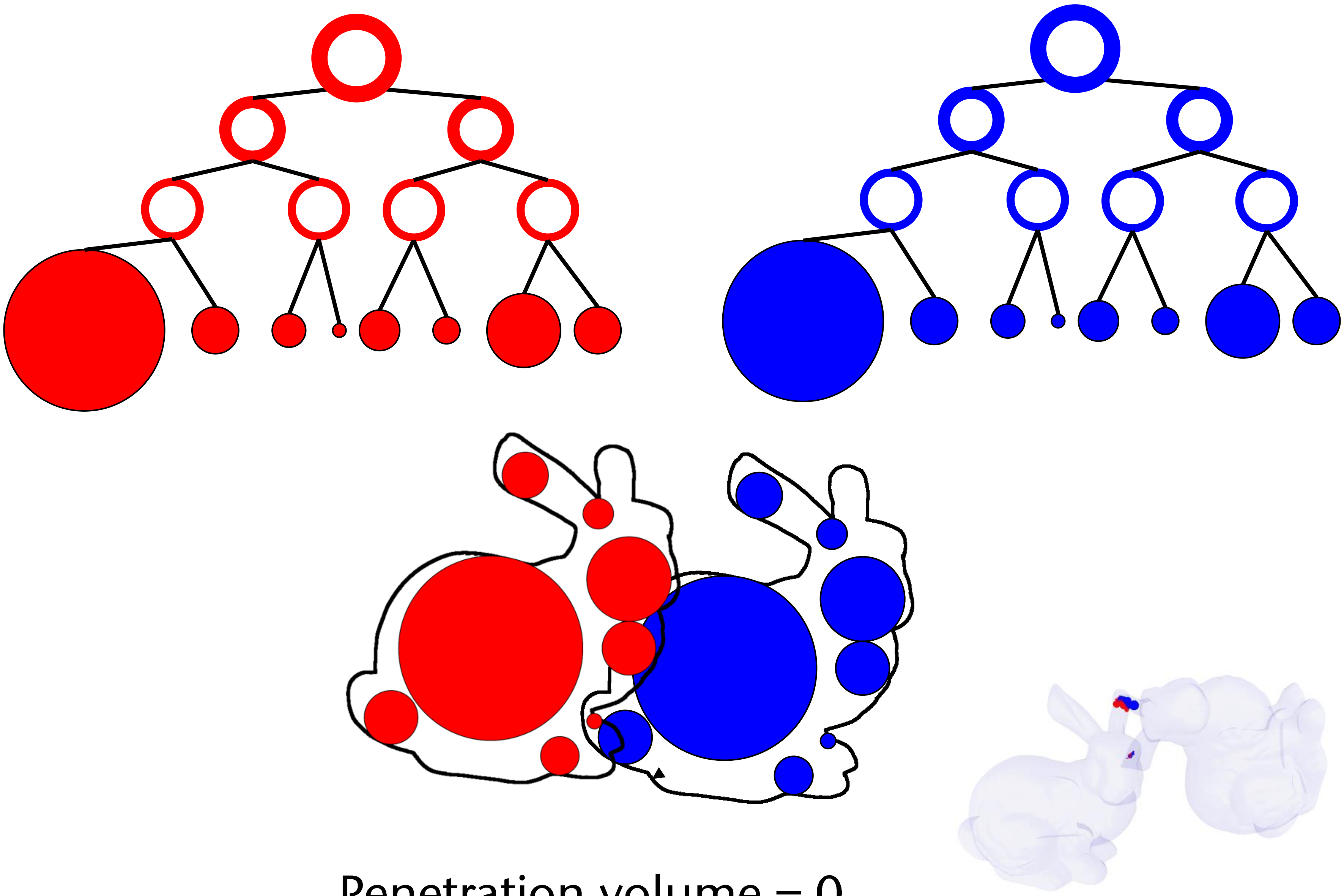
BVH Traversal: Penetration Volume Queries





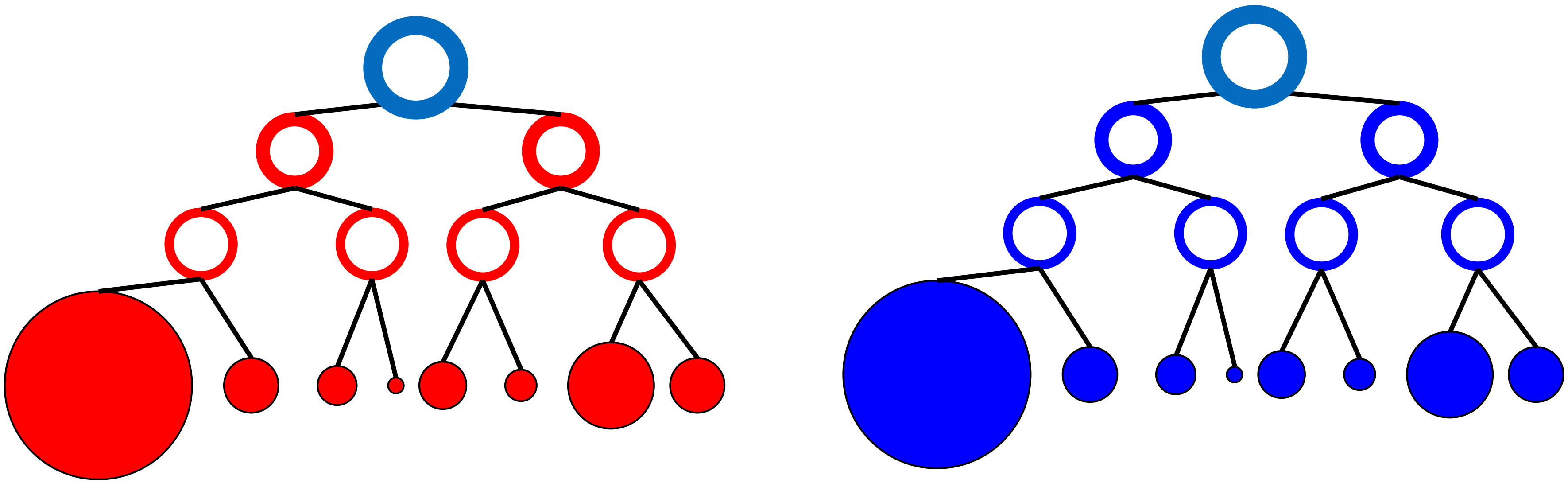


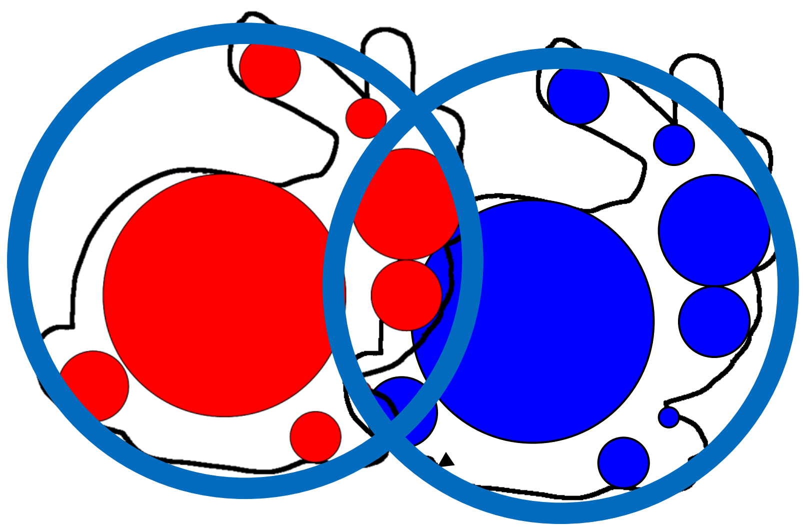
BVH Traversal: Penetration Volume Queries

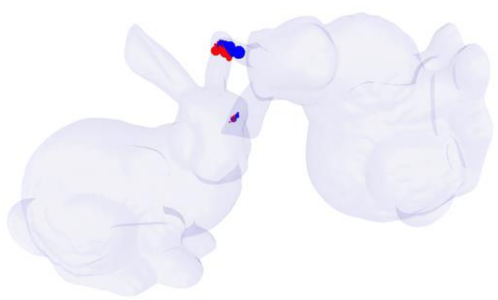


Penetration volume = 0

BVH Traversal: Penetration Volume Queries

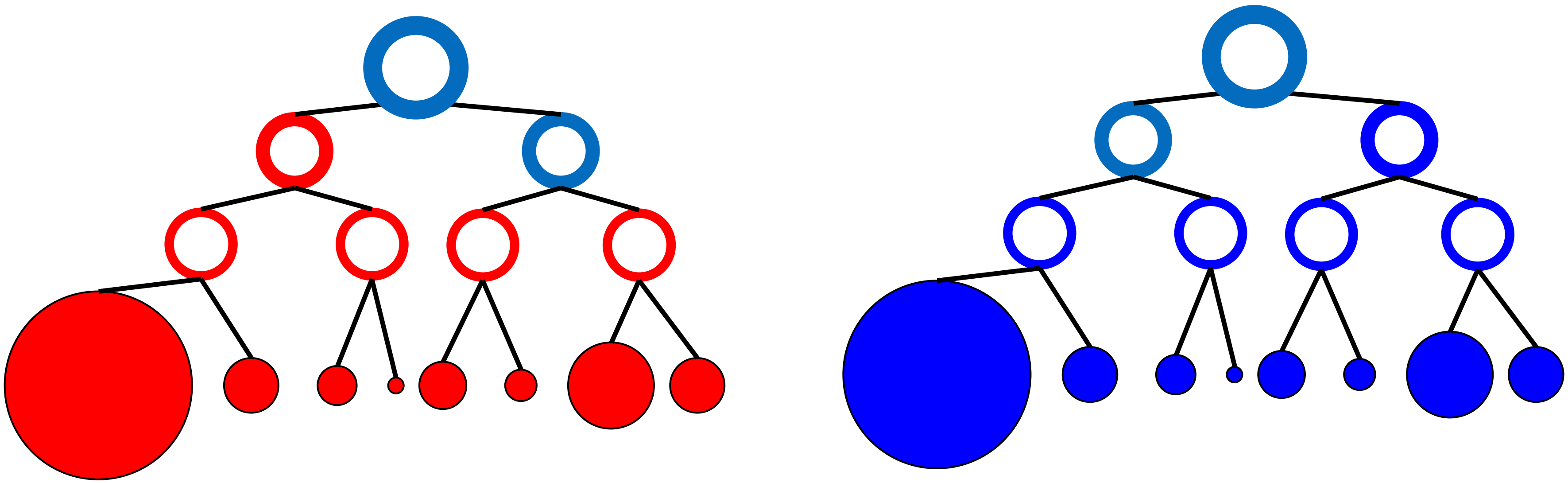


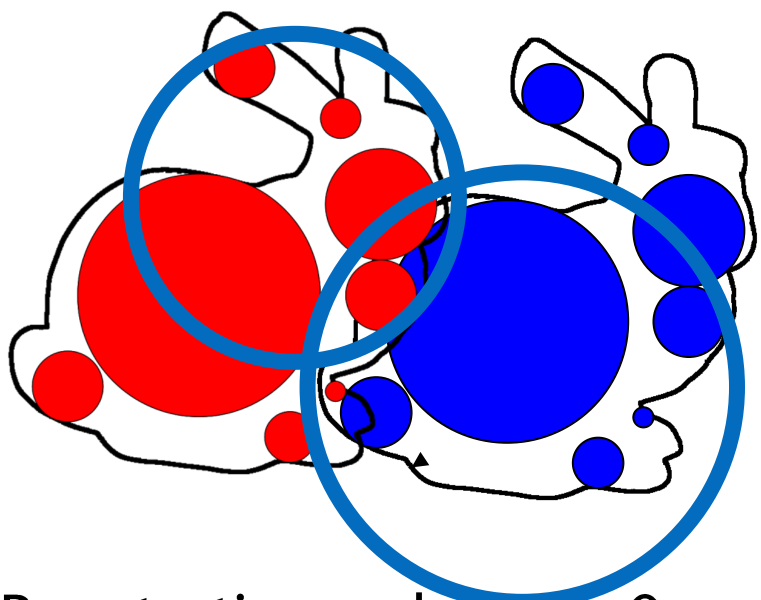


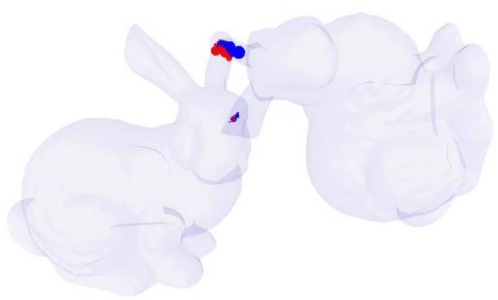


Penetration volume = 0

BVH Traversal: Penetration Volume Queries

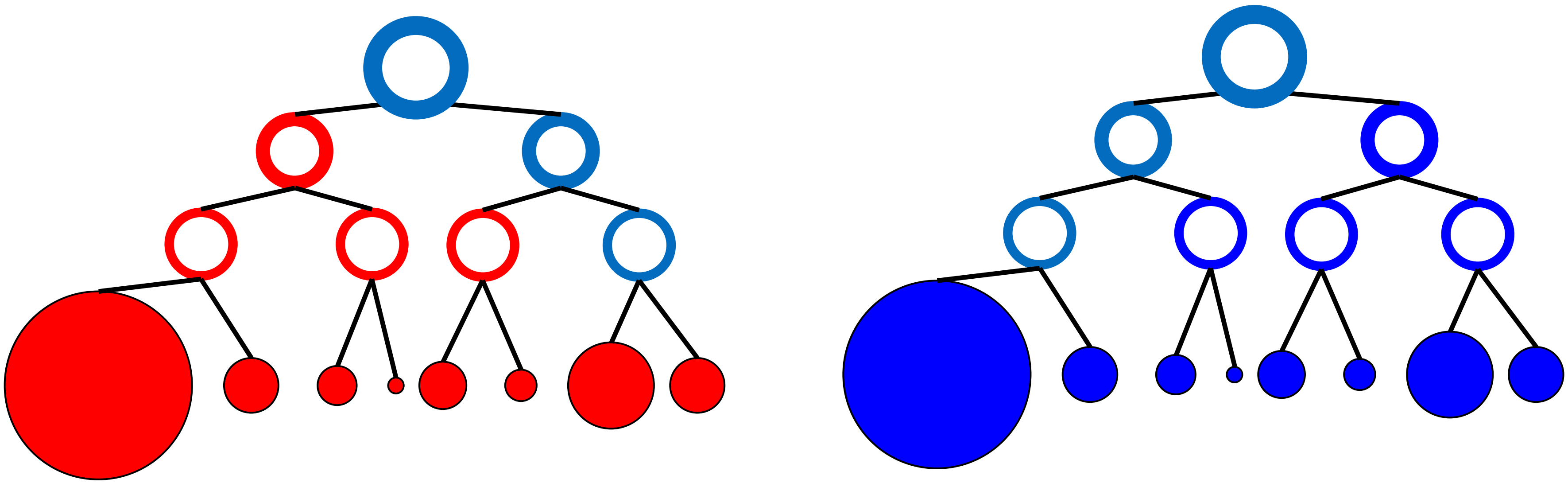


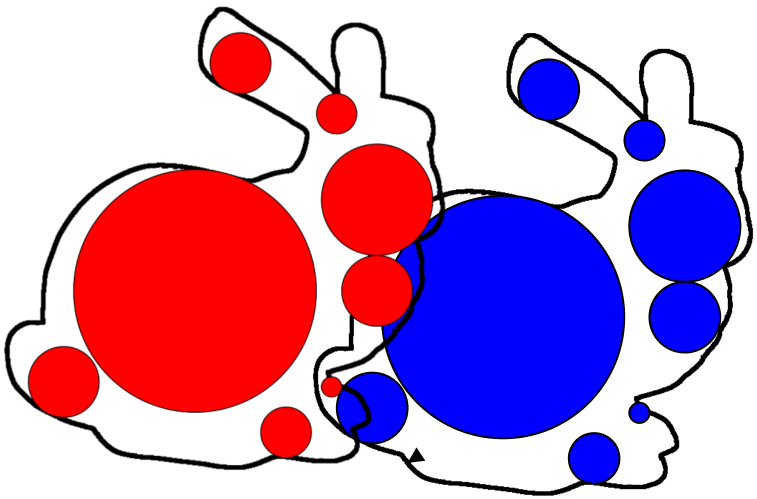


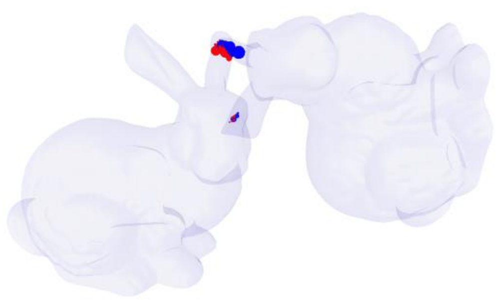


Penetration volume = 0

BVH Traversal: Penetration Volume Queries

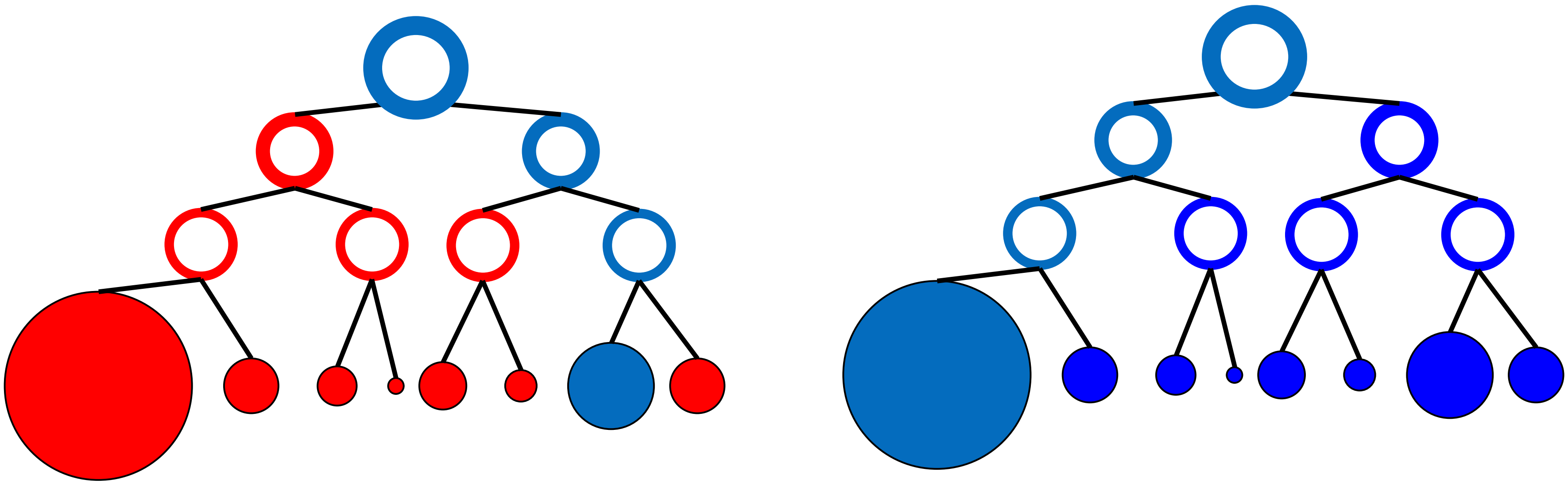


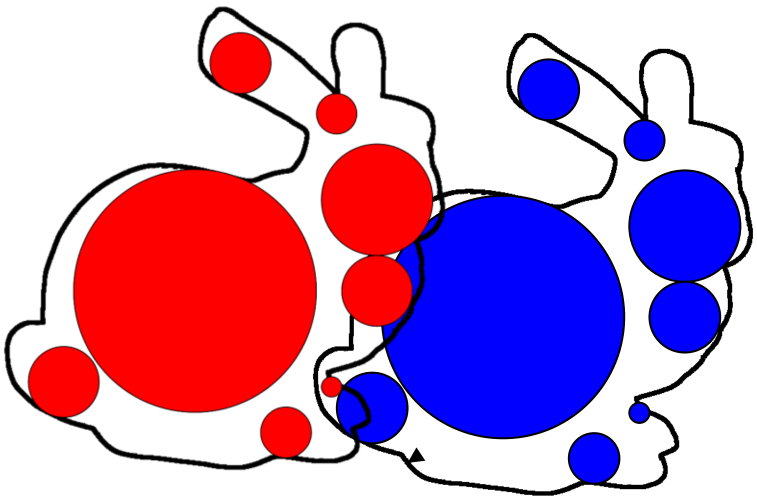




Penetration volume = 0

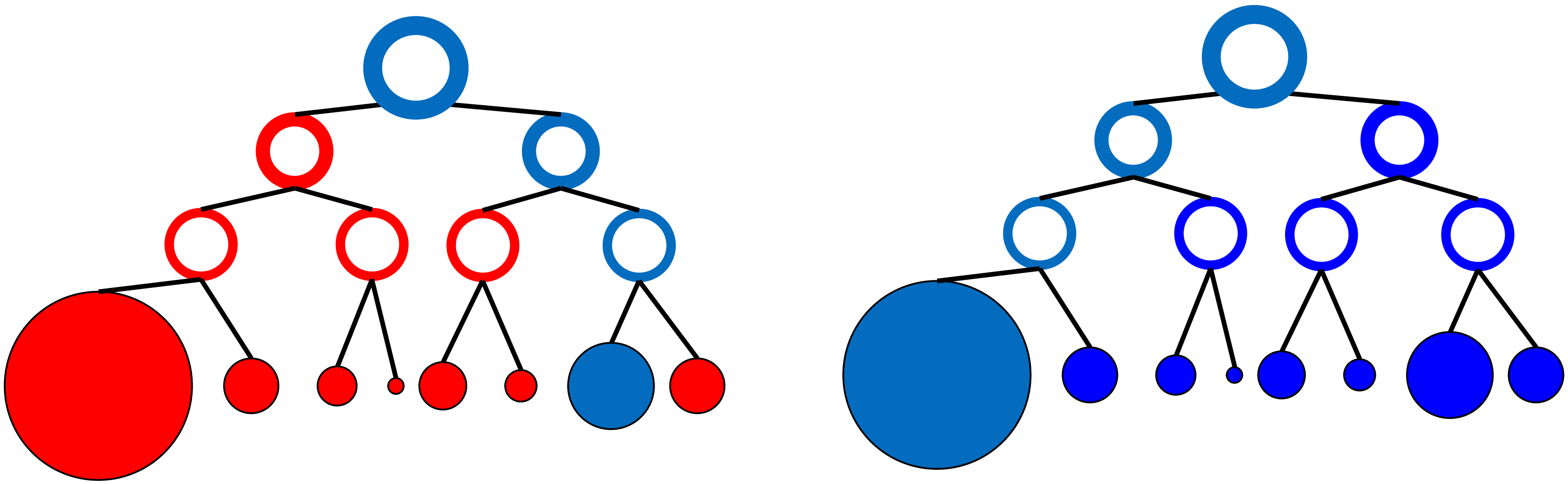
BVH Traversal: Penetration Volume Queries

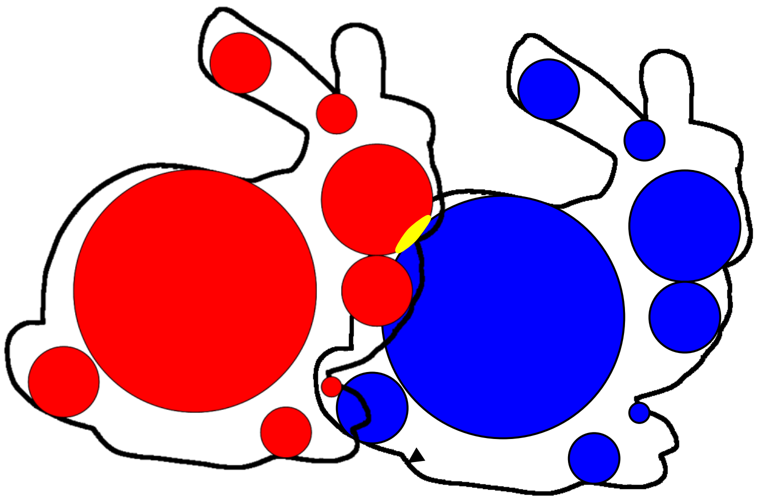


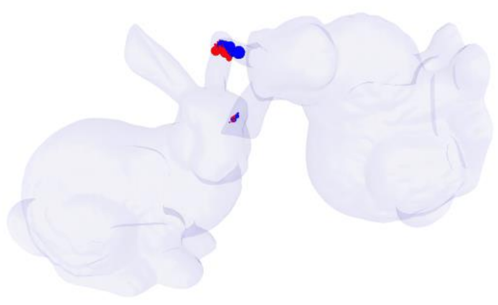


Penetration volume = 0

BVH Traversal: Penetration Volume Queries

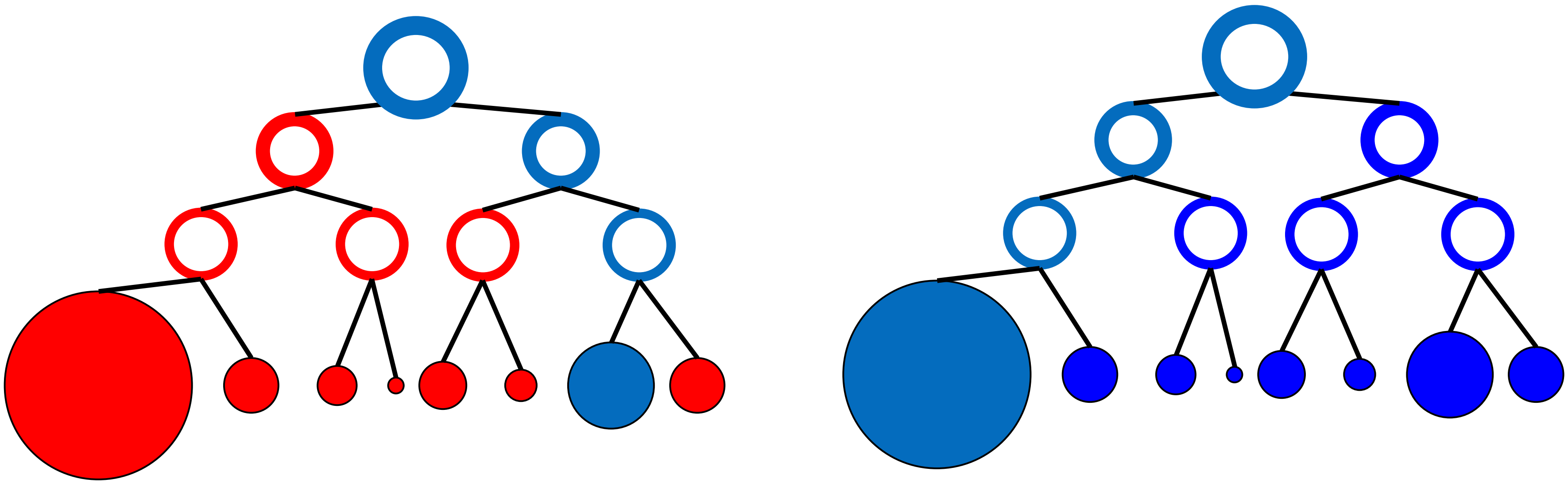


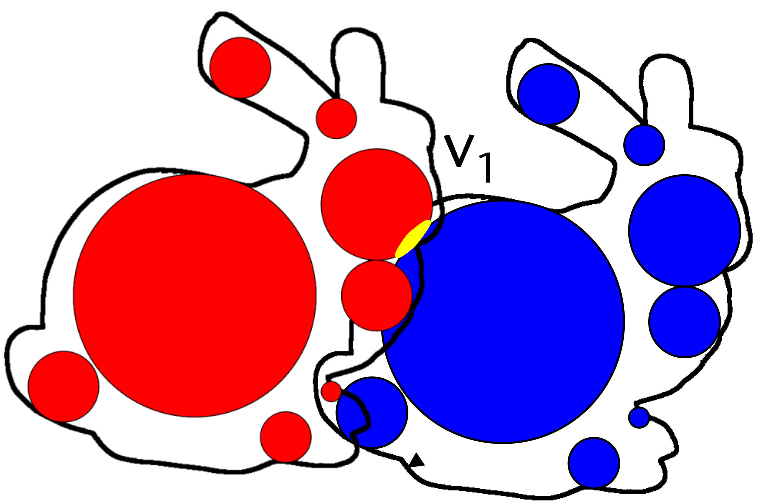


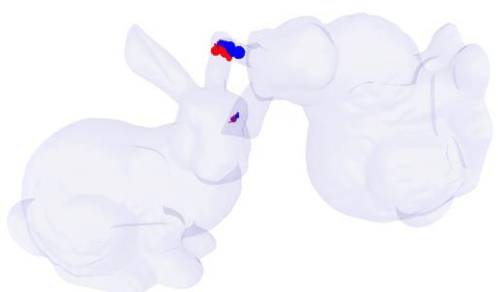


Penetration volume = 0

BVH Traversal: Penetration Volume Queries

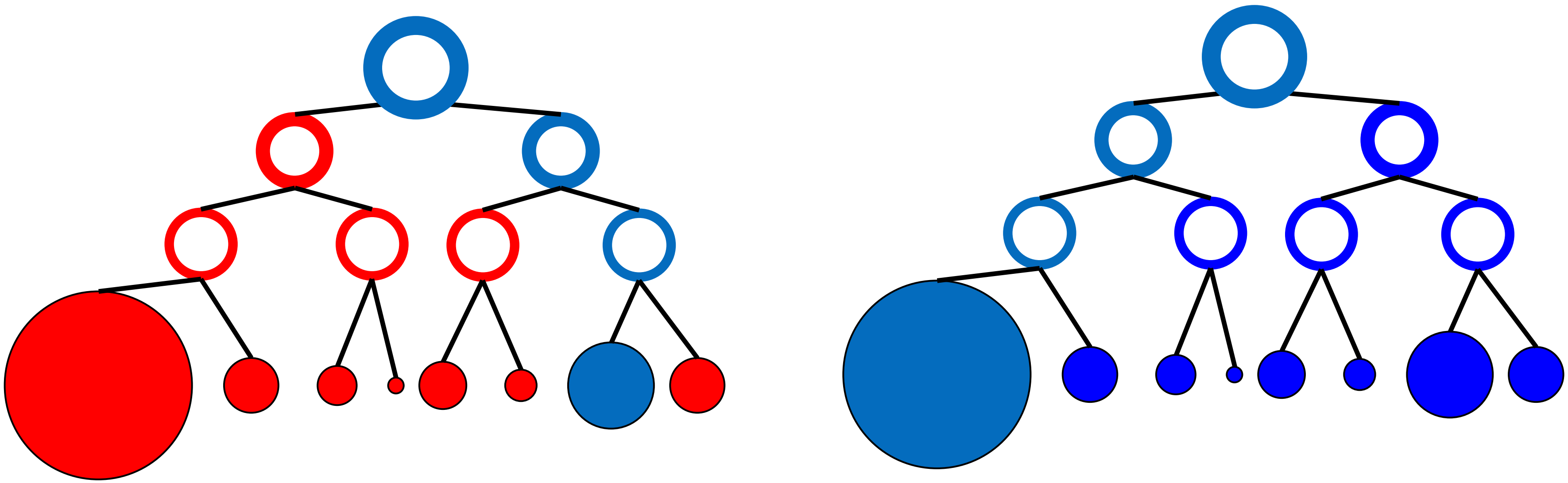


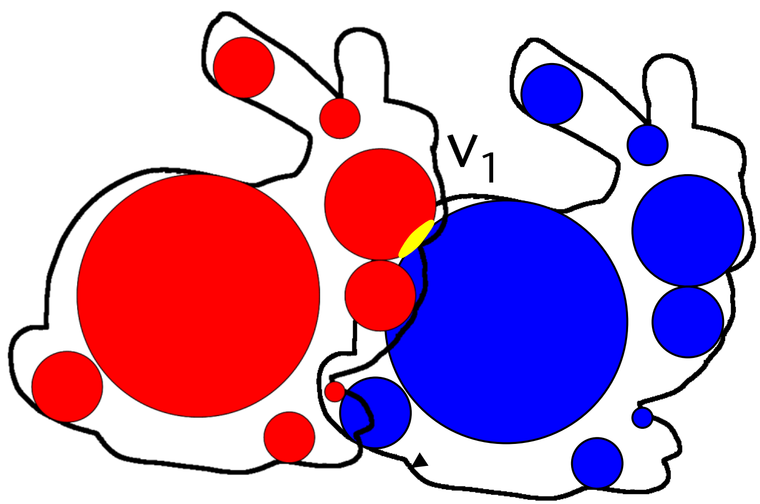


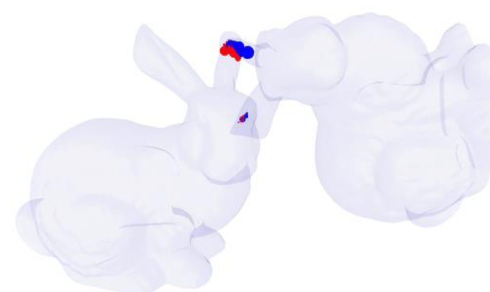


Penetration volume = 0

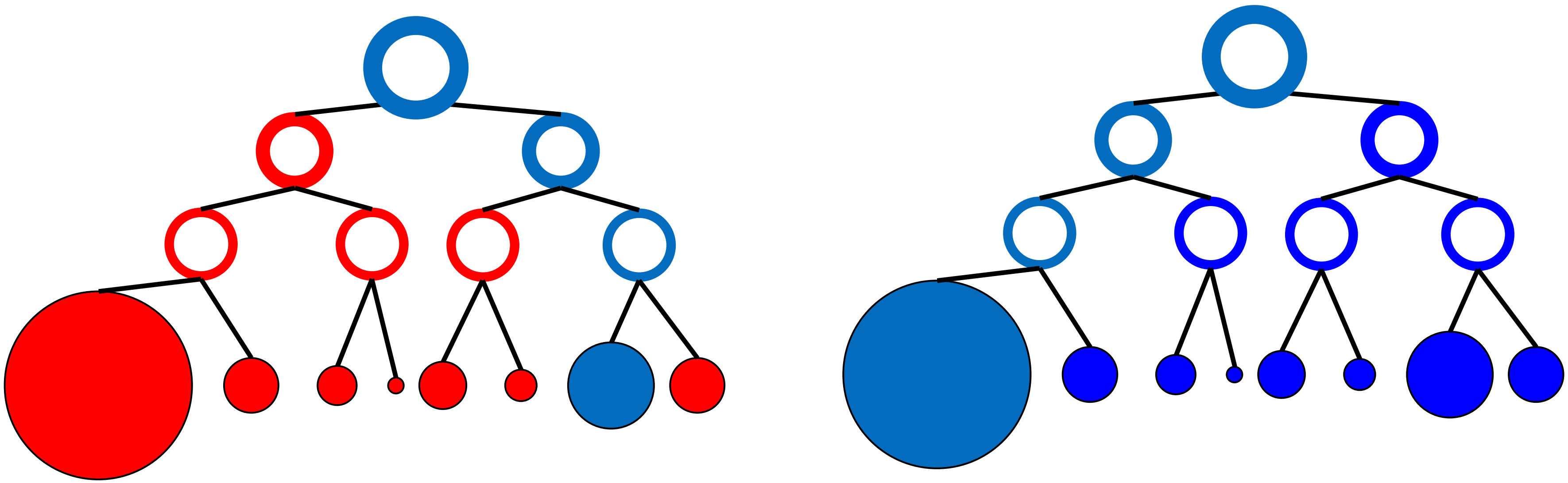
BVH Traversal: Penetration Volume Queries

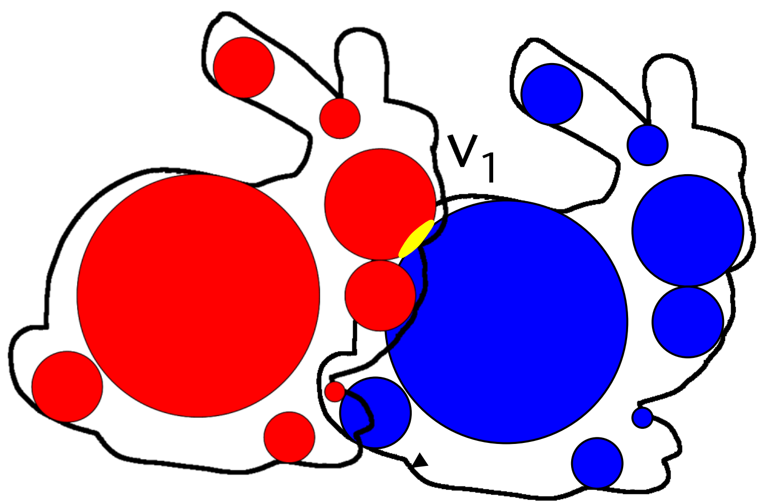






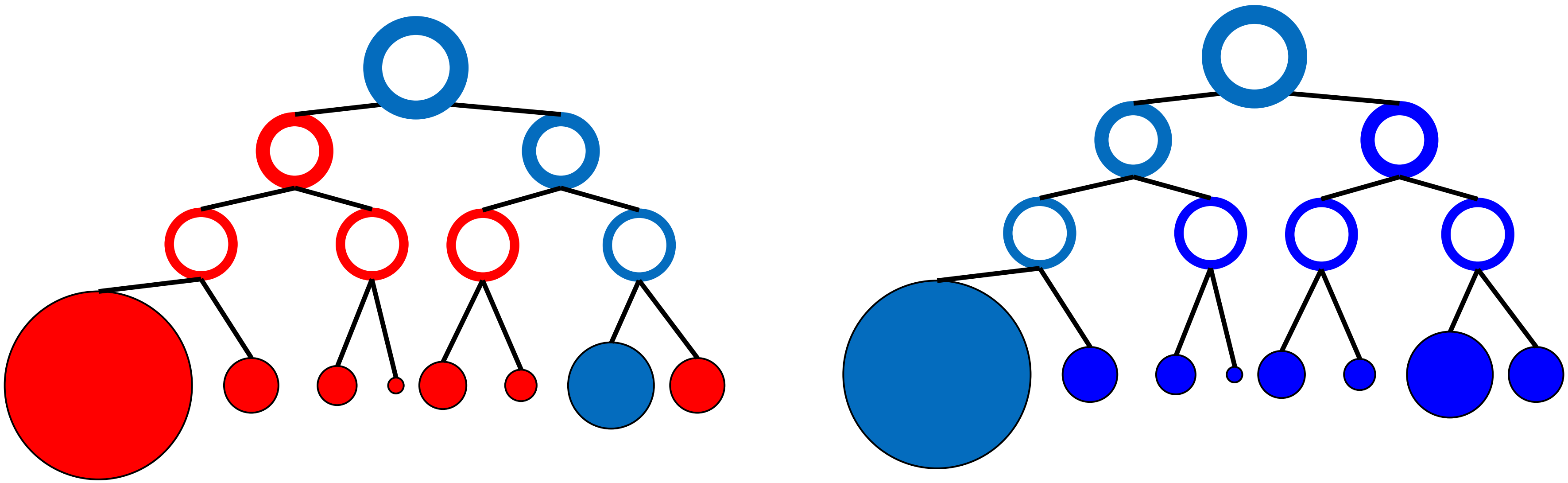
BVH Traversal: Penetration Volume Queries

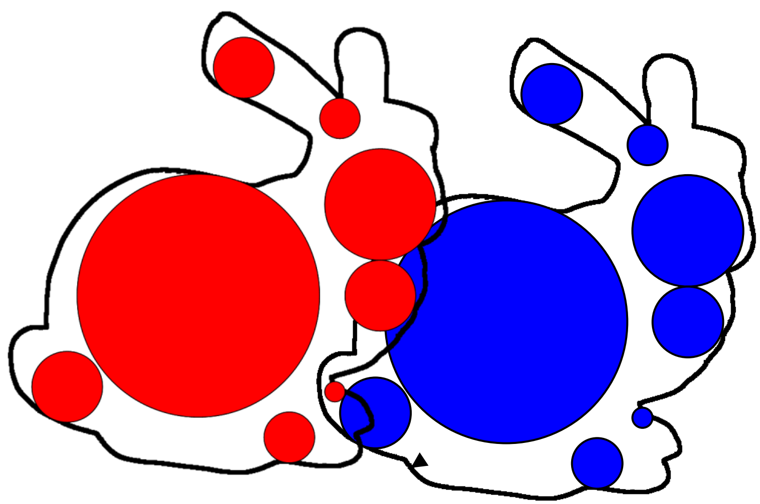


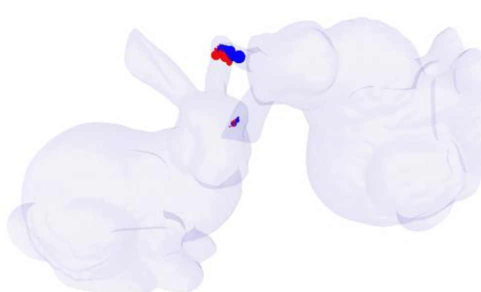


Penetration volume = v_1

BVH Traversal: Penetration Volume Queries

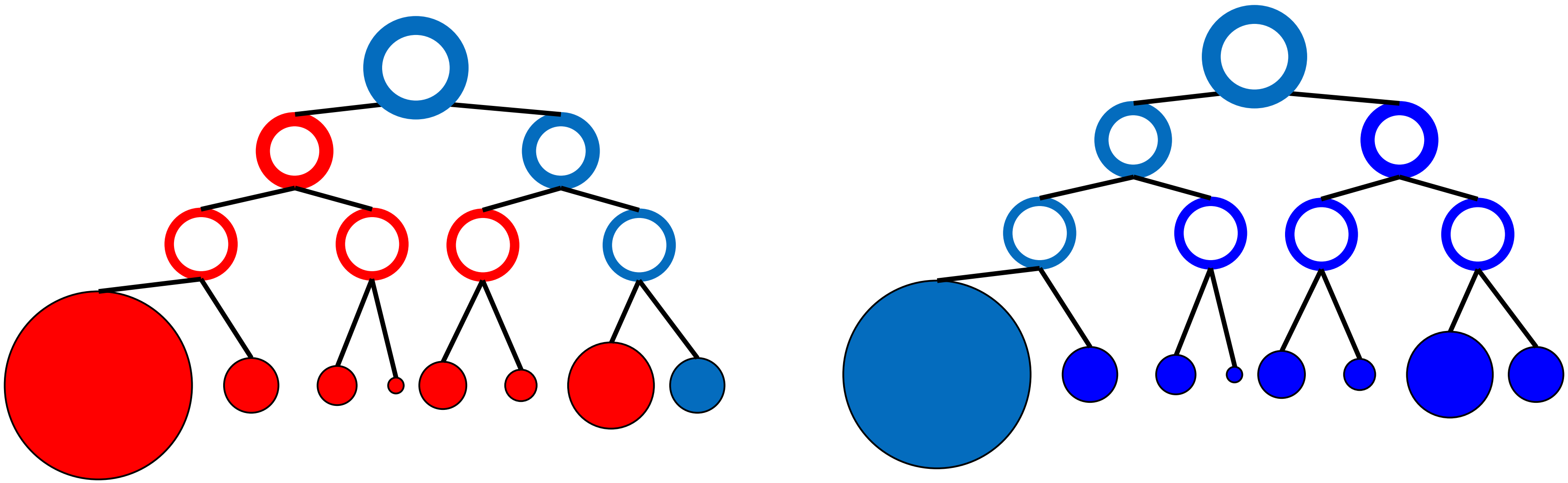


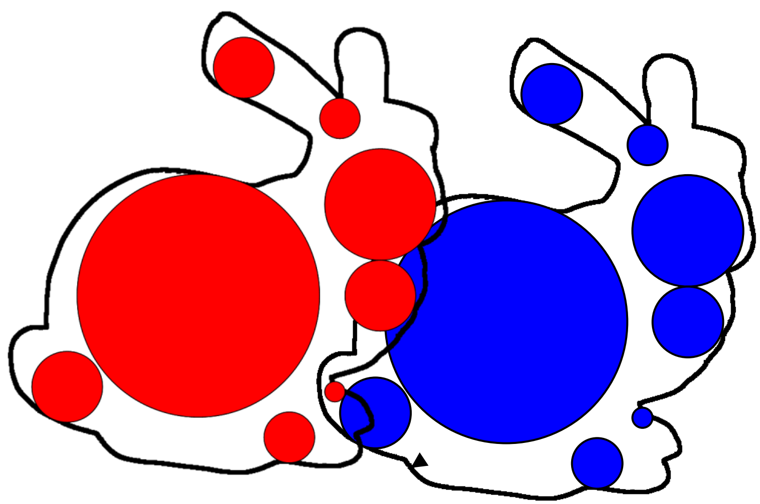


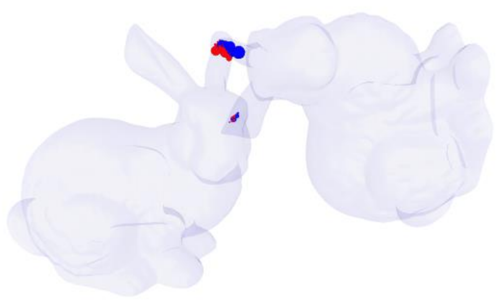


Penetration volume = v_1

BVH Traversal: Penetration Volume Queries

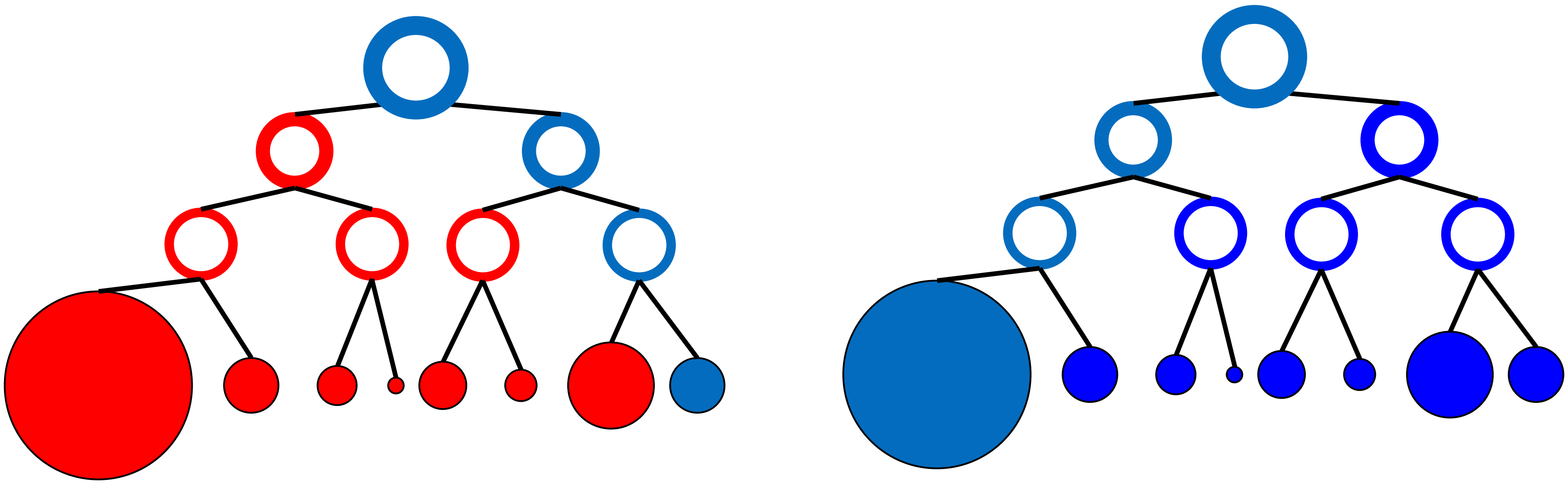


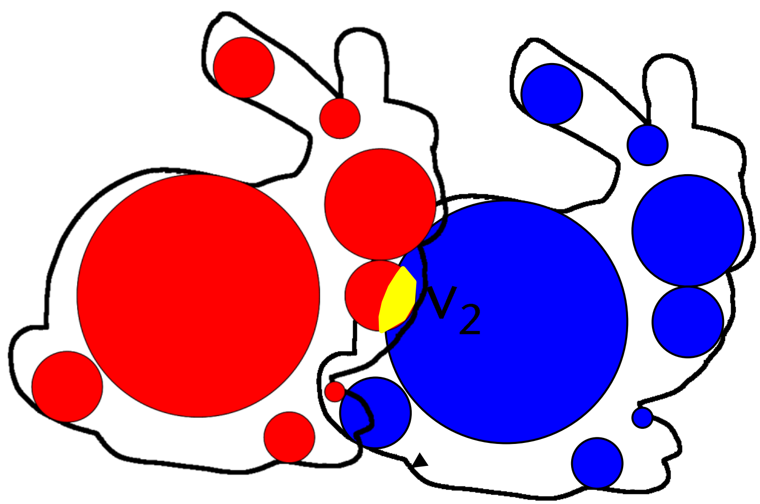


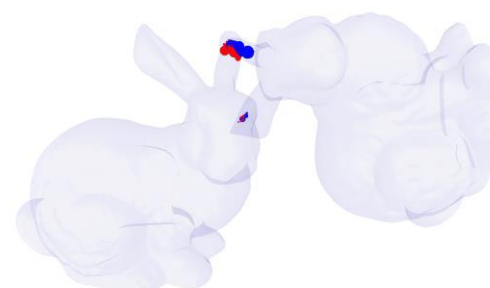


Penetration volume = v_1

BVH Traversal: Penetration Volume Queries

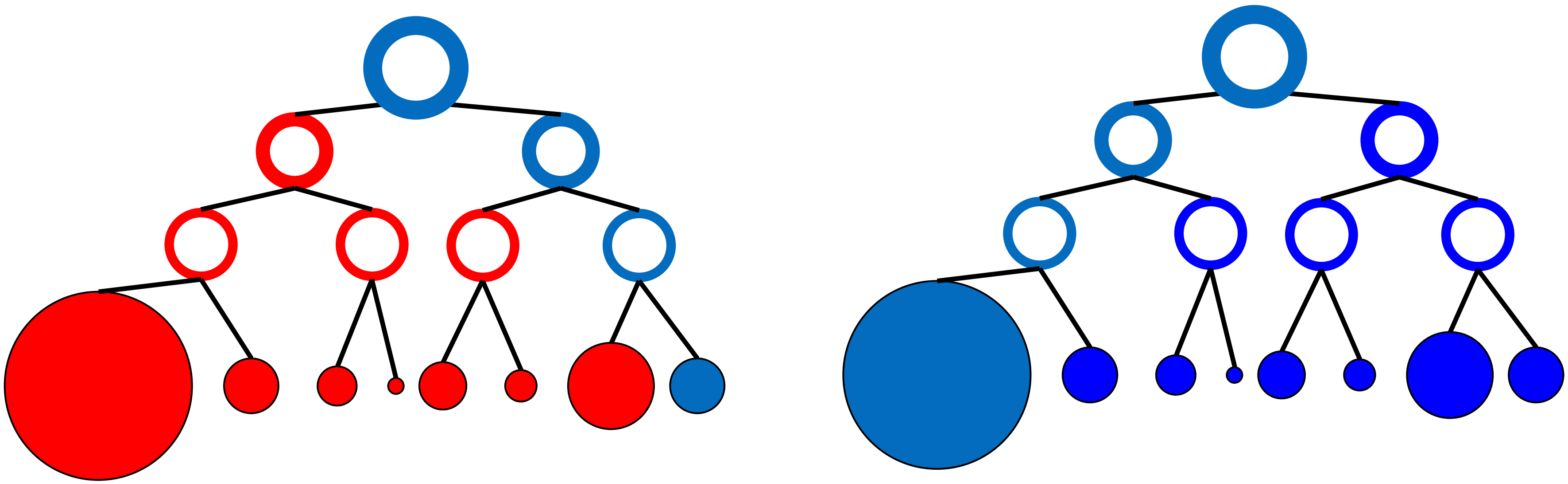


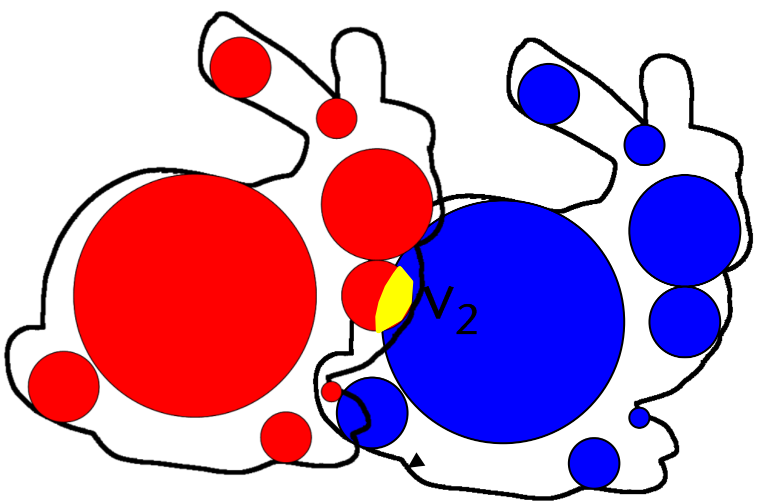


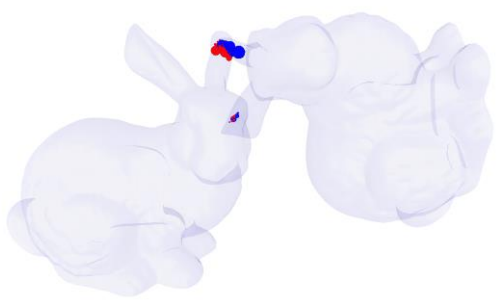


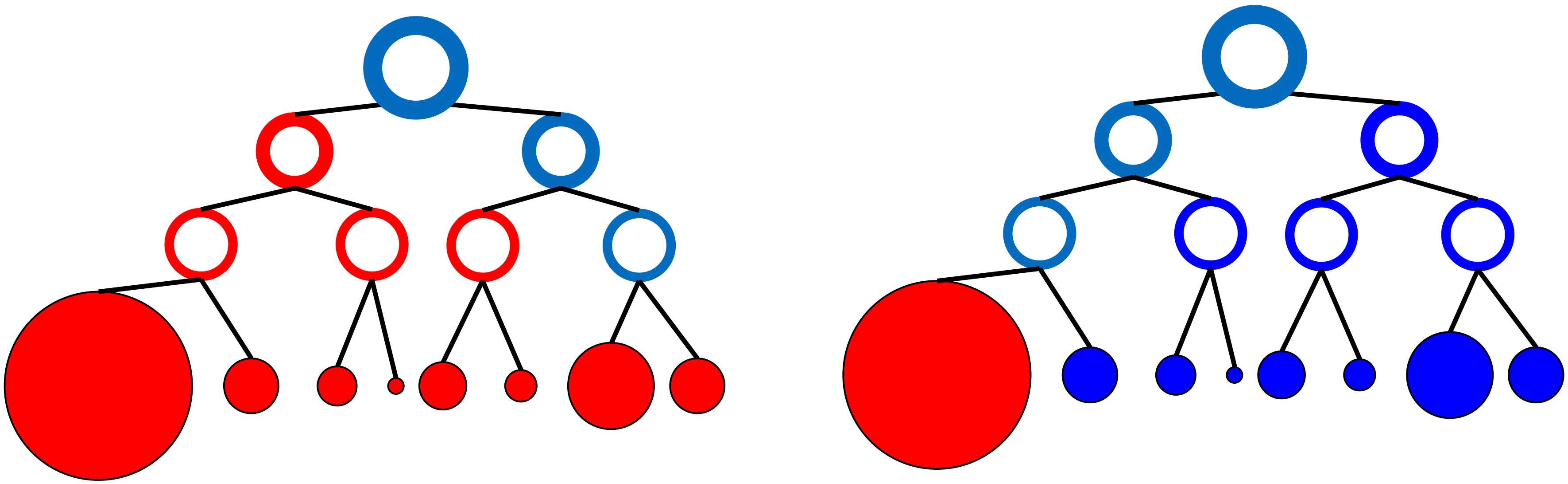
Penetration volume = v_1

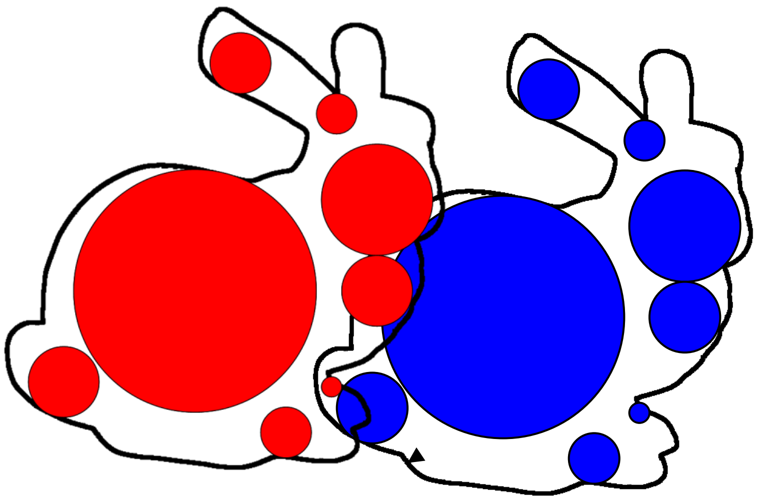
BVH Traversal: Penetration Volume Queries

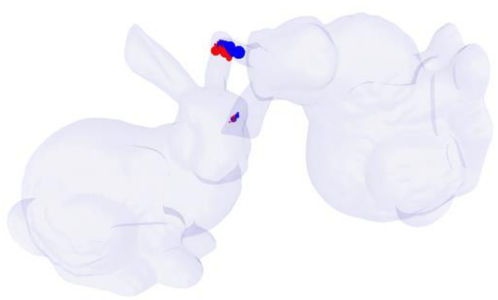






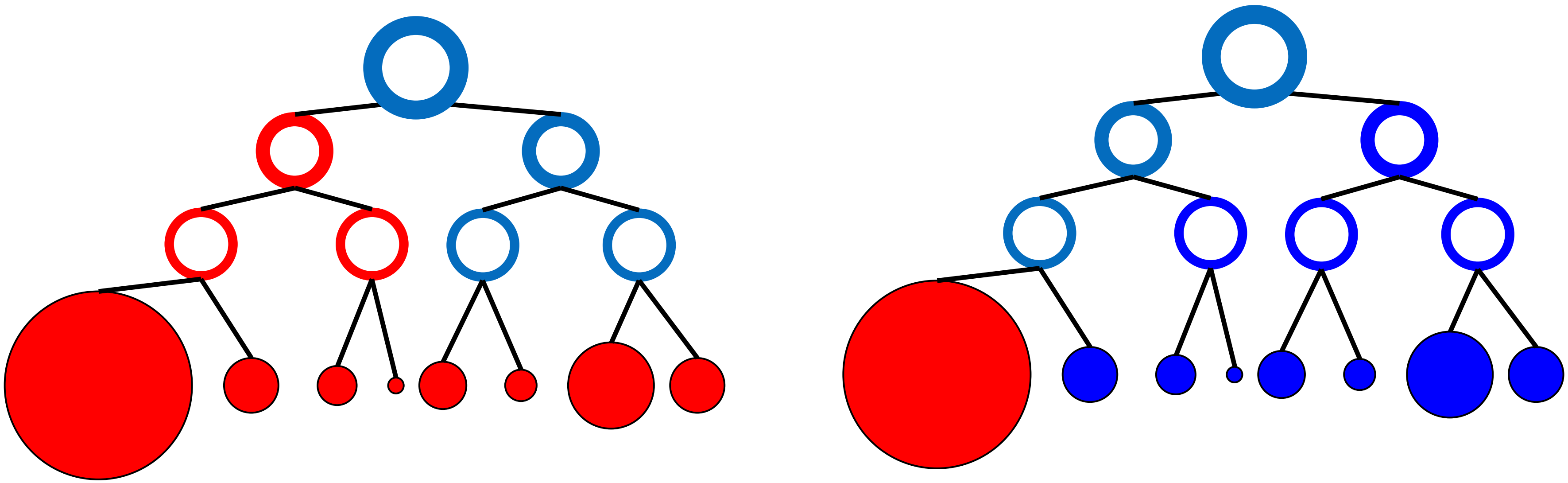


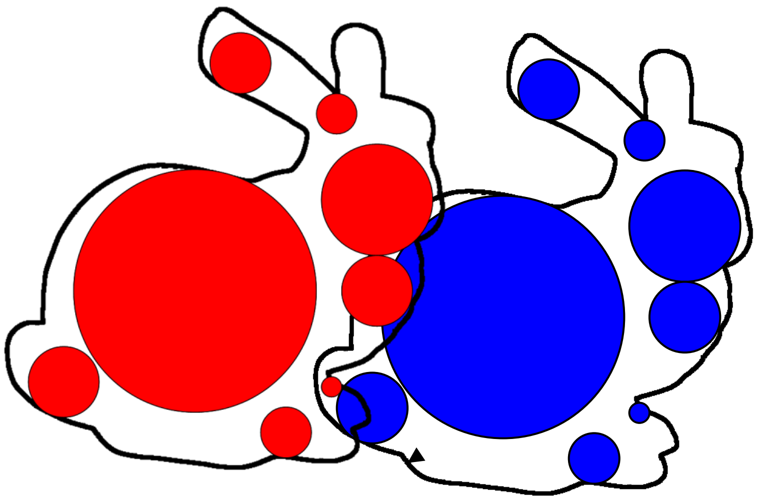


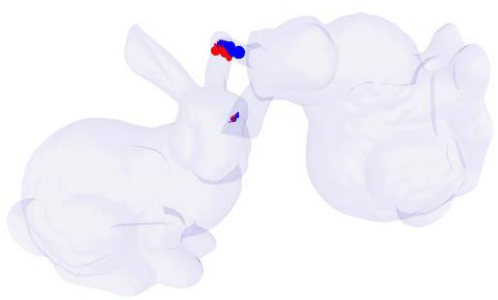


Penetration volume = $v_1 + v_2$

BVH Traversal: Penetration Volume Queries

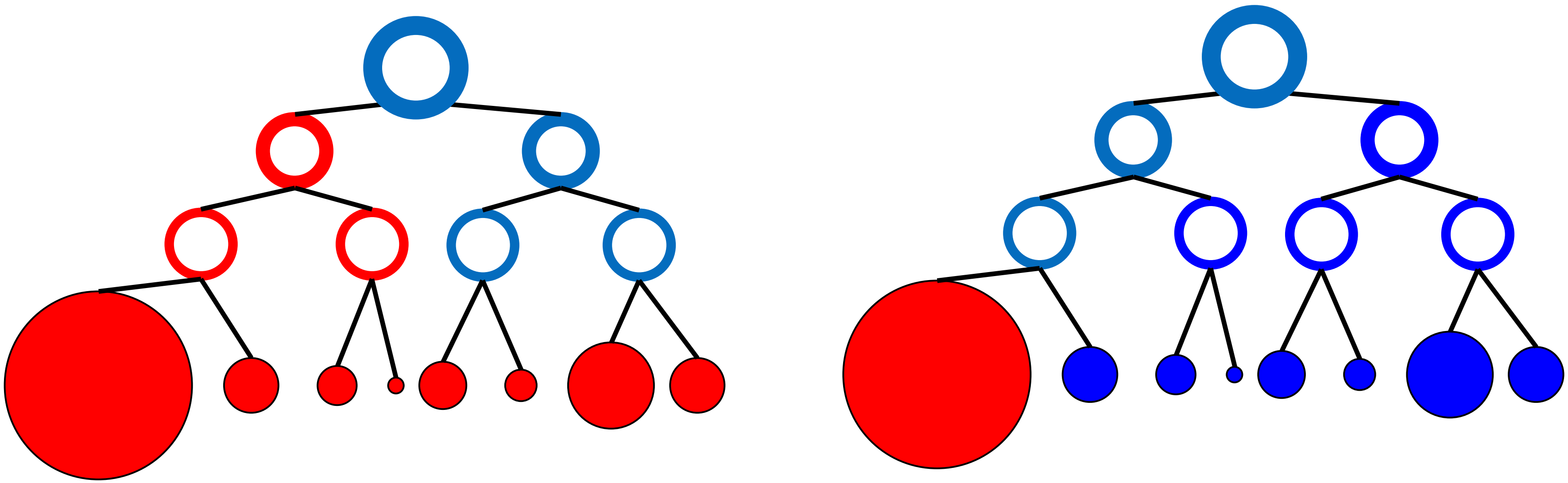


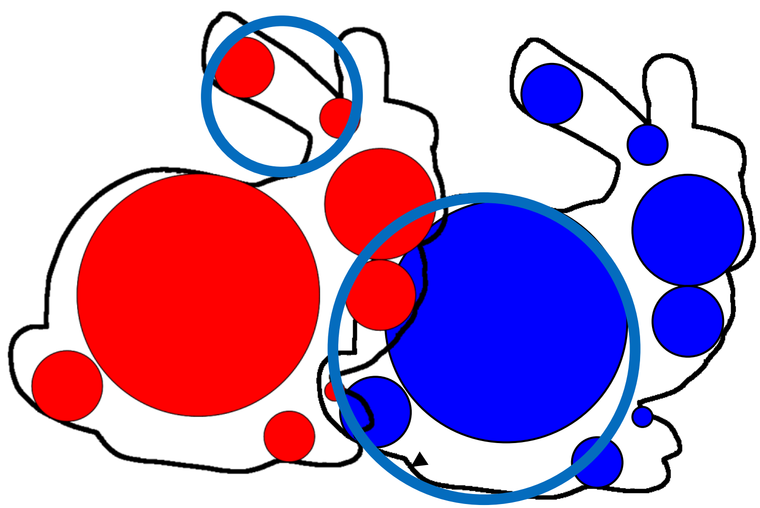


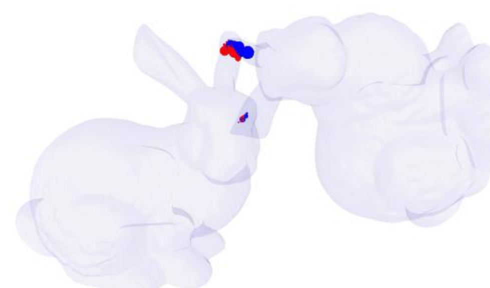


Penetration volume = $v_1 + v_2$

BVH Traversal: Penetration Volume Queries

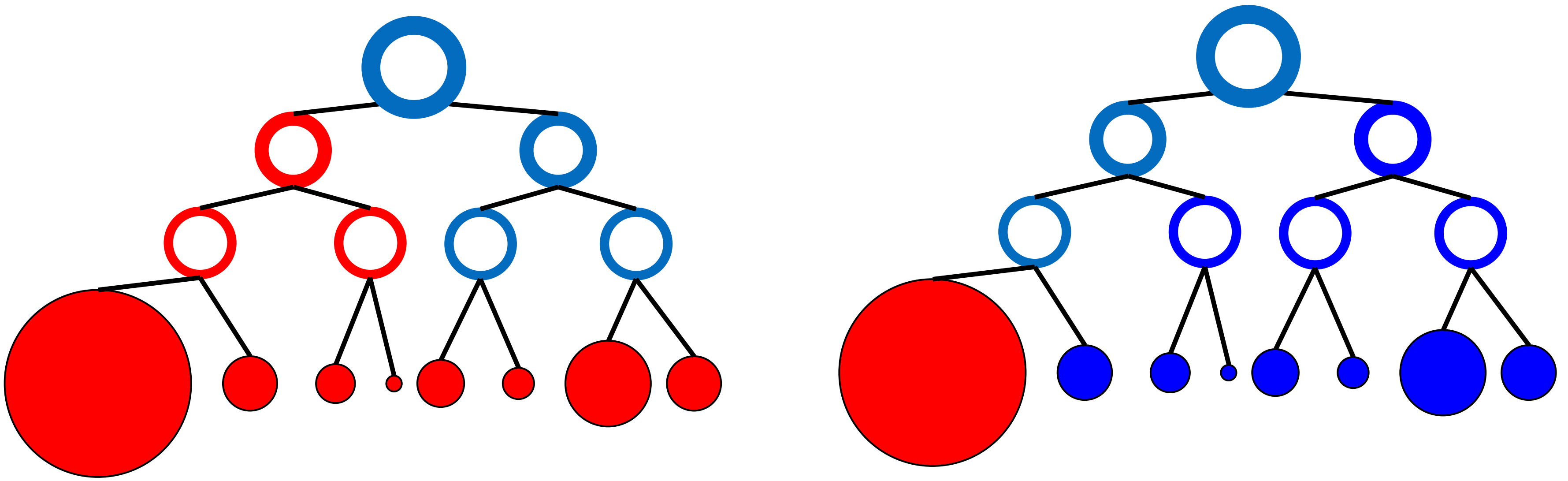


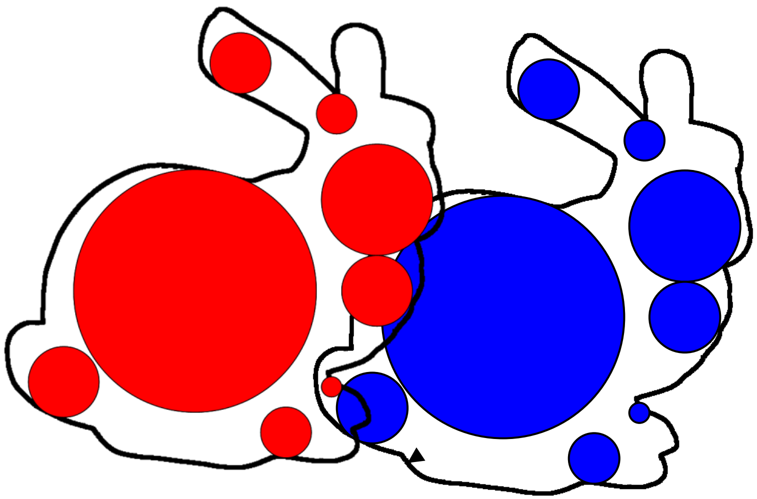


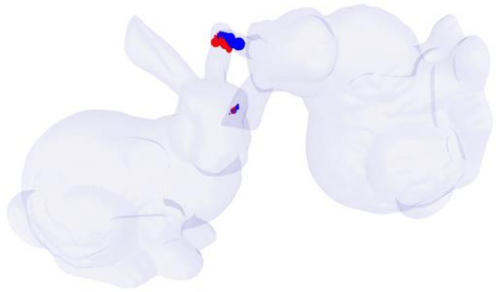


Penetration volume = $v_1 + v_2$

BVH Traversal: Penetration Volume Queries

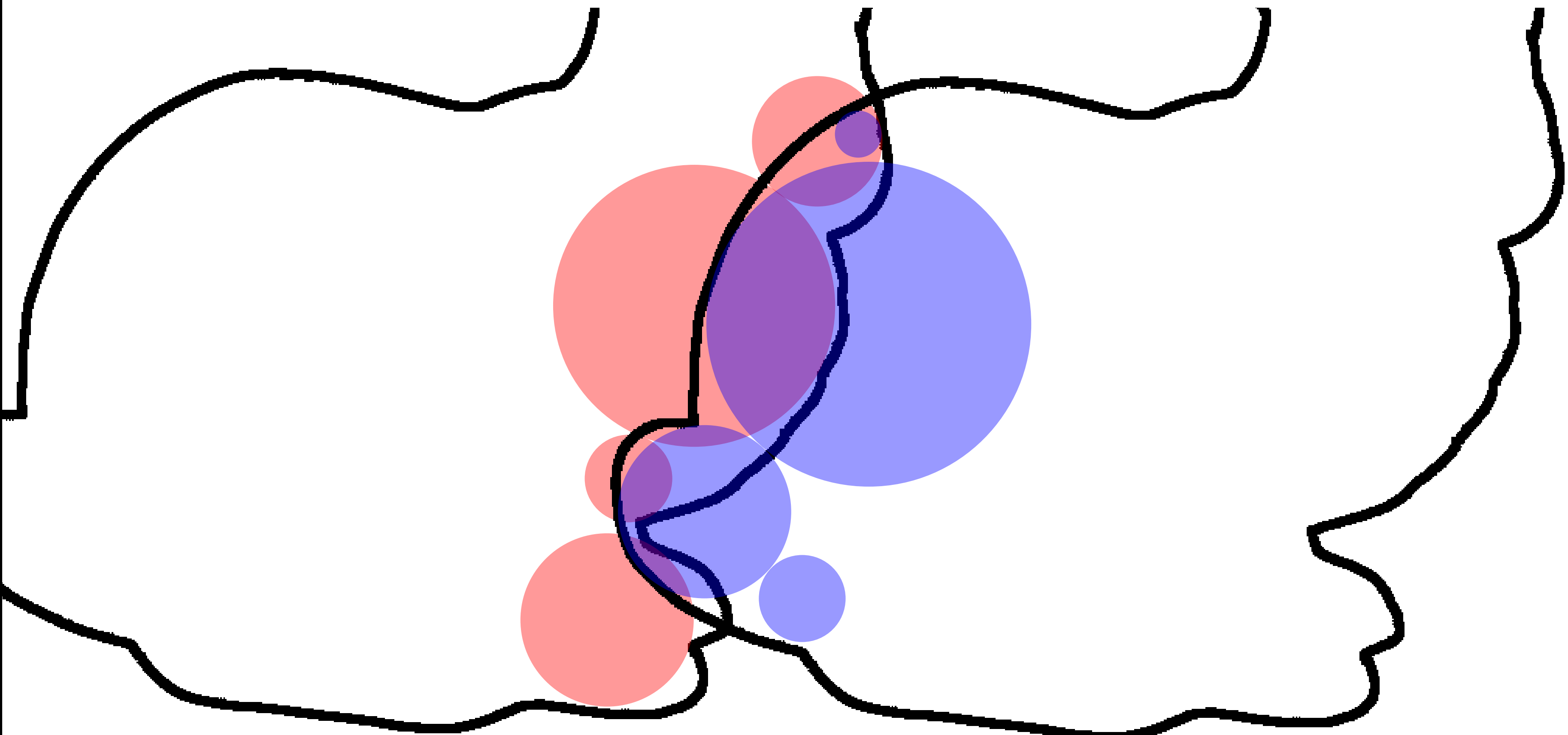




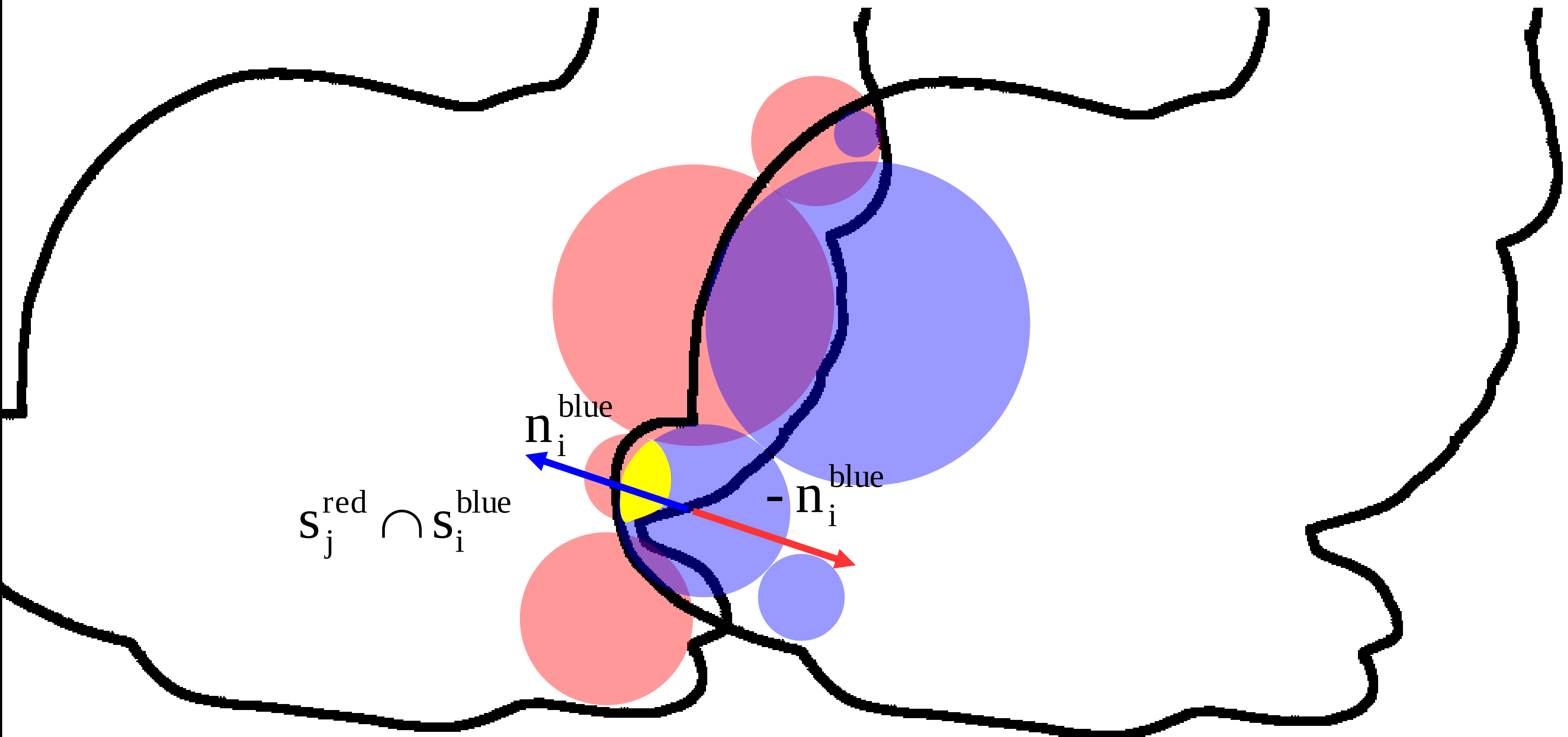


Penetration volume = $v_1 + v_2$

Collision Response: Forces and Torques



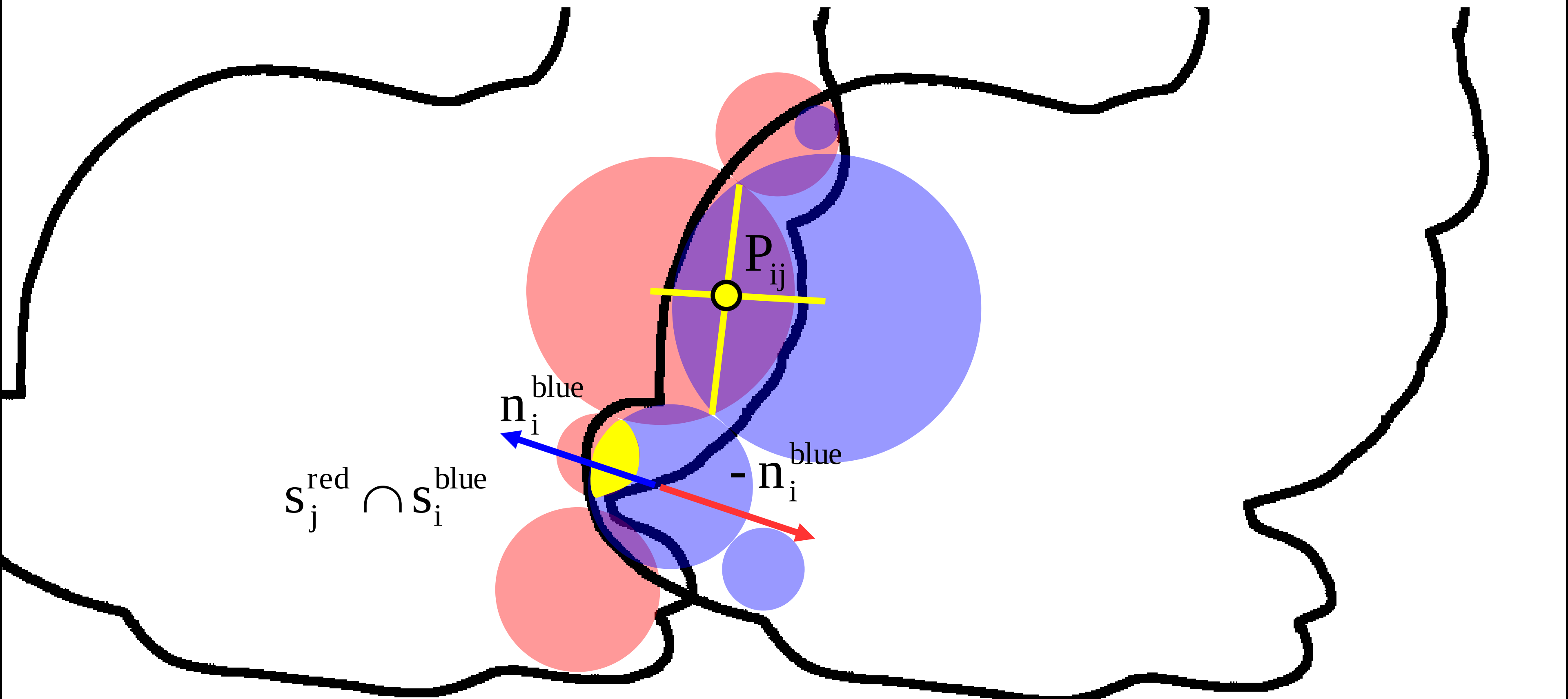
Collision Response: Forces and Torques



$$f_{ij}^{blue} = (s_j^{red} \cap s_i^{blue})(-n_i^{blue})$$

$$f_{total}^{blue} = \sum f_{ij}^{blue}$$

Collision Response: Forces and Torques



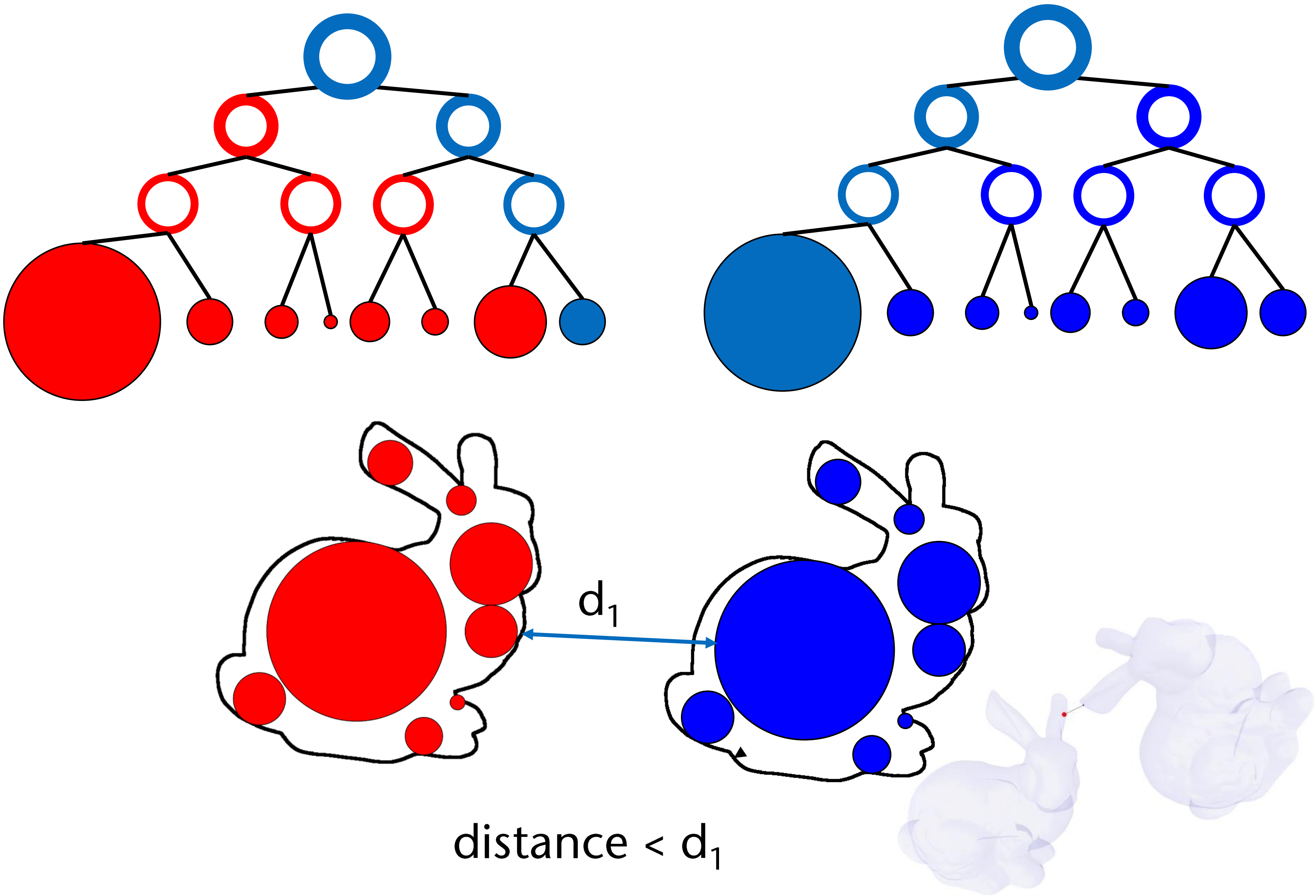
$$f_{ij}^{blue} = (s_j^{red} \cap s_i^{blue})(-n_i^{blue})$$

$$\tau_{ij}^{blue} = (P_{ij} - C_m) \times (f_{ij}^{blue})$$

$$f_{total}^{blue} = \sum f_{ij}^{blue}$$

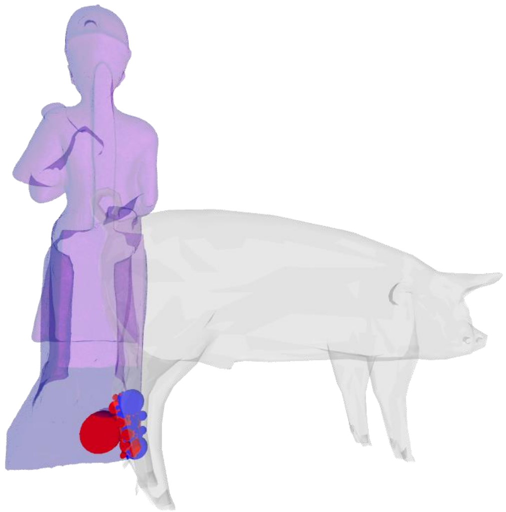
$$\tau_{total}^{blue} = \sum \tau_{ij}^{blue}$$

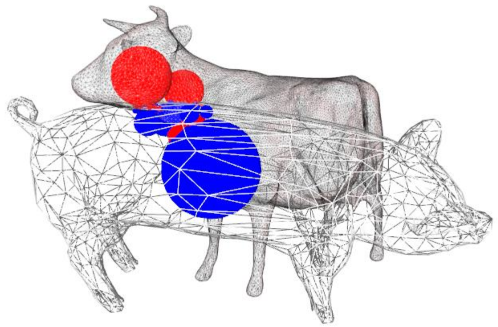
BVH Traversal: Proximity Queries

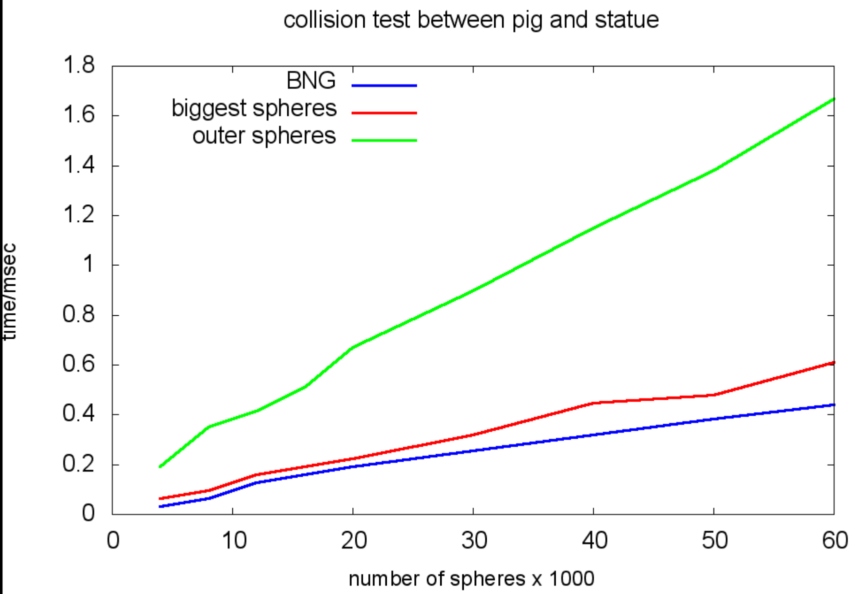


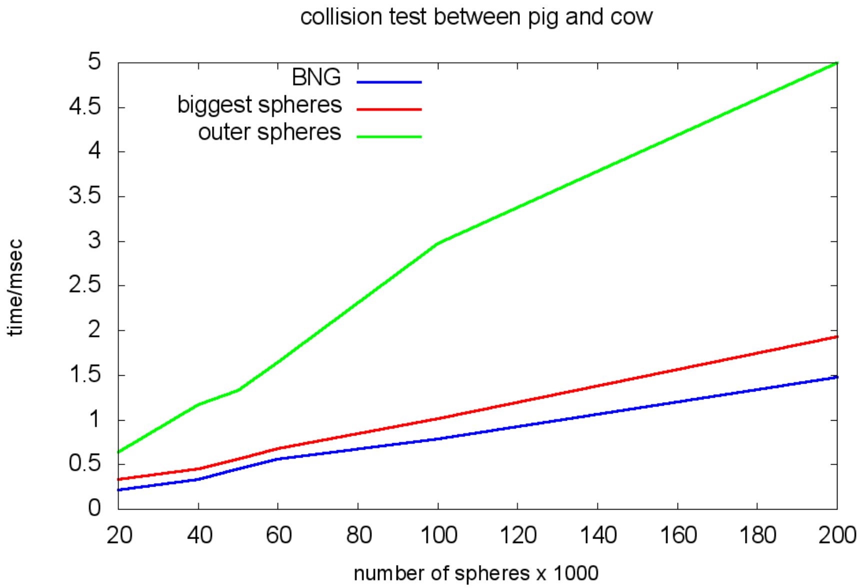
distance < d_1

Results: Runtime Performance









Motivation: Other Object Representations

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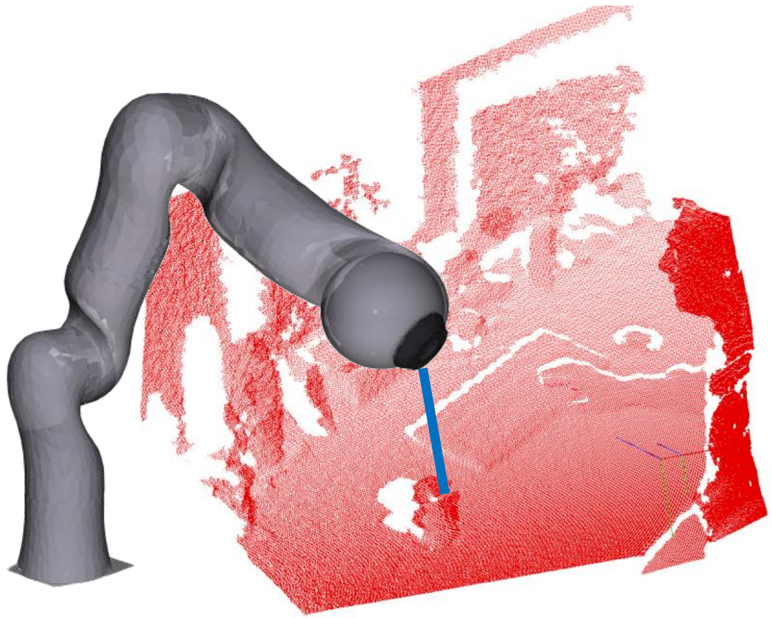

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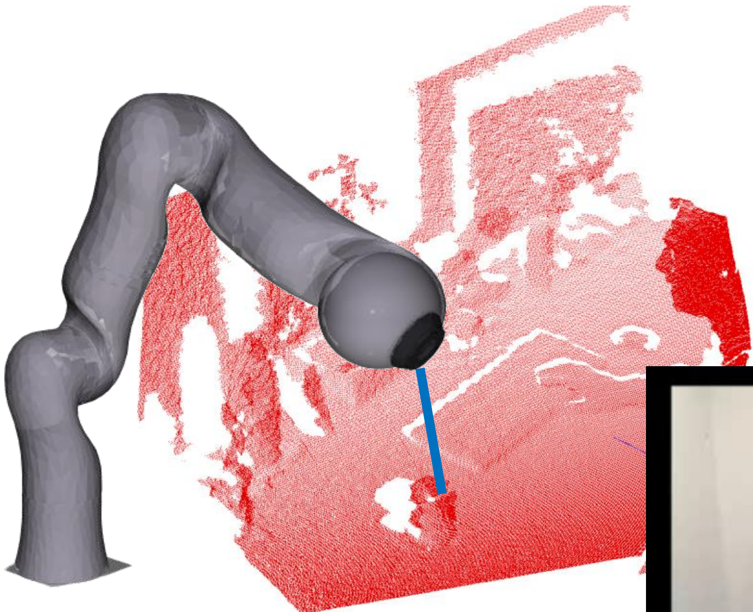
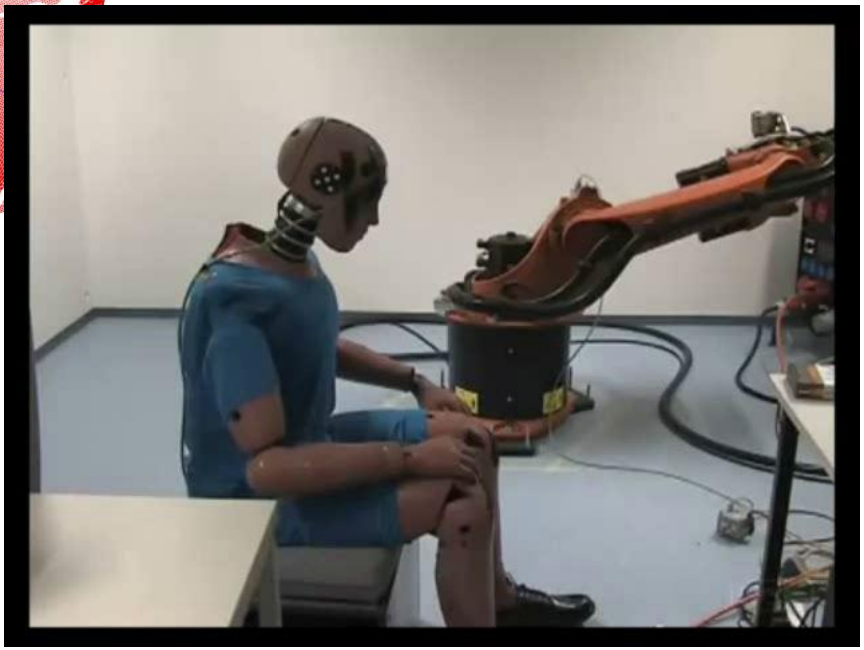
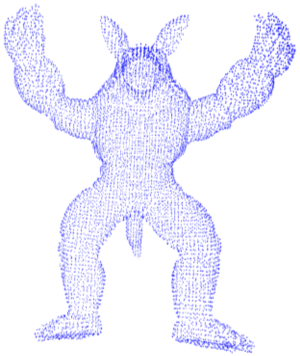
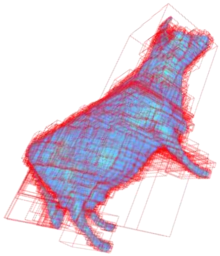




Previous Works





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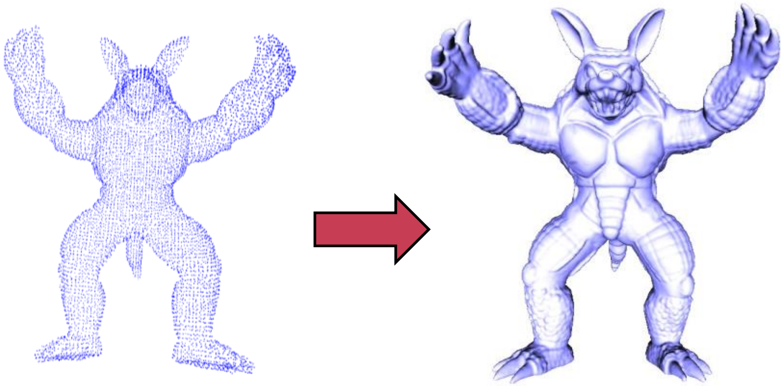

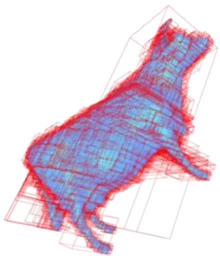

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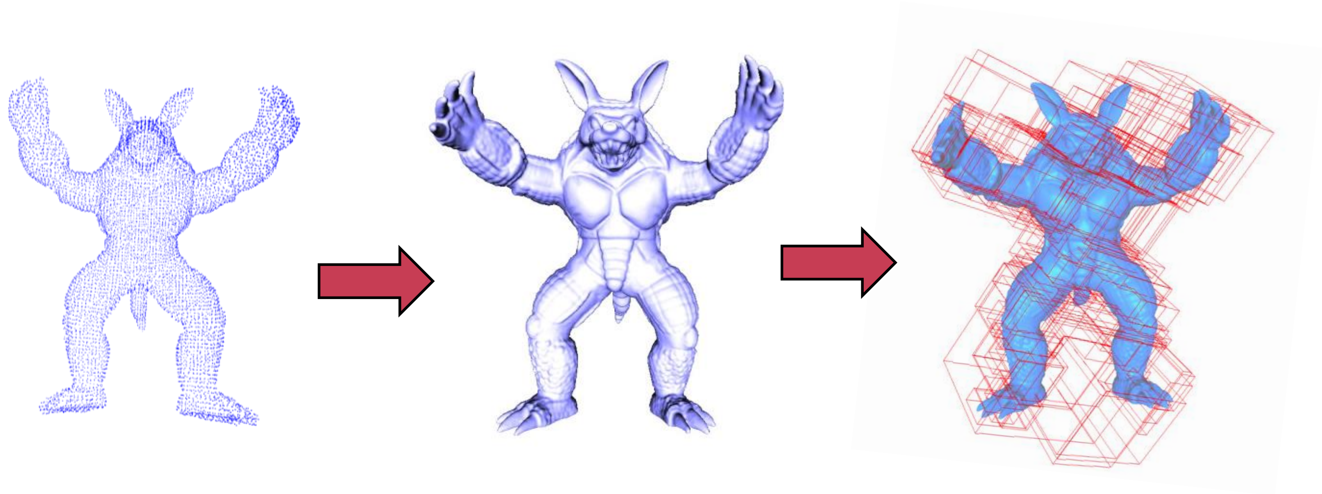

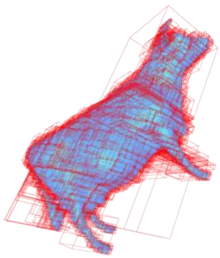

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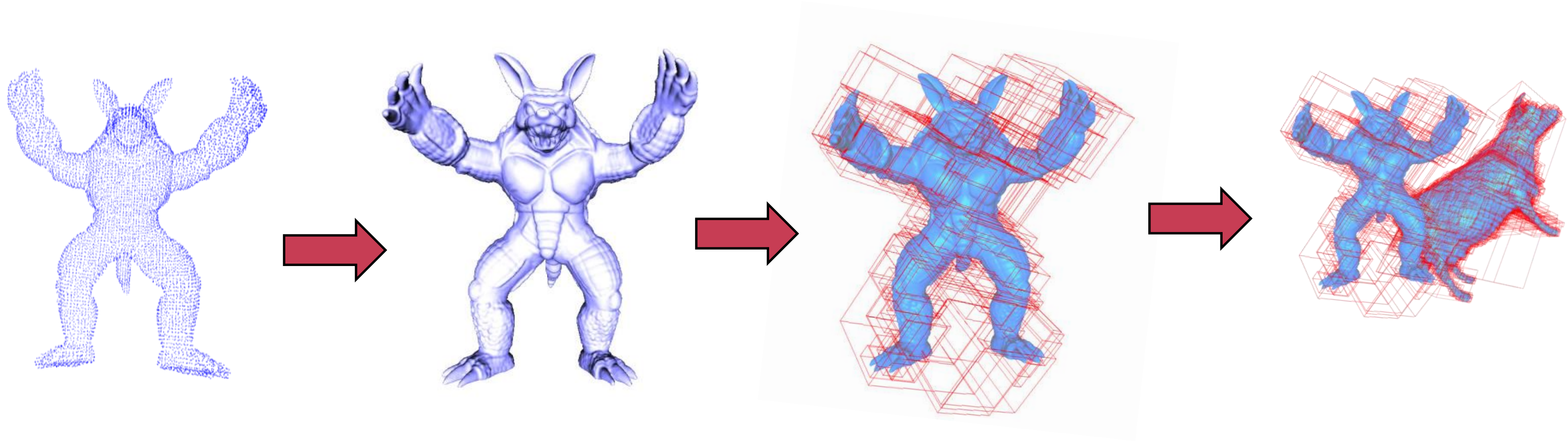

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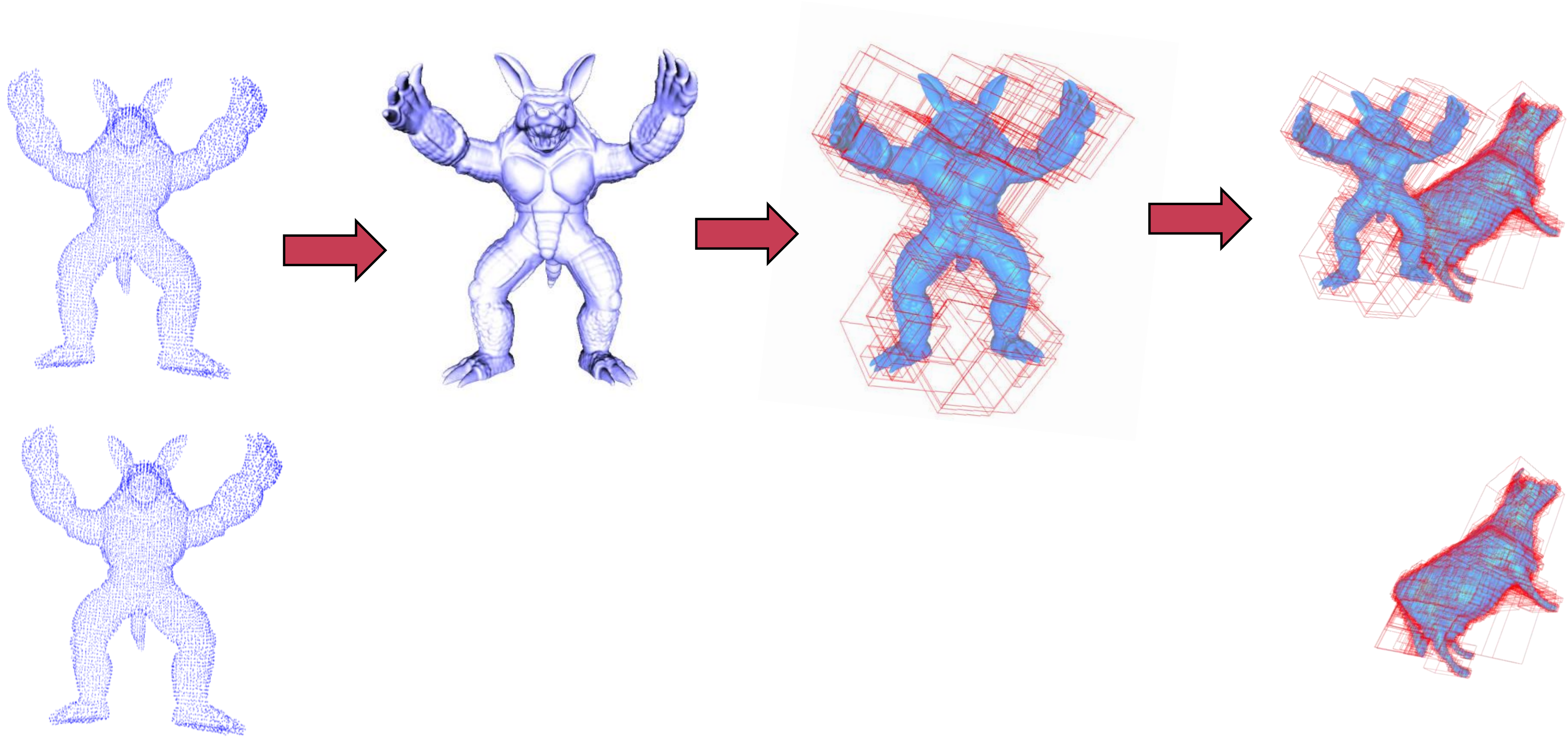

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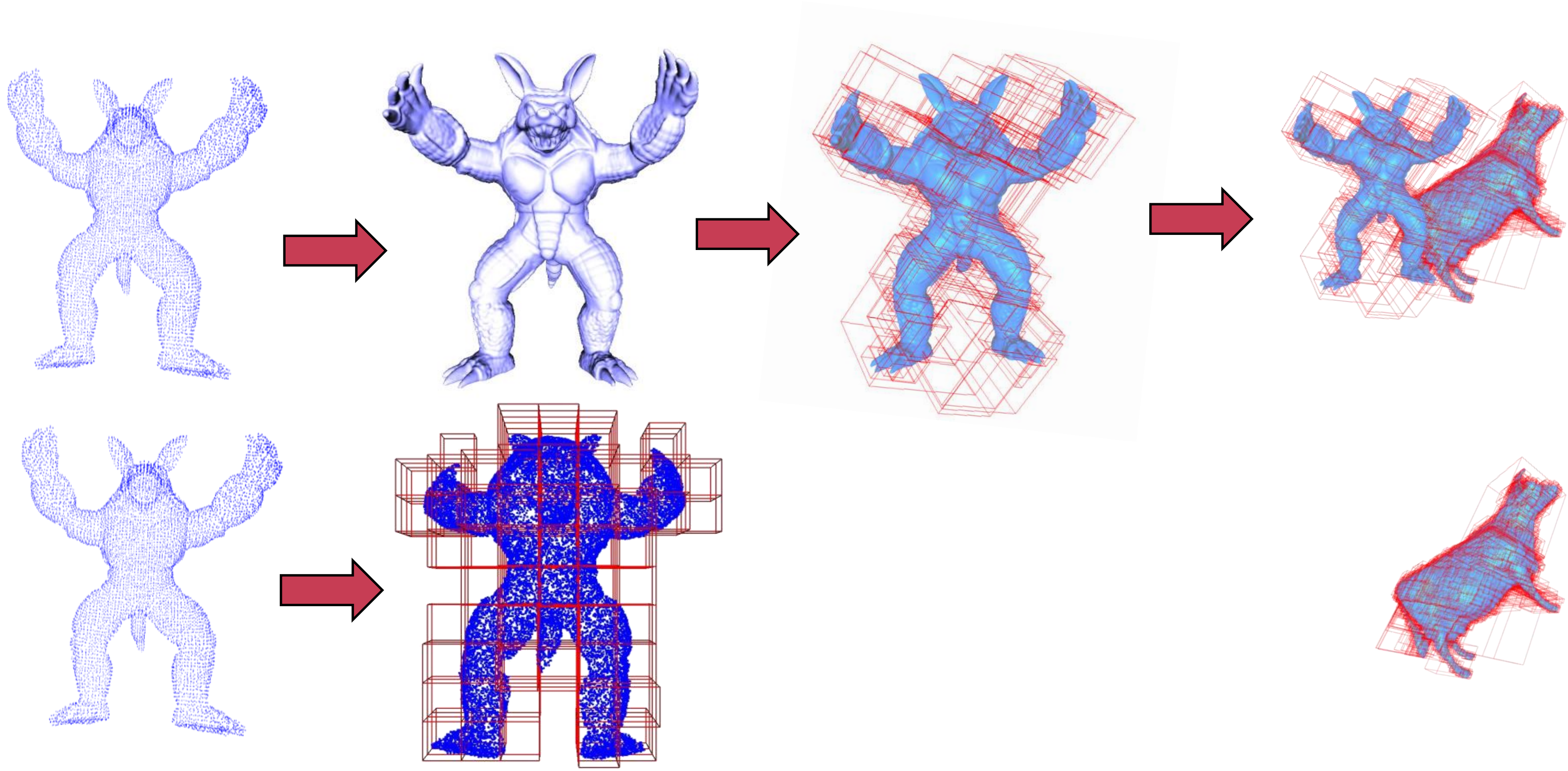

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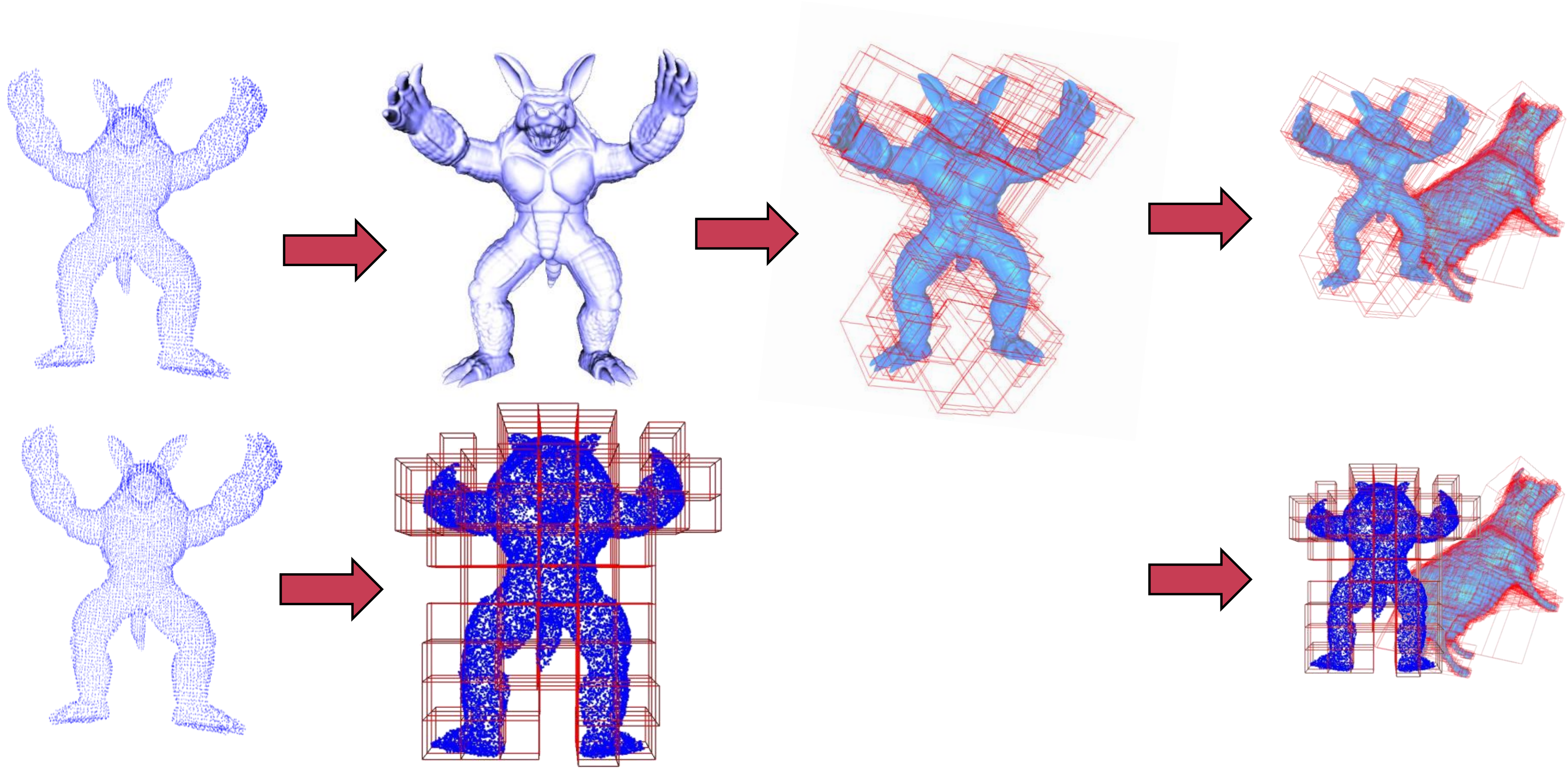

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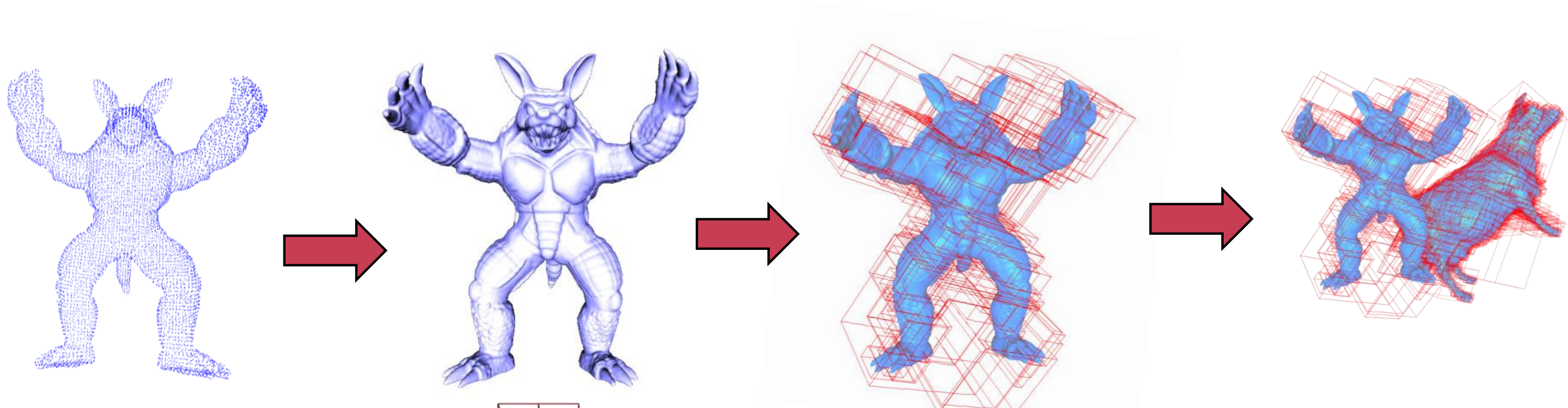

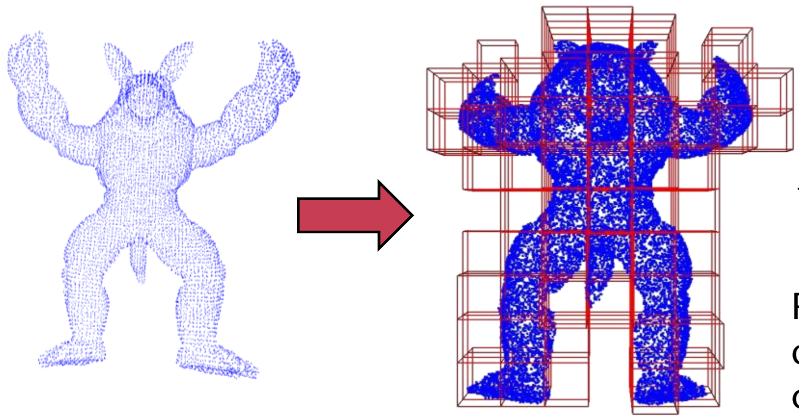

Point cloud collision detection
[Klein and Zachmann, 2004]

New Geometric Data Structures
for Collision Detection
[Weller, 2012]

Real-time collision detection and
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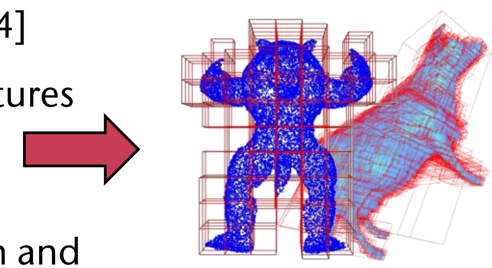

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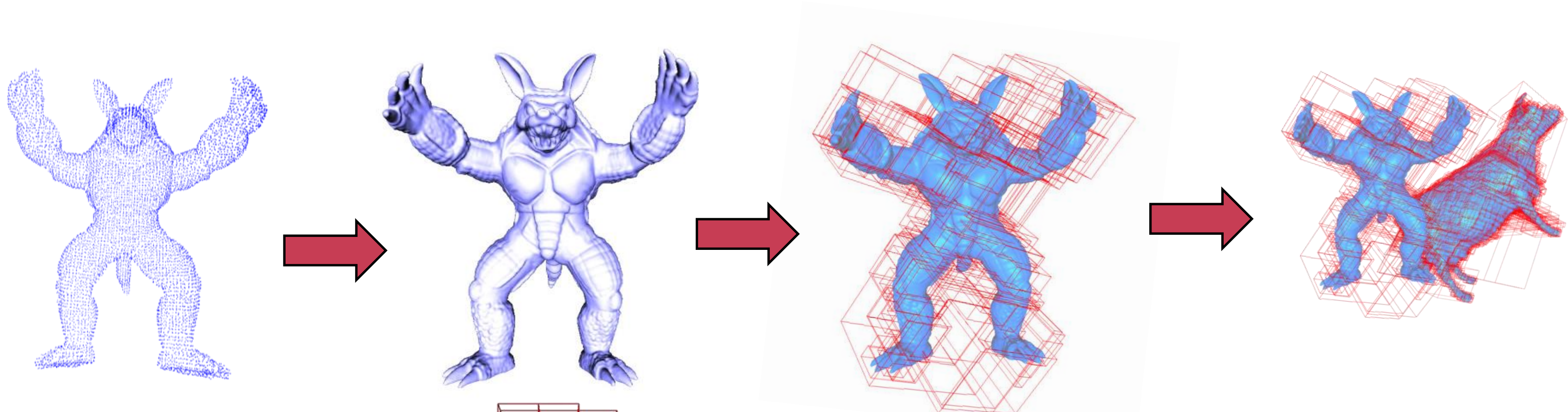

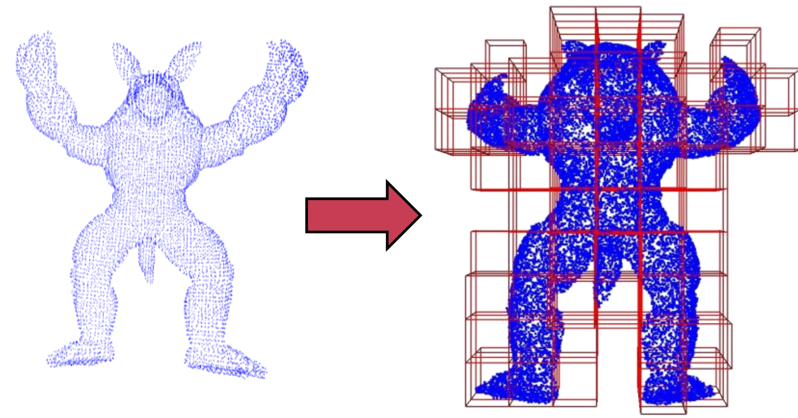

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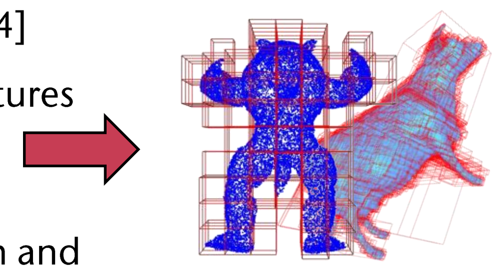

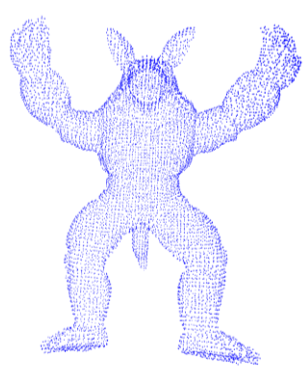

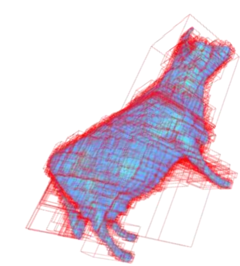

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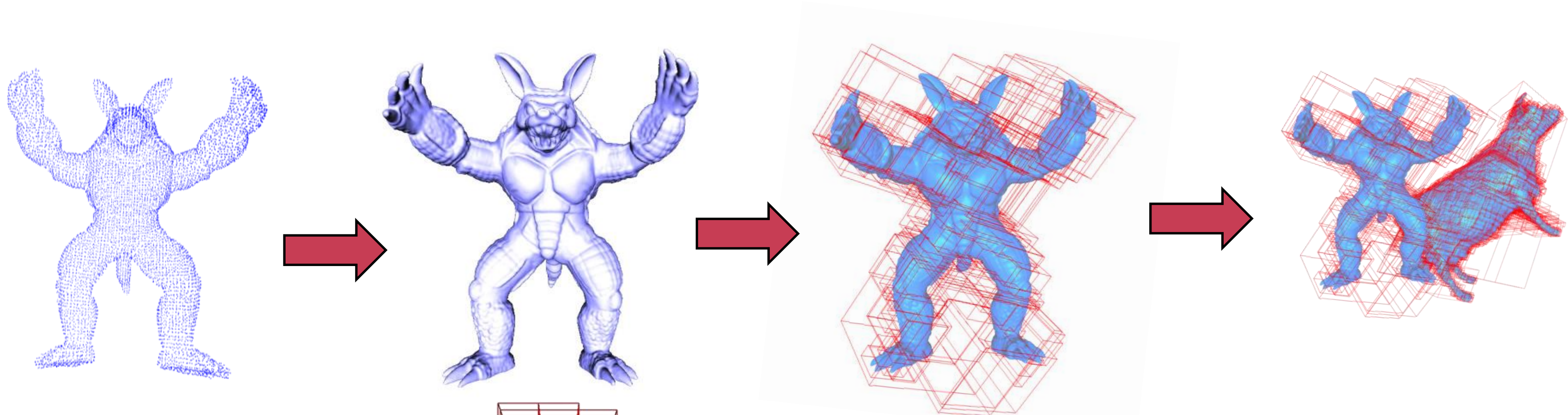

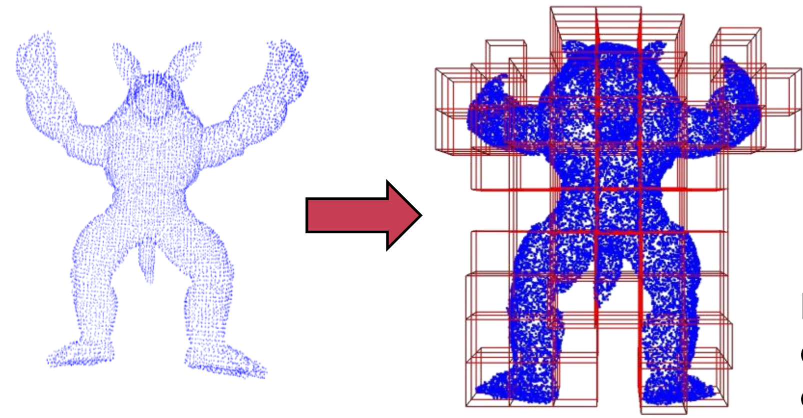

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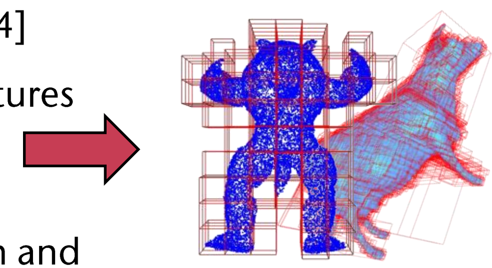
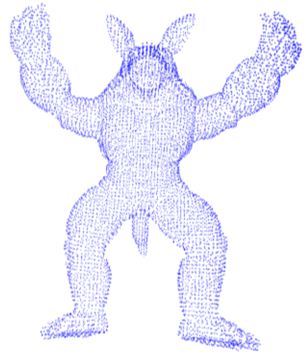
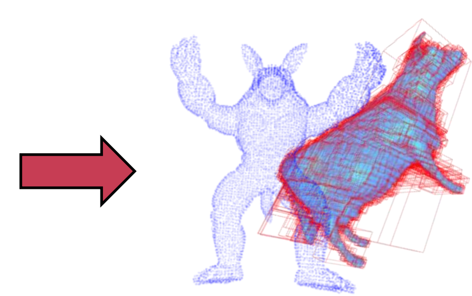
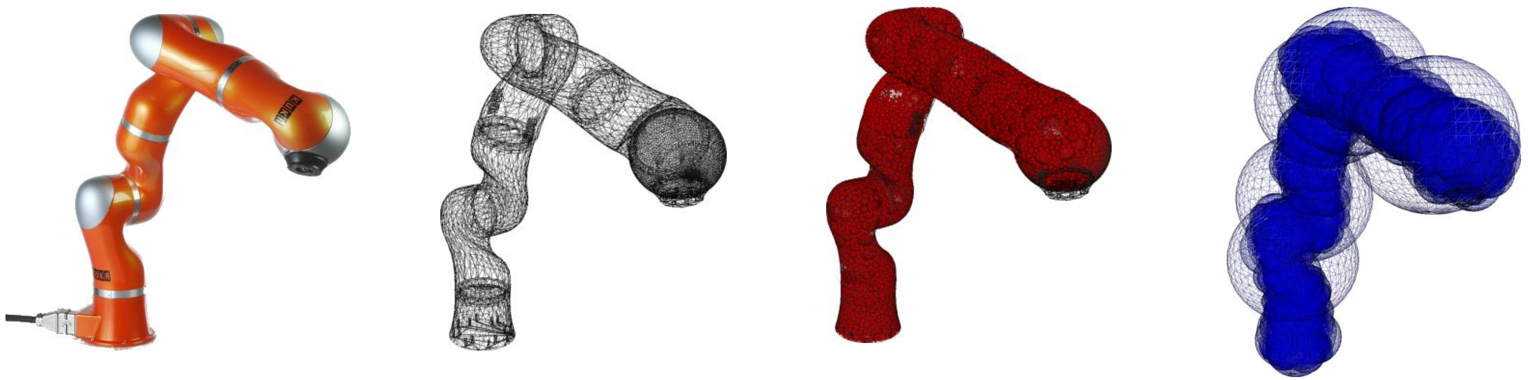






Precondition

- Polygonal object representation: Inner Sphere Trees (ISTs)



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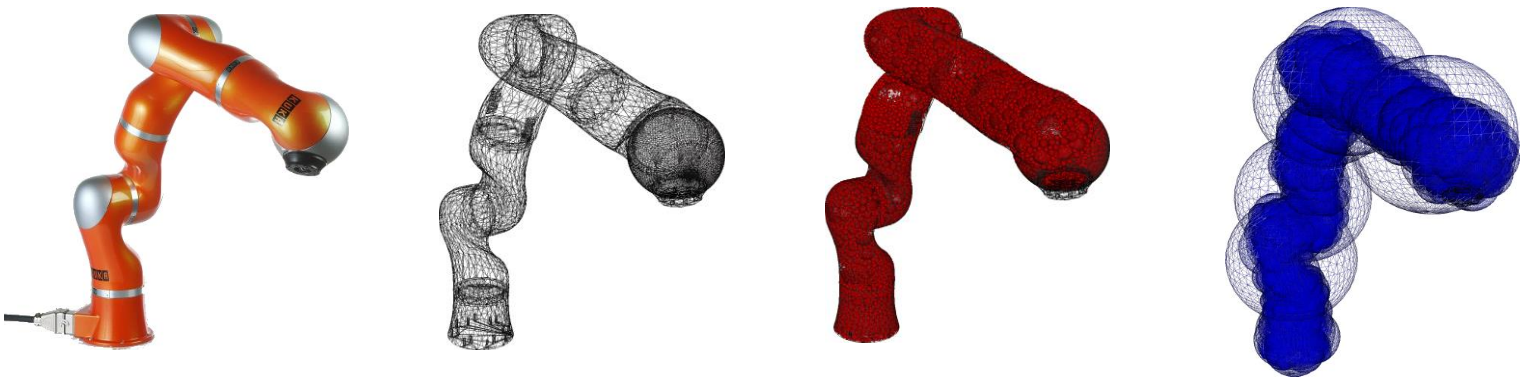

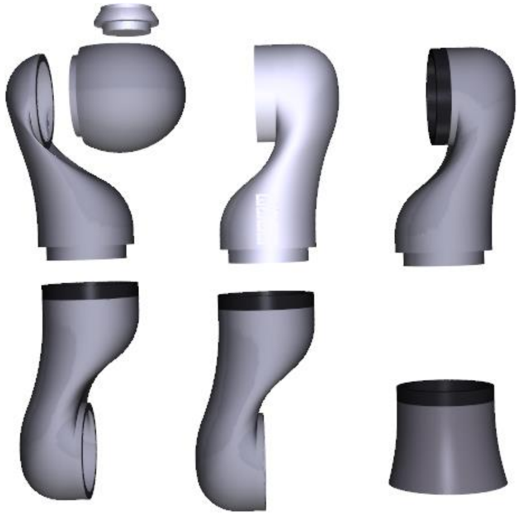

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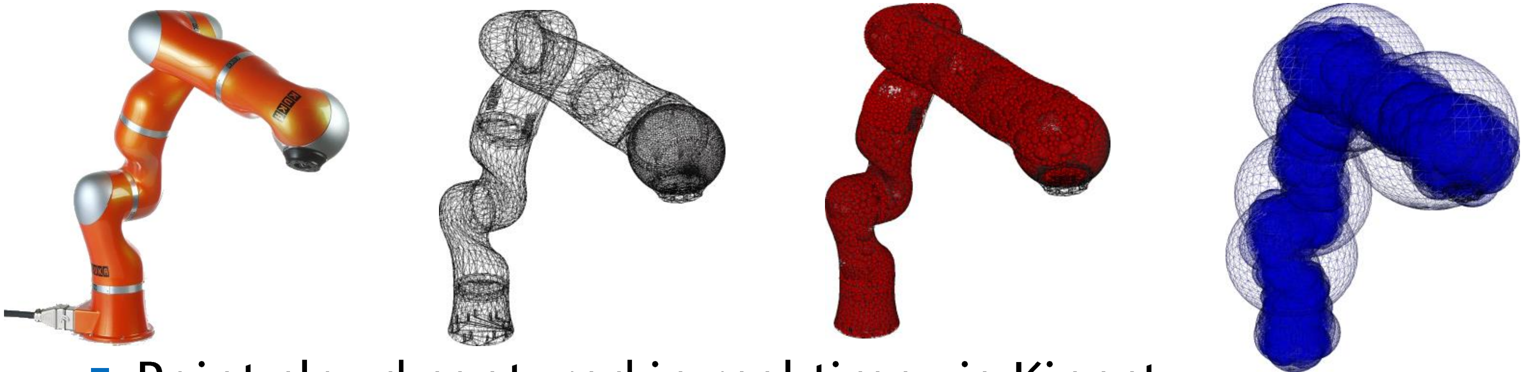
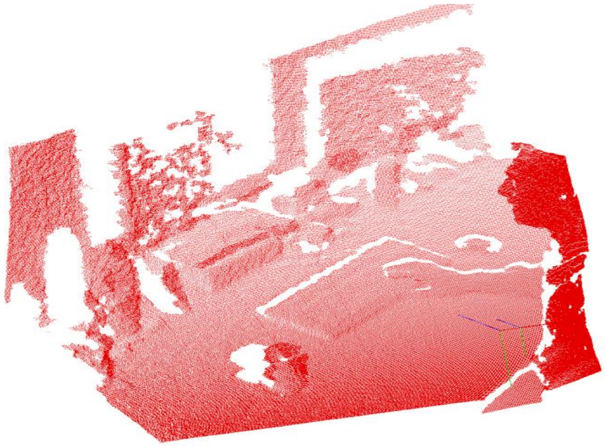
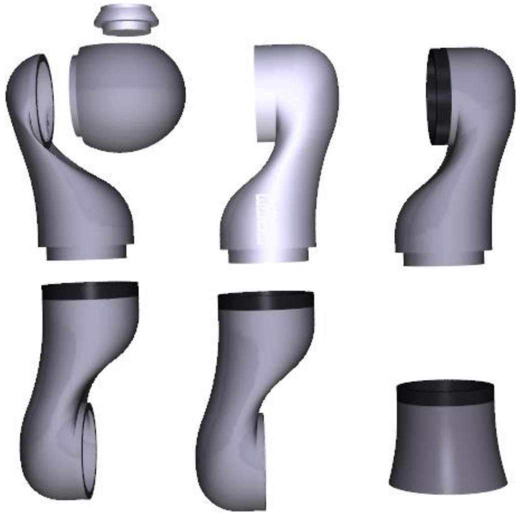
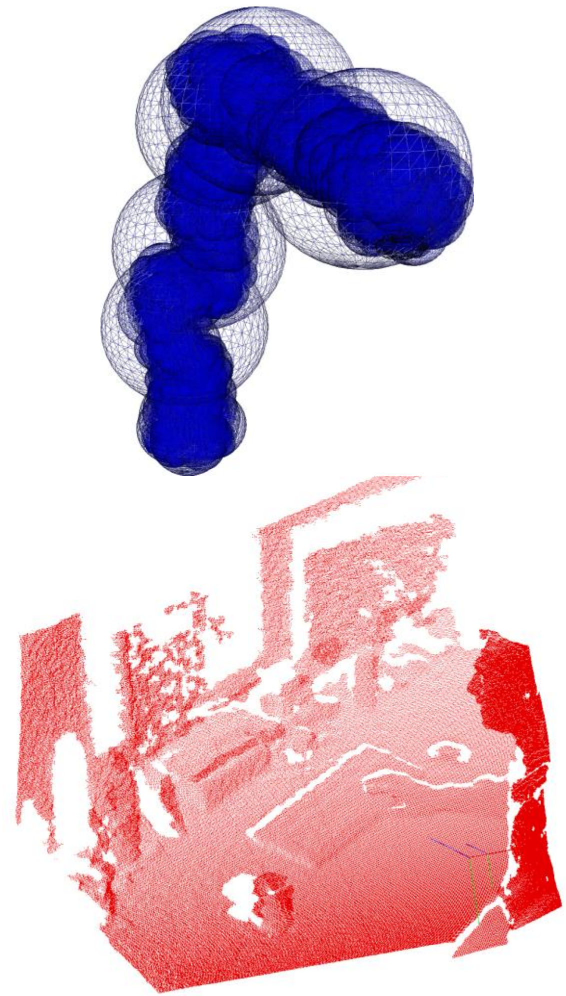


- Point cloud captured in real-time via Kinect



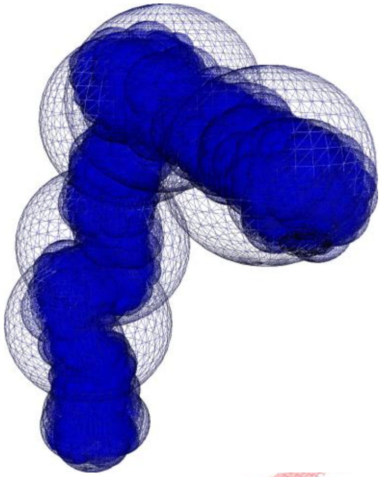


Basic Algorithm



Basic Algorithm

```
minDist = ∞
For each IST ∈ Robot
 For each point p ∈ Point Cloud
   getDistance( Root(IST), p, minDist)
```



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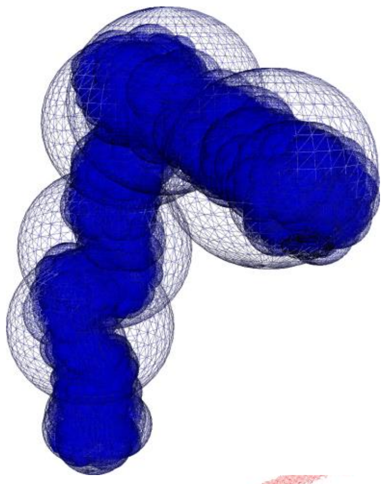

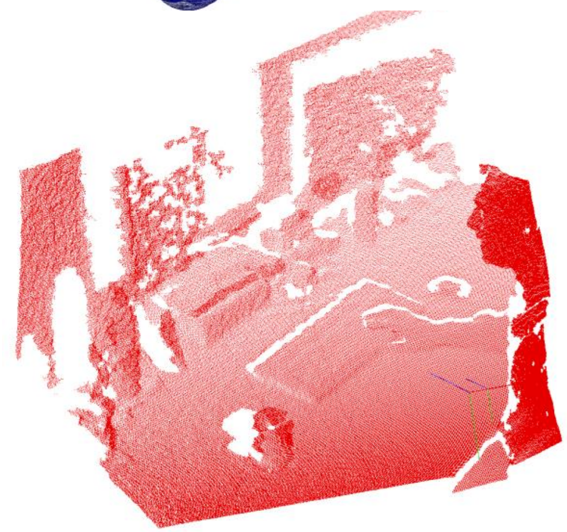

```
getDistance( Sphere s, Point p, d )
  forall the Children sc of s do
    d = distance( sc, p )
    if d < minDist then
      getDistance( sc, p, d )
```

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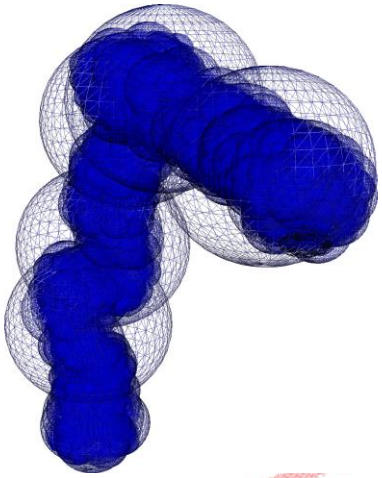
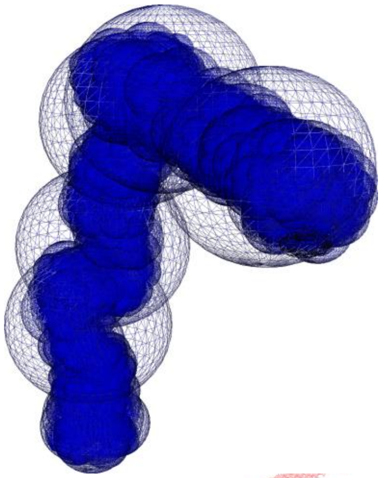
      getDistance( sc, p, d )
  if s is Leaf then
    minDist = min( d, minDist )
```



Parallel Algorithm

```
minDist = ∞
For each IST ∈ Robot
 For each point p ∈ Point Cloud
   getDistance( Root(IST), p, minDist)
```



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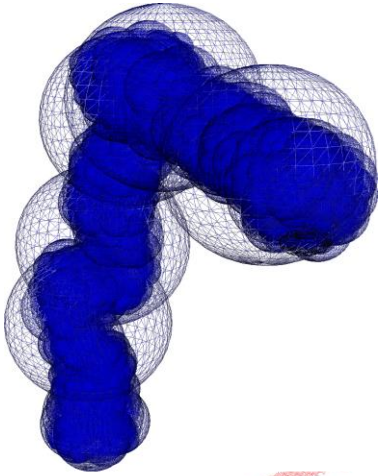

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```

Parallel Algorithm

```
minDist = ∞
In Parallel ISTs ∈ Robot:
  For each point p ∈ Point Cloud
    getDistance( Root(IST), p, minDist)
```

```
getDistance( Sphere s, Point p, d )
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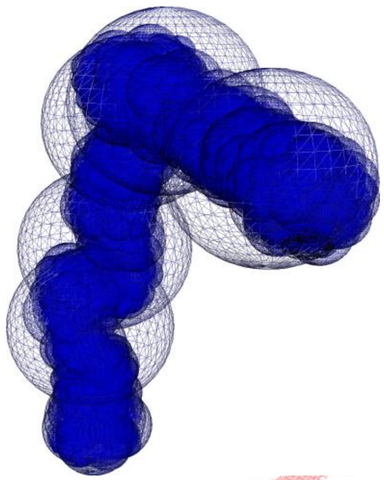

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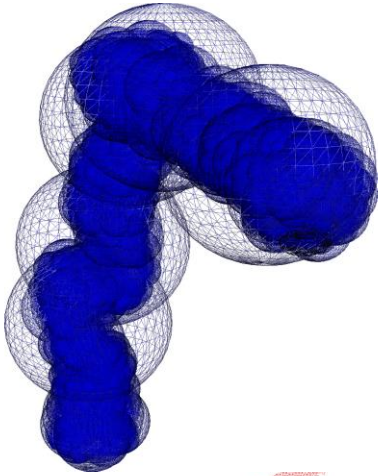

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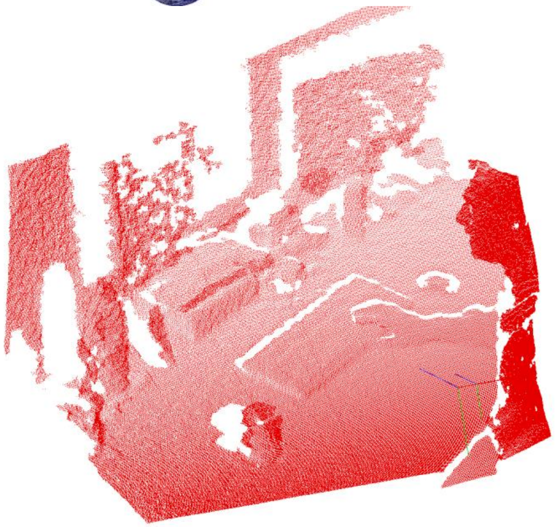

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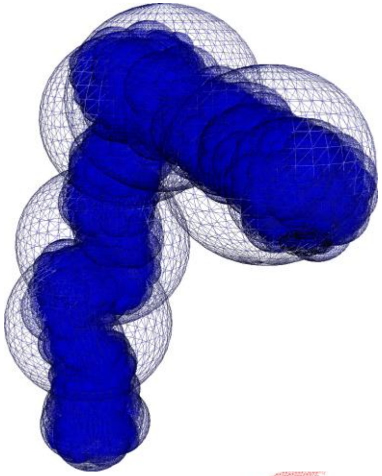

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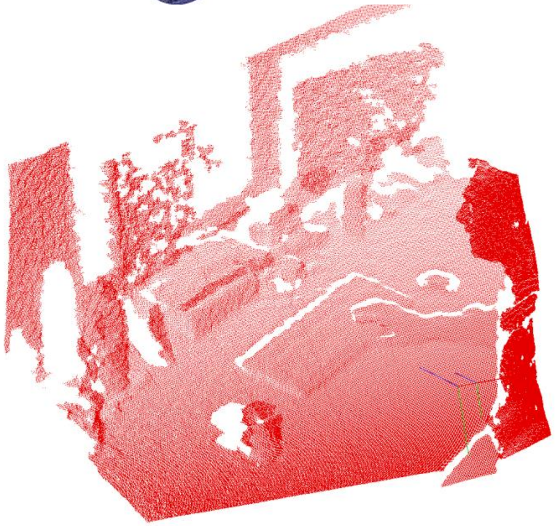

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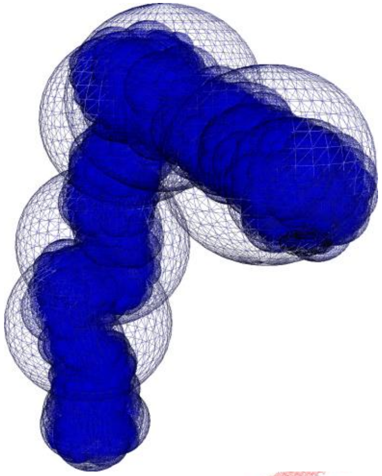
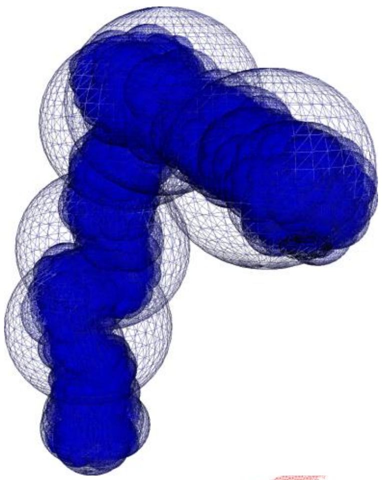


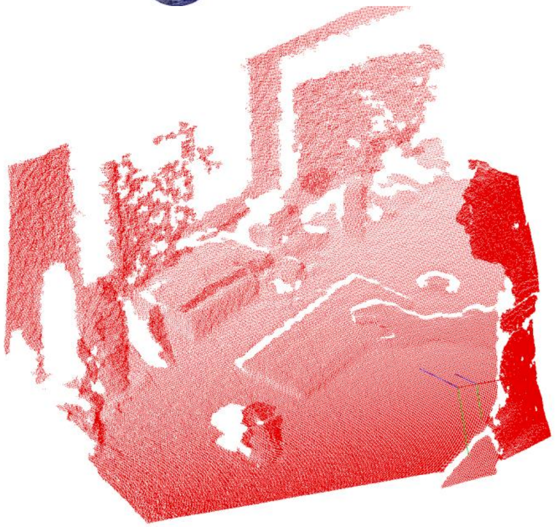
```
getDistance( Sphere s, Point p, d )
  forall the Children sc of s do
    d = distance( sc, p )
    if d < minDist then
      getDistance( sc, p, d )
  if s is Leaf then
    minDist = min( d, minDist )
```

Parallel Algorithm

```
minDist = ∞
In Parallel ISTs ∈ Robot:
 In Parallel point p ∈ Point Cloud:
   getDistance( Root(IST), p, minDist)
```

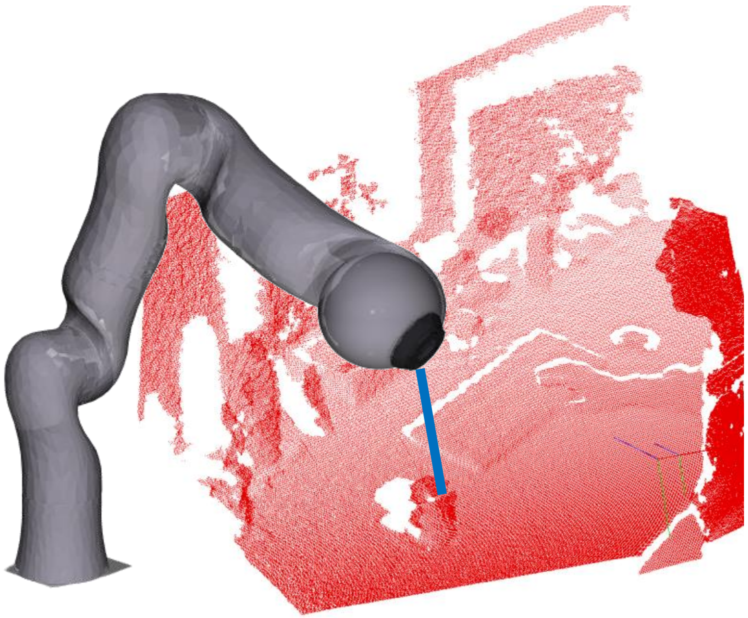
```
getDistance( Sphere s, Point p, d )
  forall the Children sc of s do
    d = distance( sc, p )
    if d < minDist then
      getDistance( sc, p, d )
  if s is Leaf then
    minDist = atomicMin( d, minDist )
```

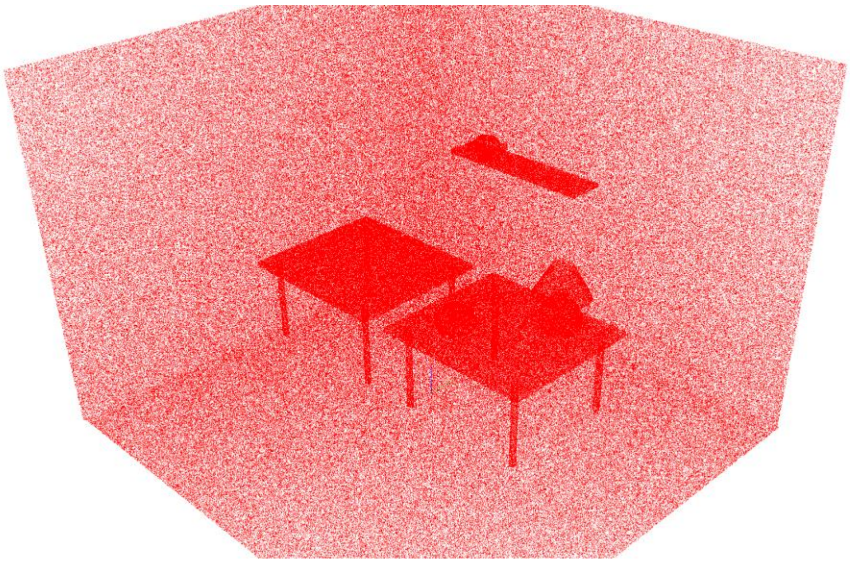




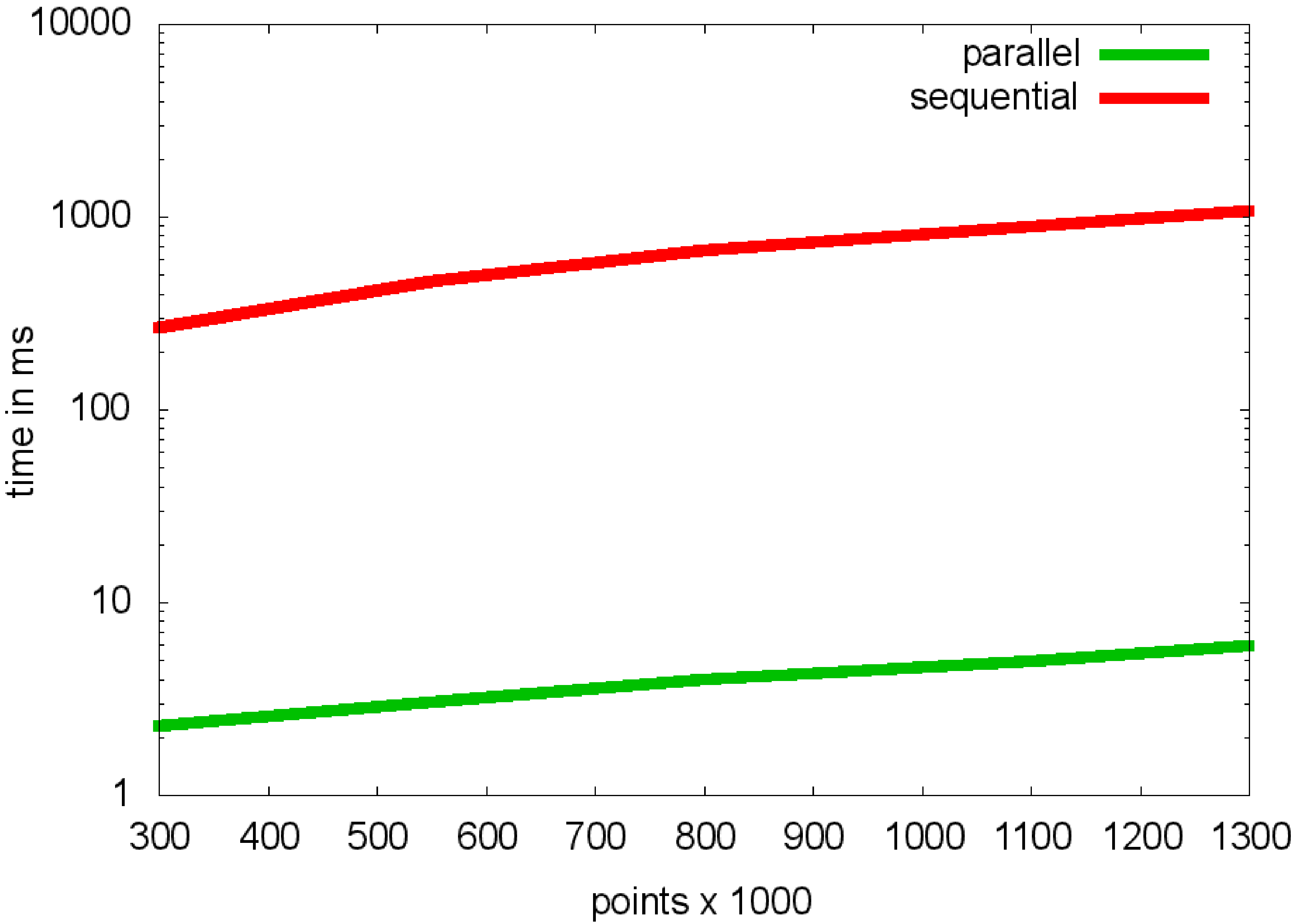
Test Scenarios

- Implemented in CUDA (5.5 & 6.0)
- Geforce GTX 780, 2GByte Memory
- Pre-recorded and artificial point clouds with up tp 5M points

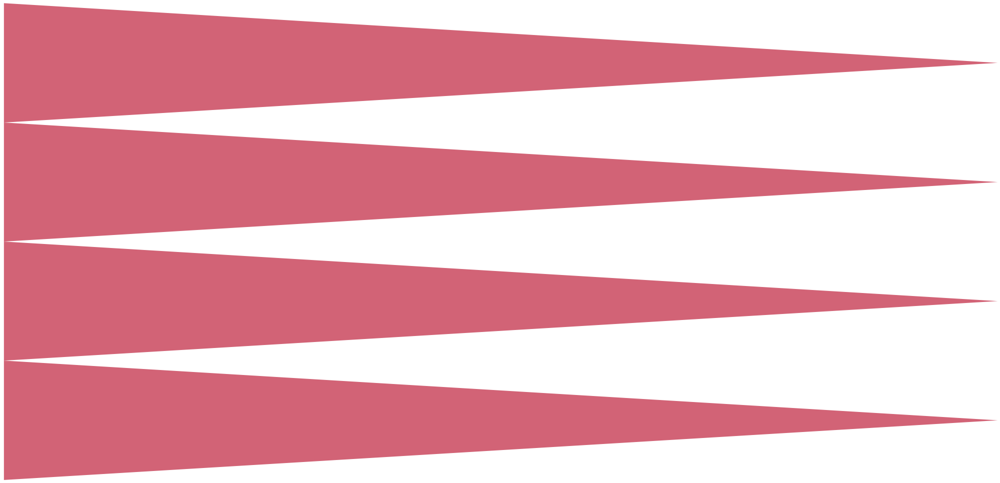




Results: Parallel vs. Sequential

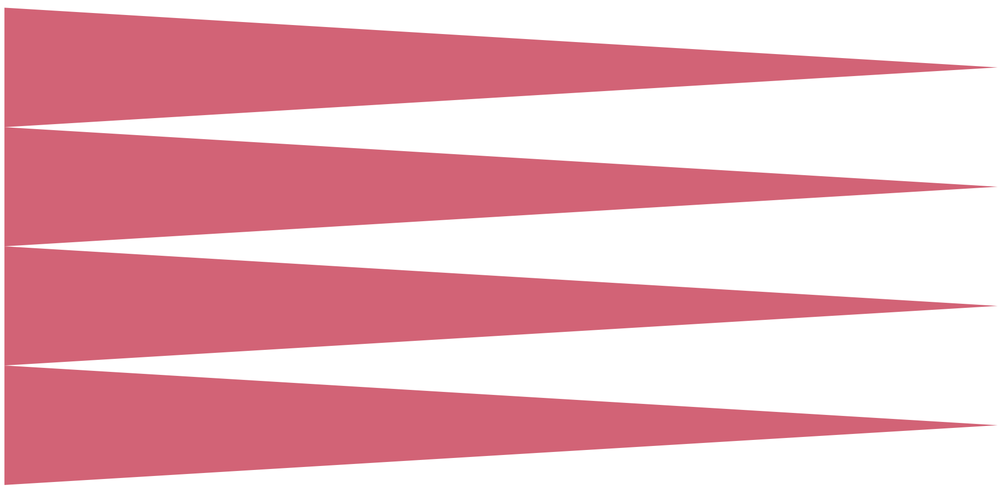


Collision Detection beyond BVHs



Collision Detection beyond BVHs

- BVHs are just a heuristic



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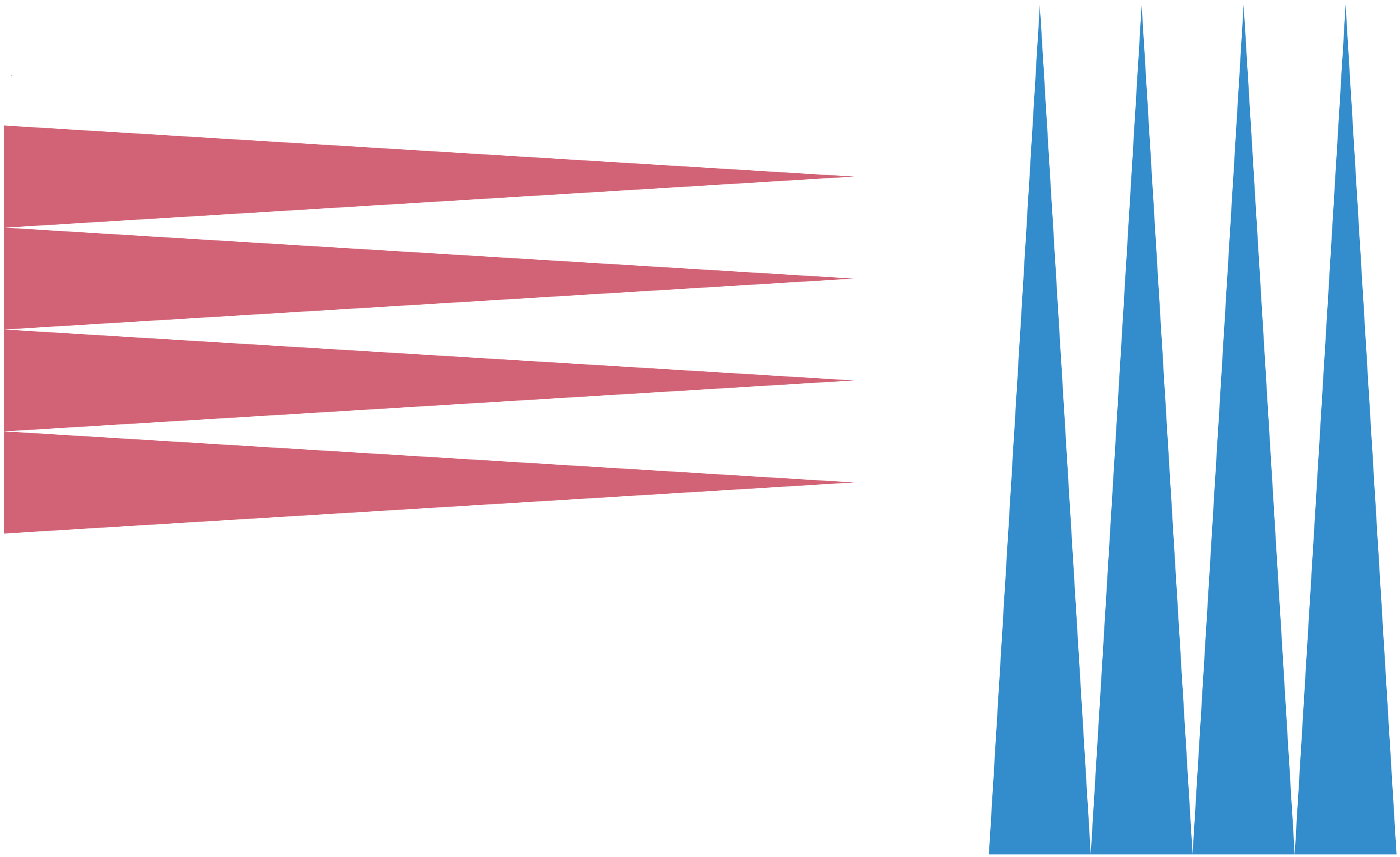

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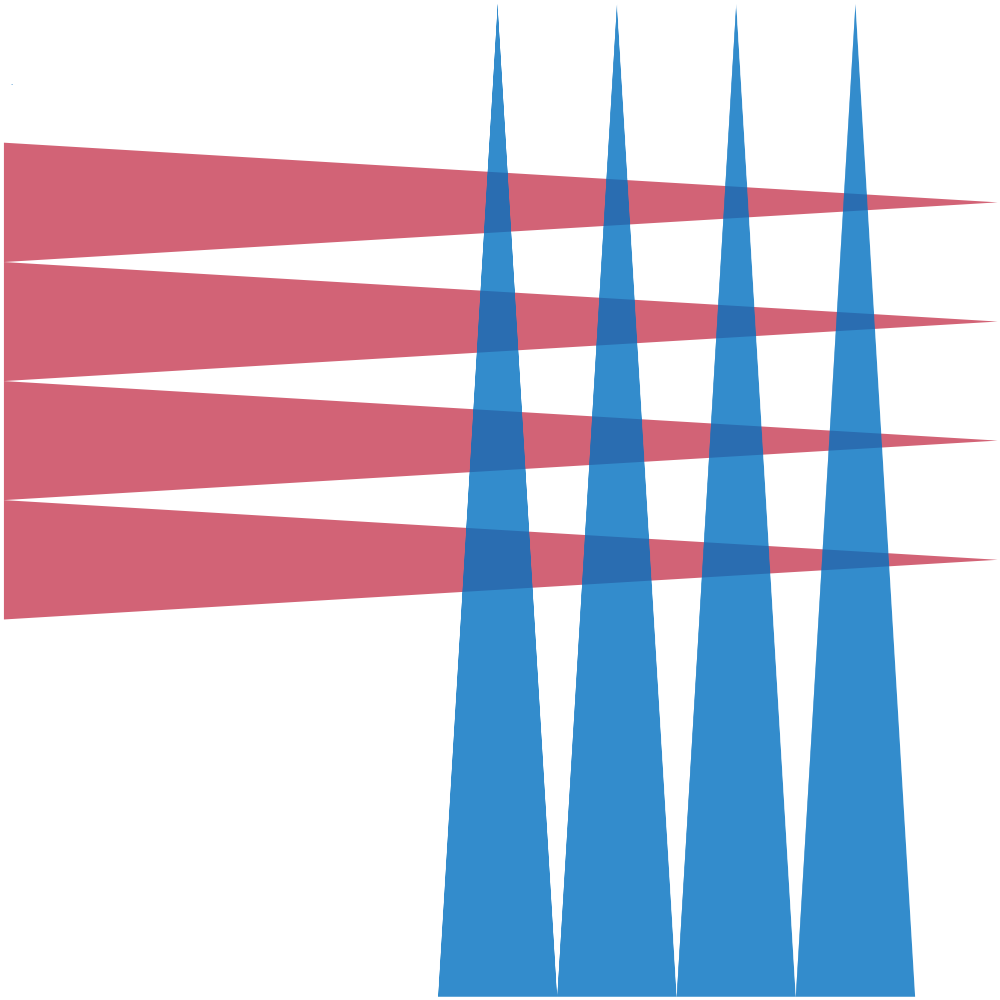

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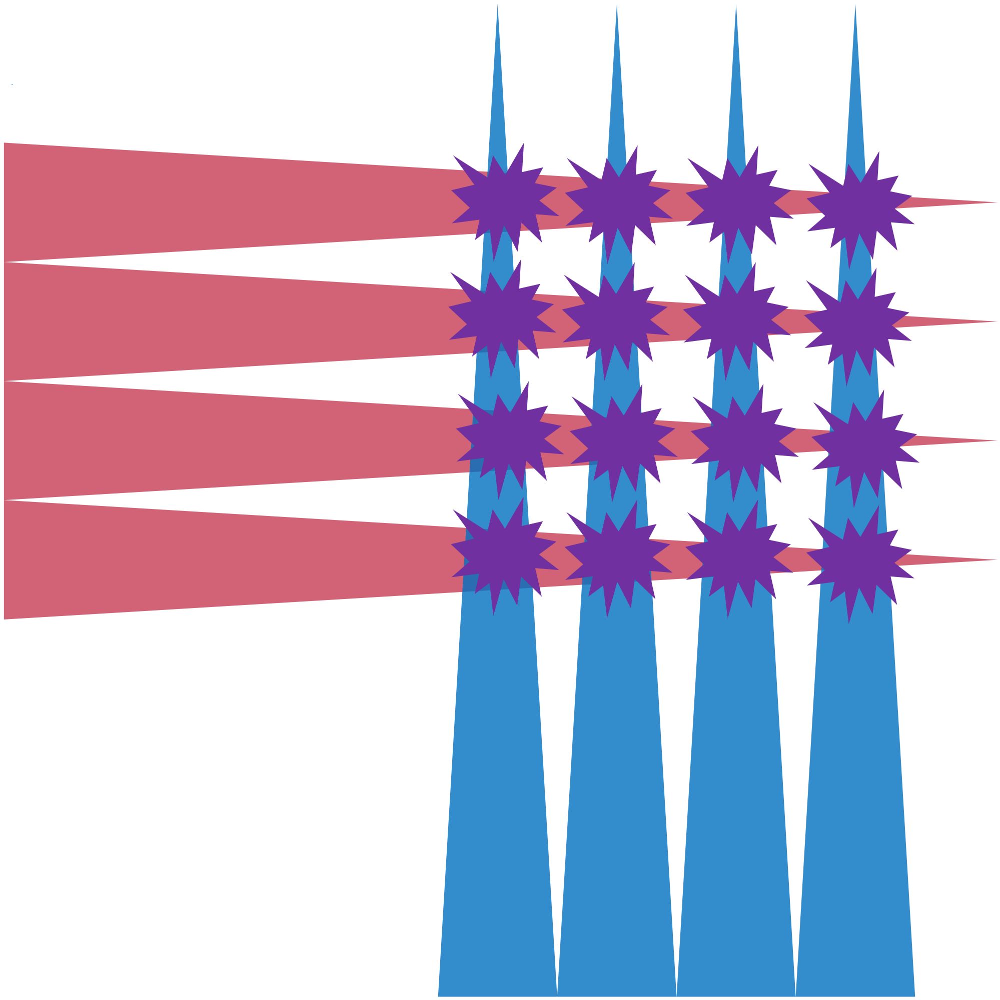
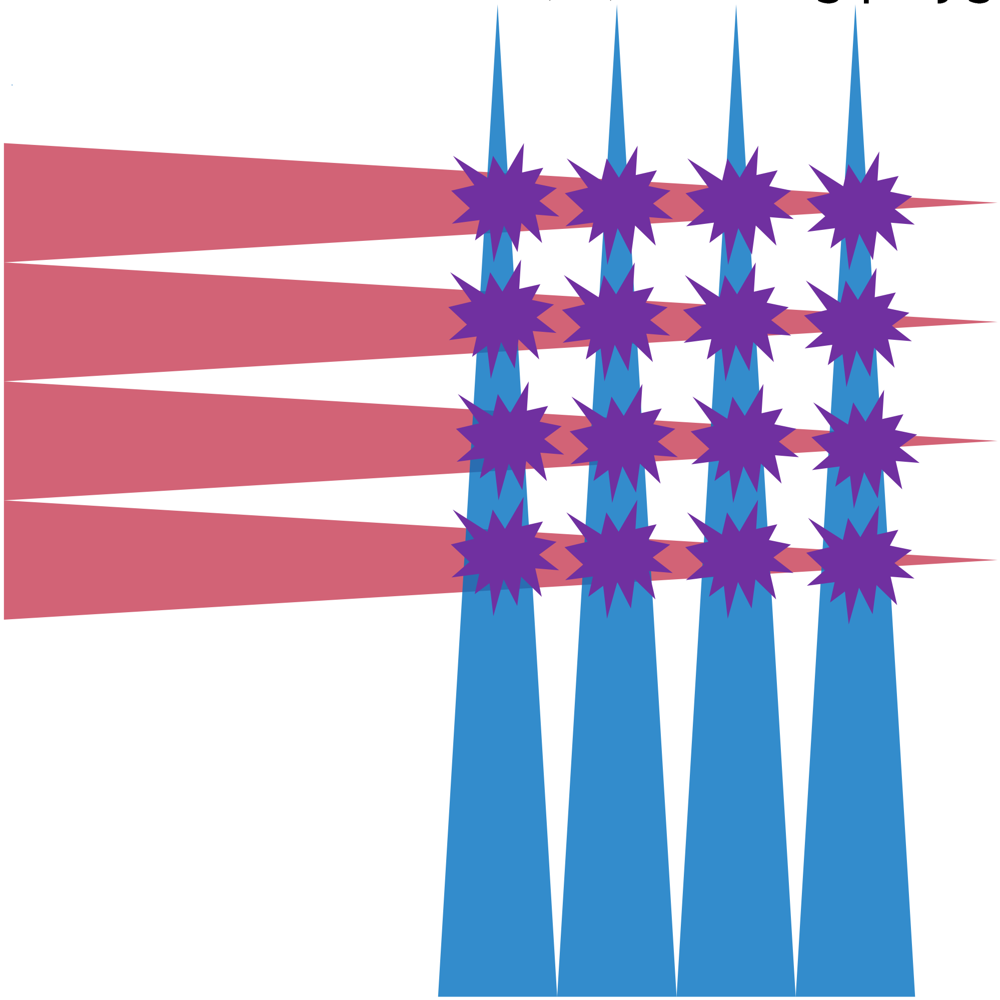


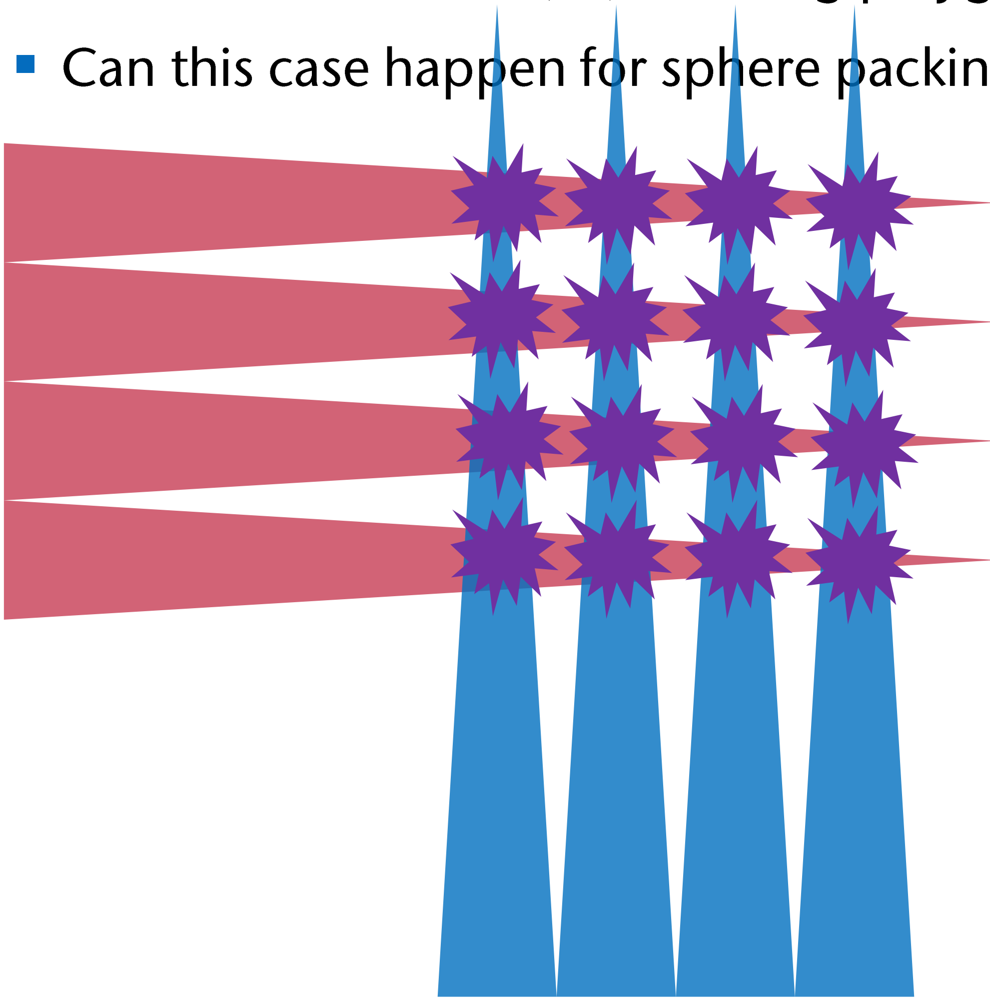
Collision Detection beyond BVHs

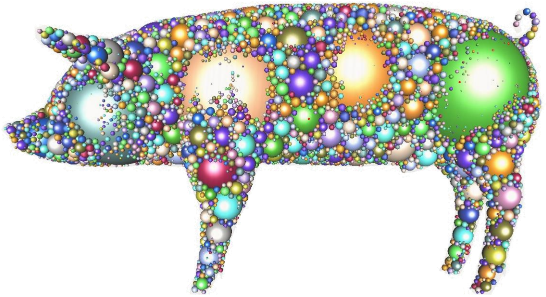
- BVHs are just a heuristic
- In the worst case: $O(n^2)$ colliding polygons



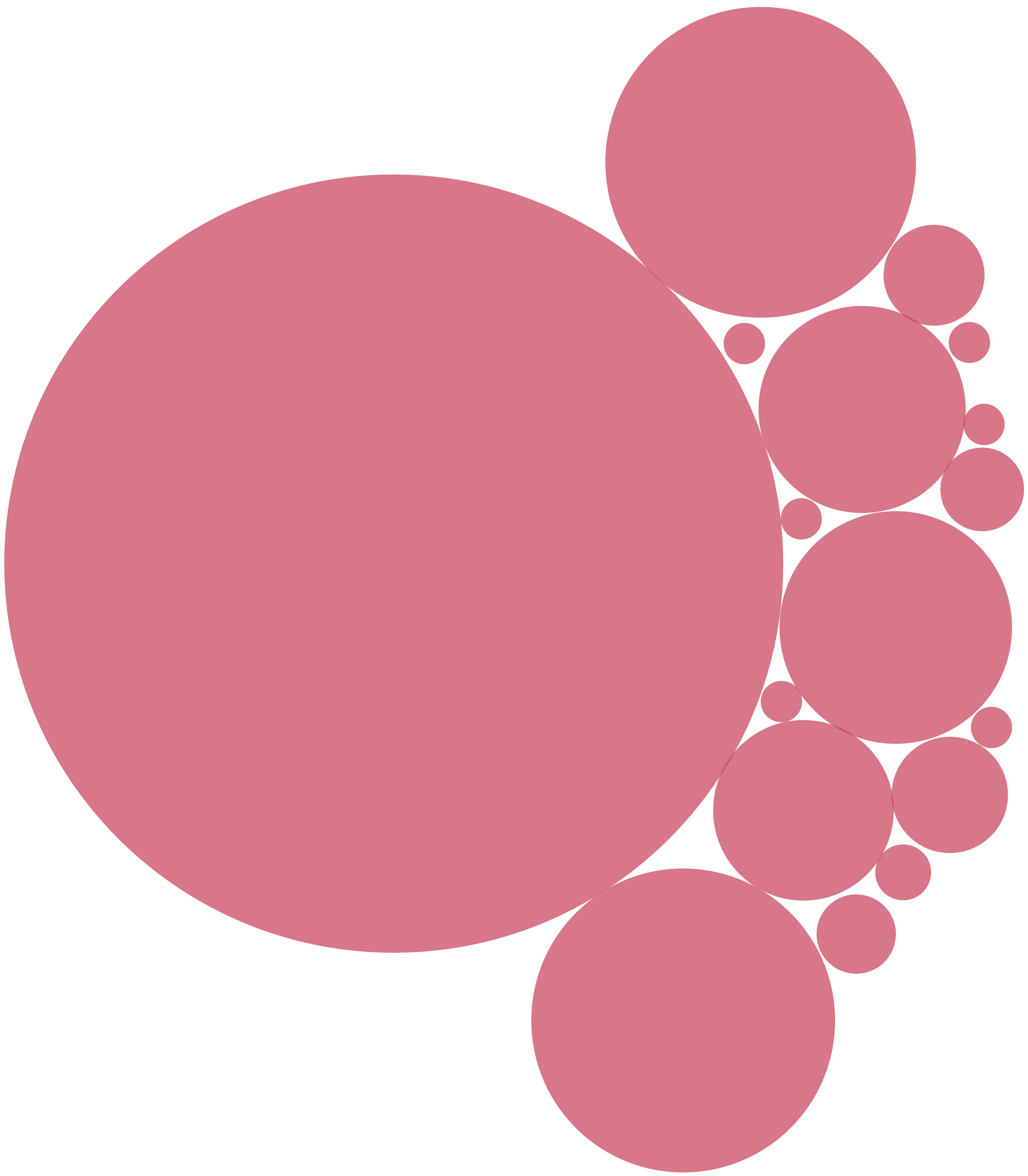
Collision Detection beyond BVHs

- BVHs are just a heuristic
- In the worst case: $O(n^2)$ colliding polygons
- Can this case happen for sphere packings?





Theoretical Foundation



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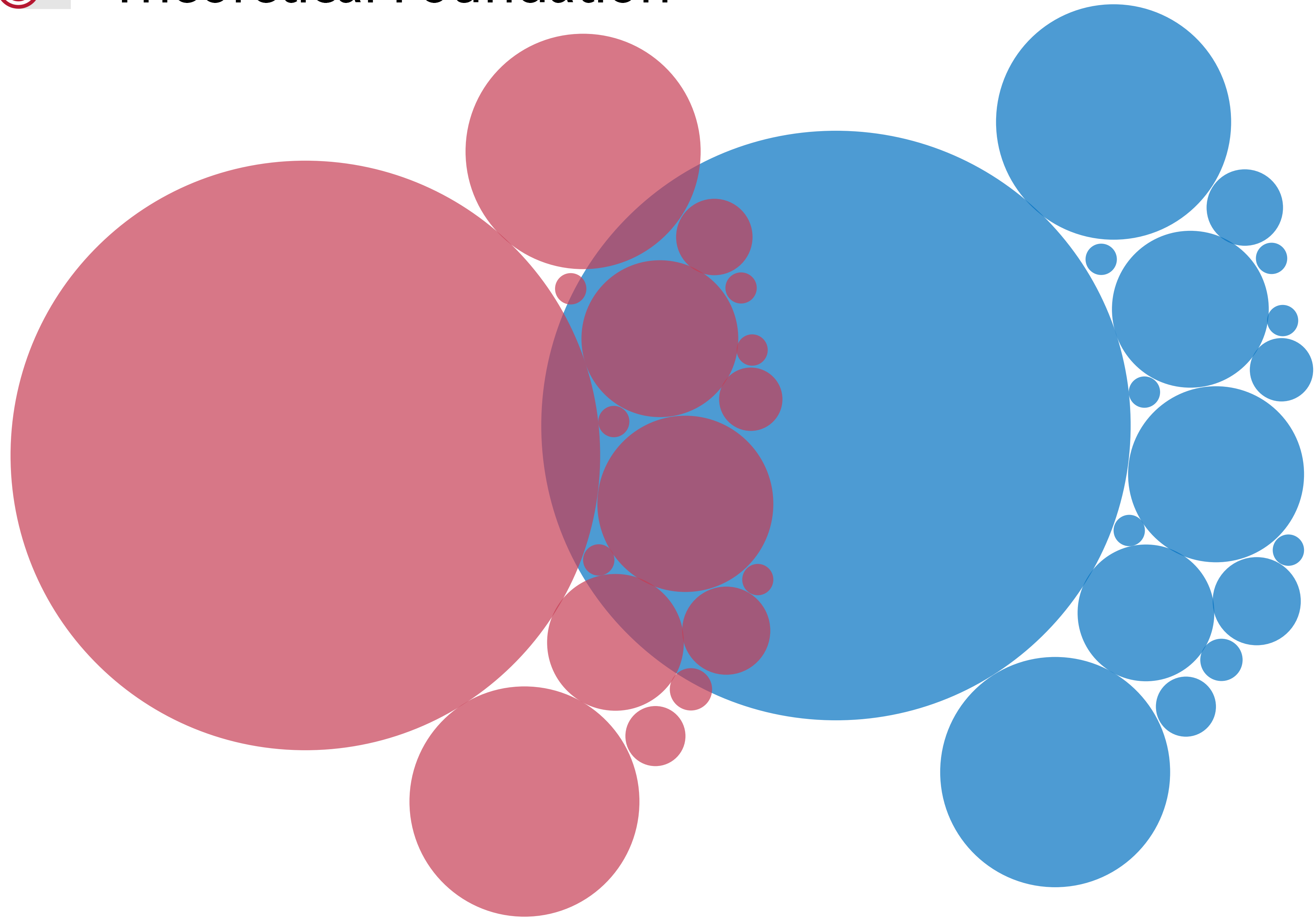

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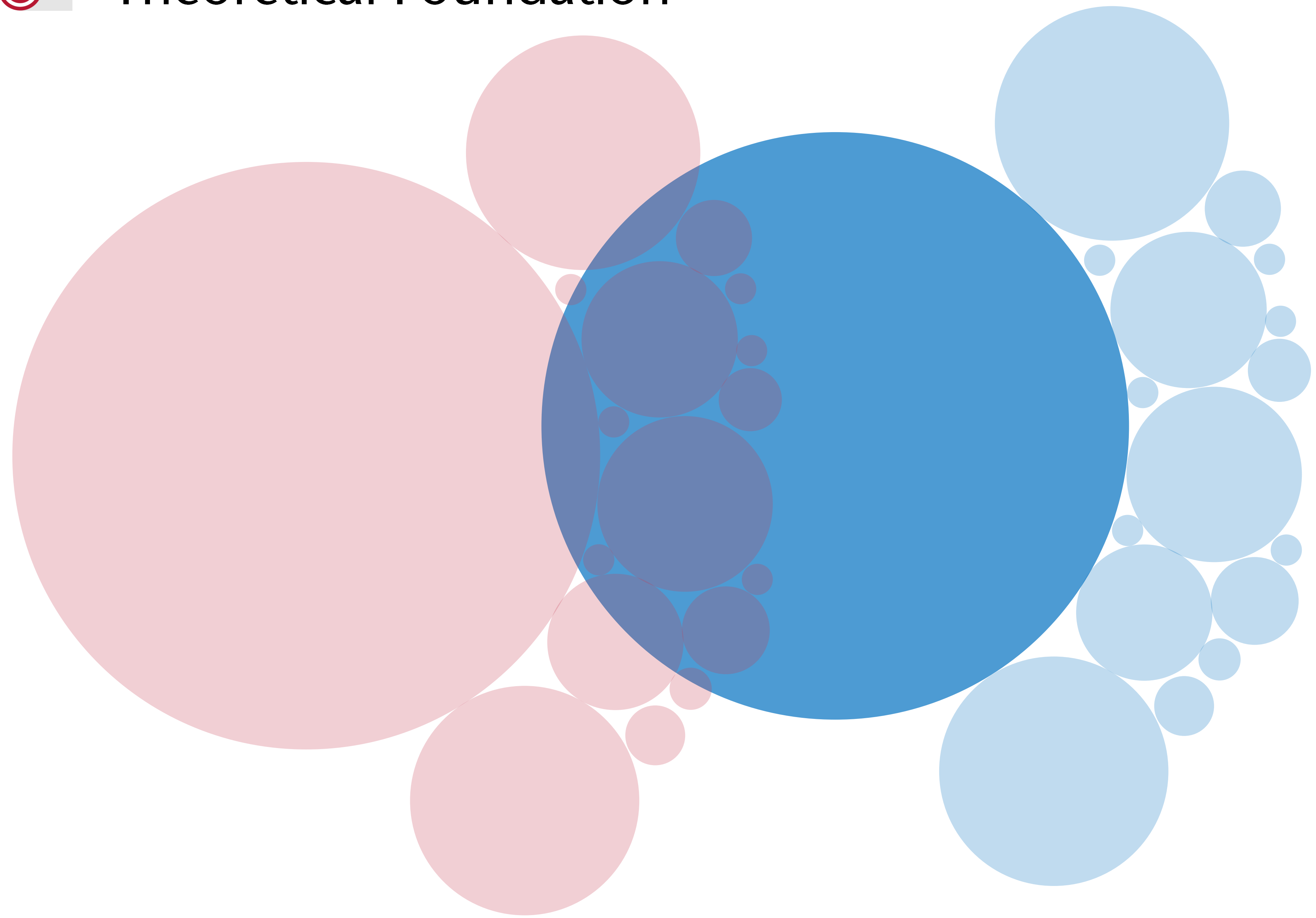

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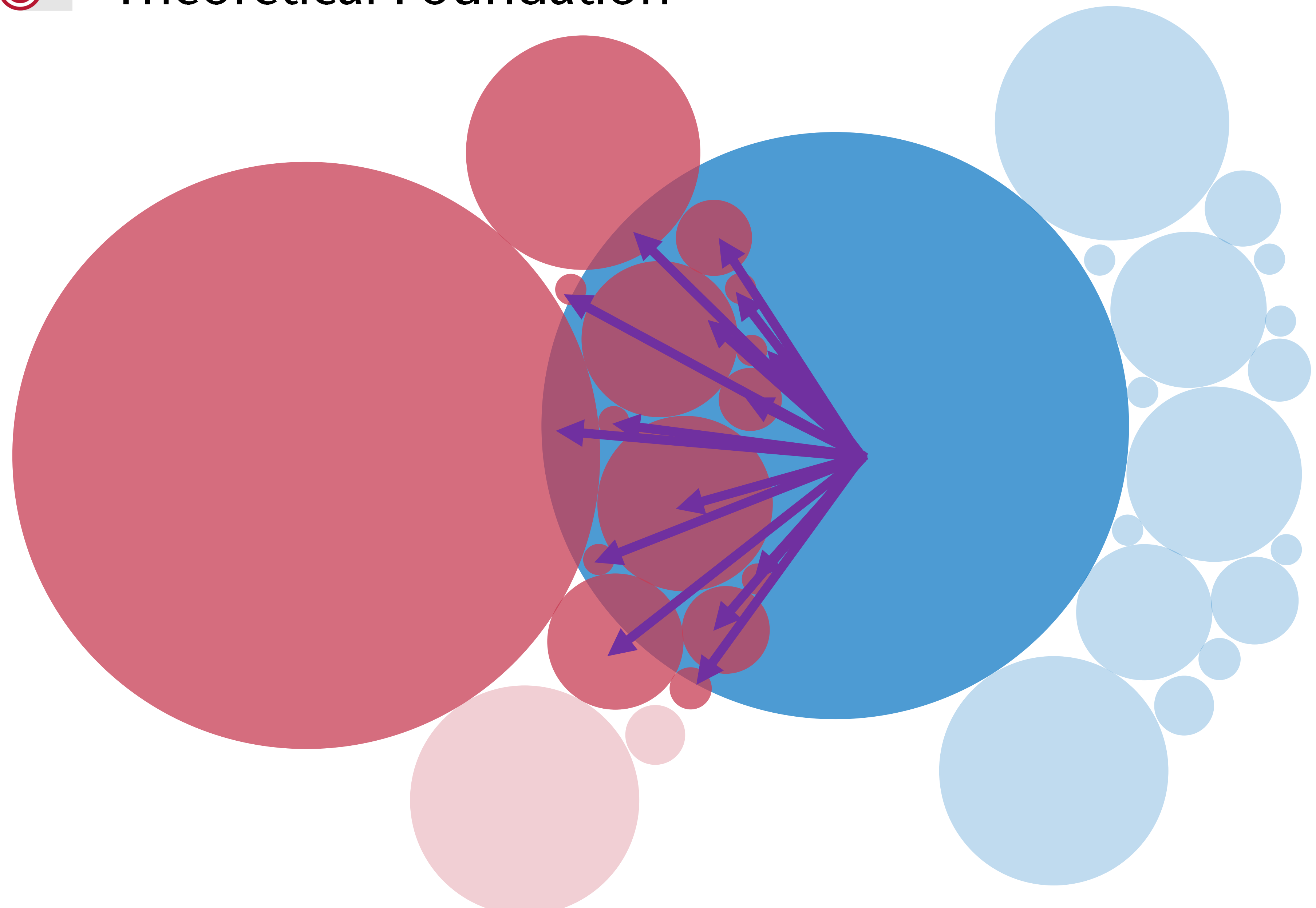
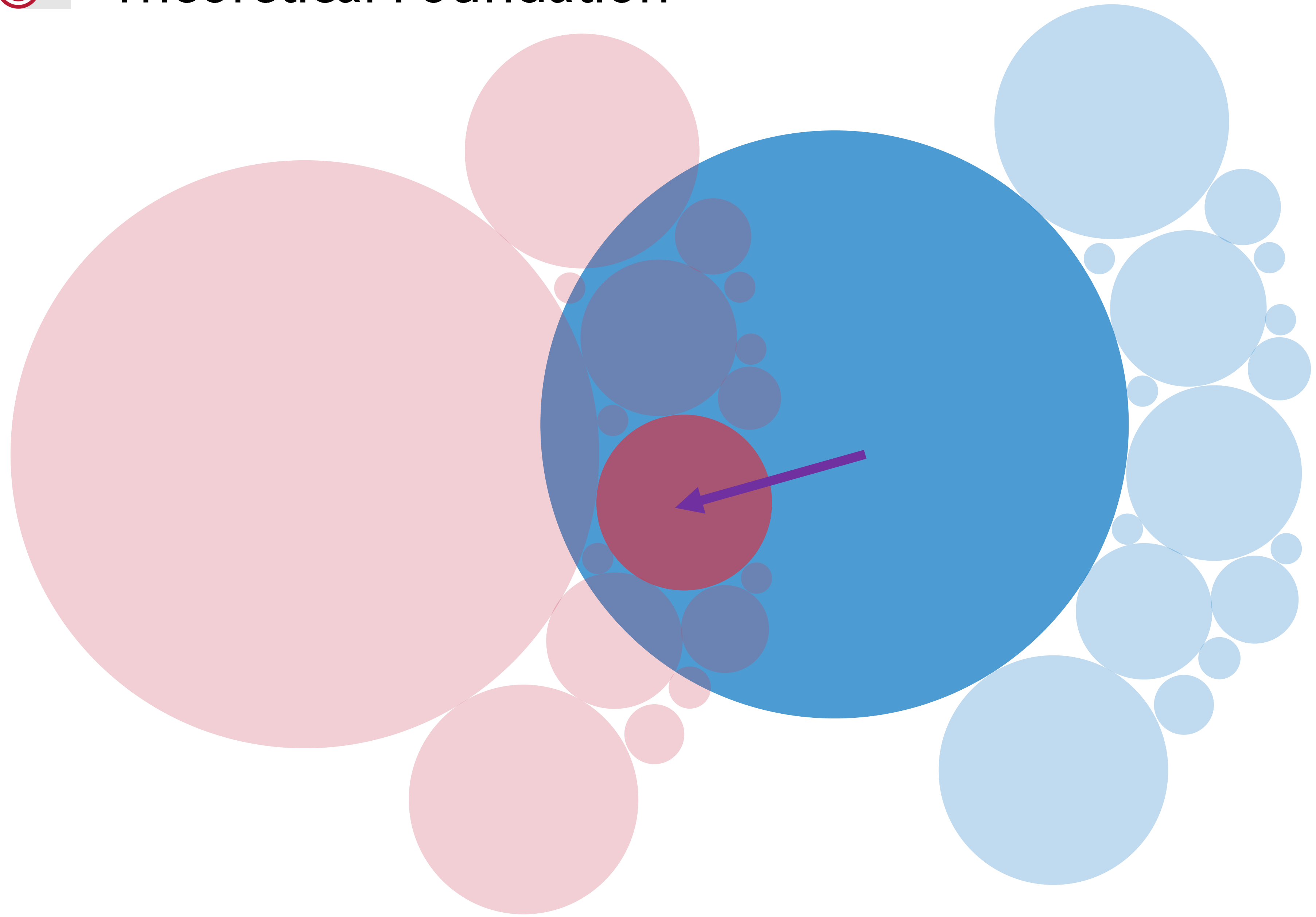


Theoretical Foundation



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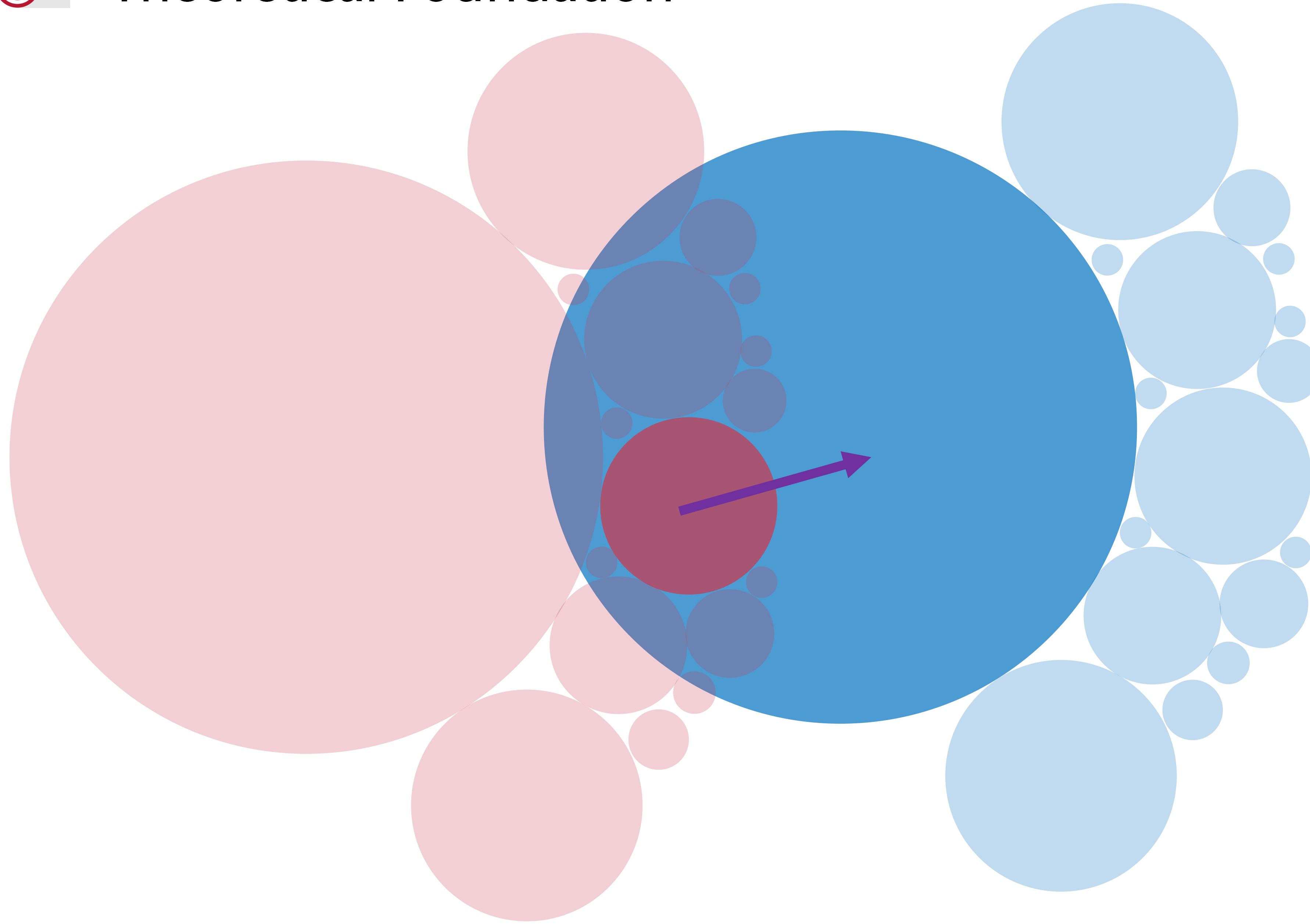

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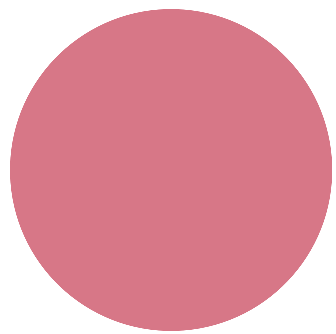

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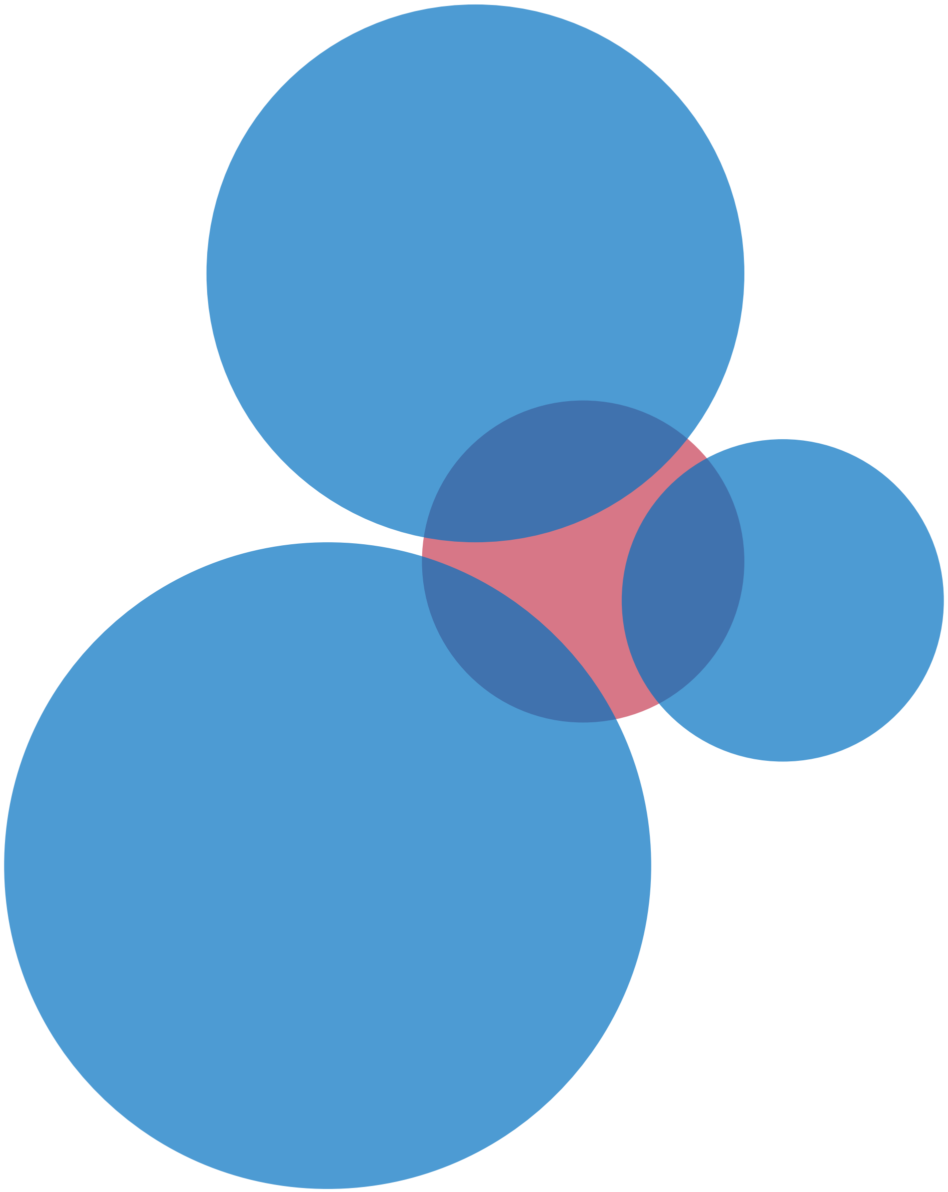

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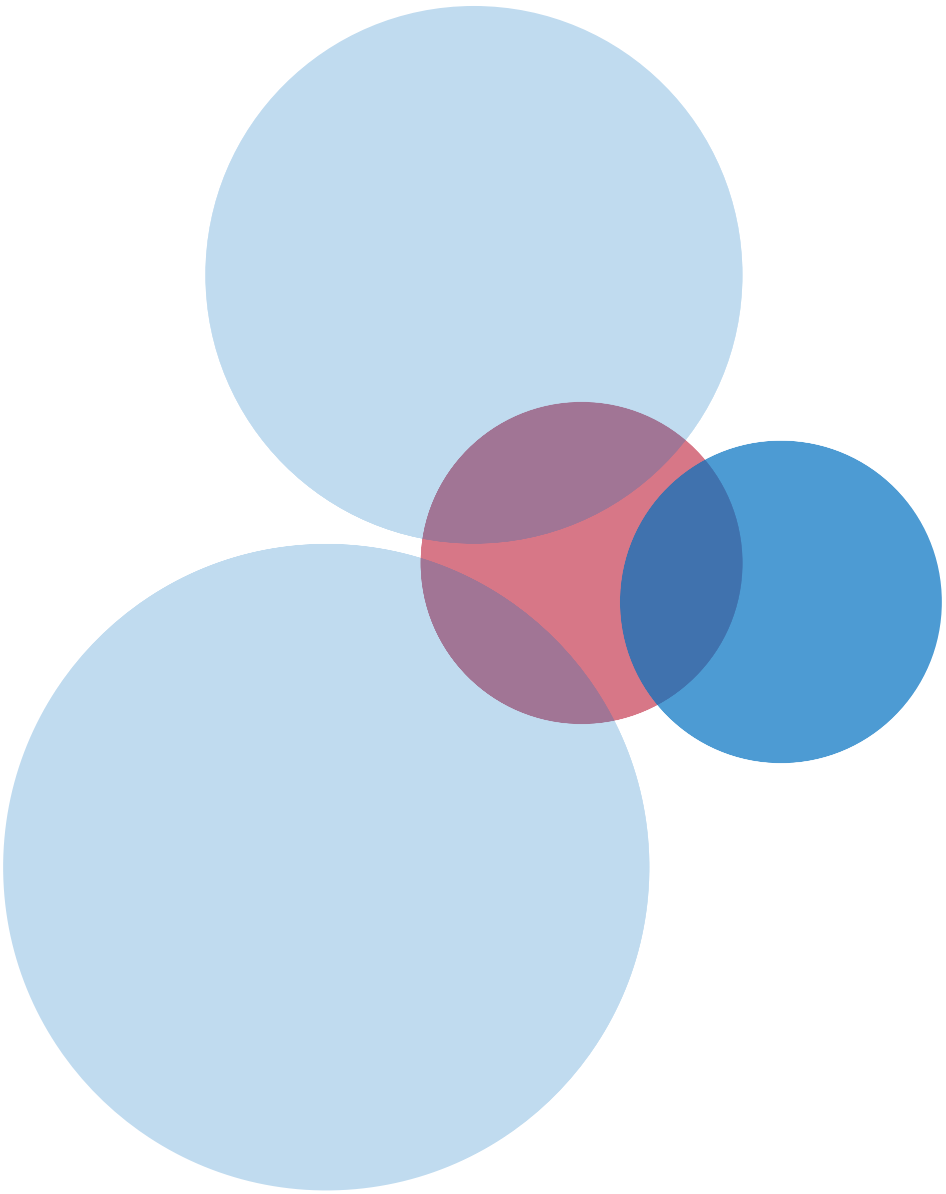

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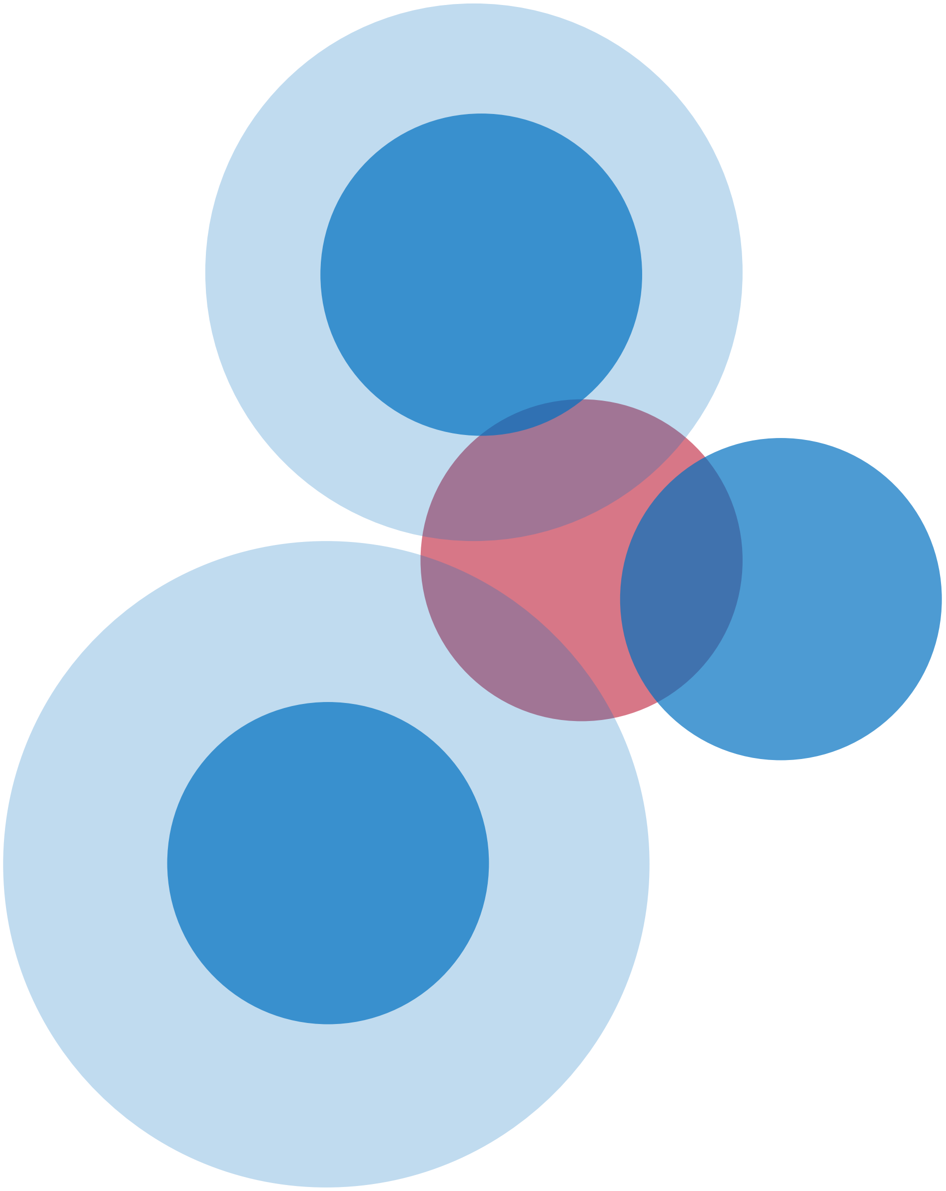

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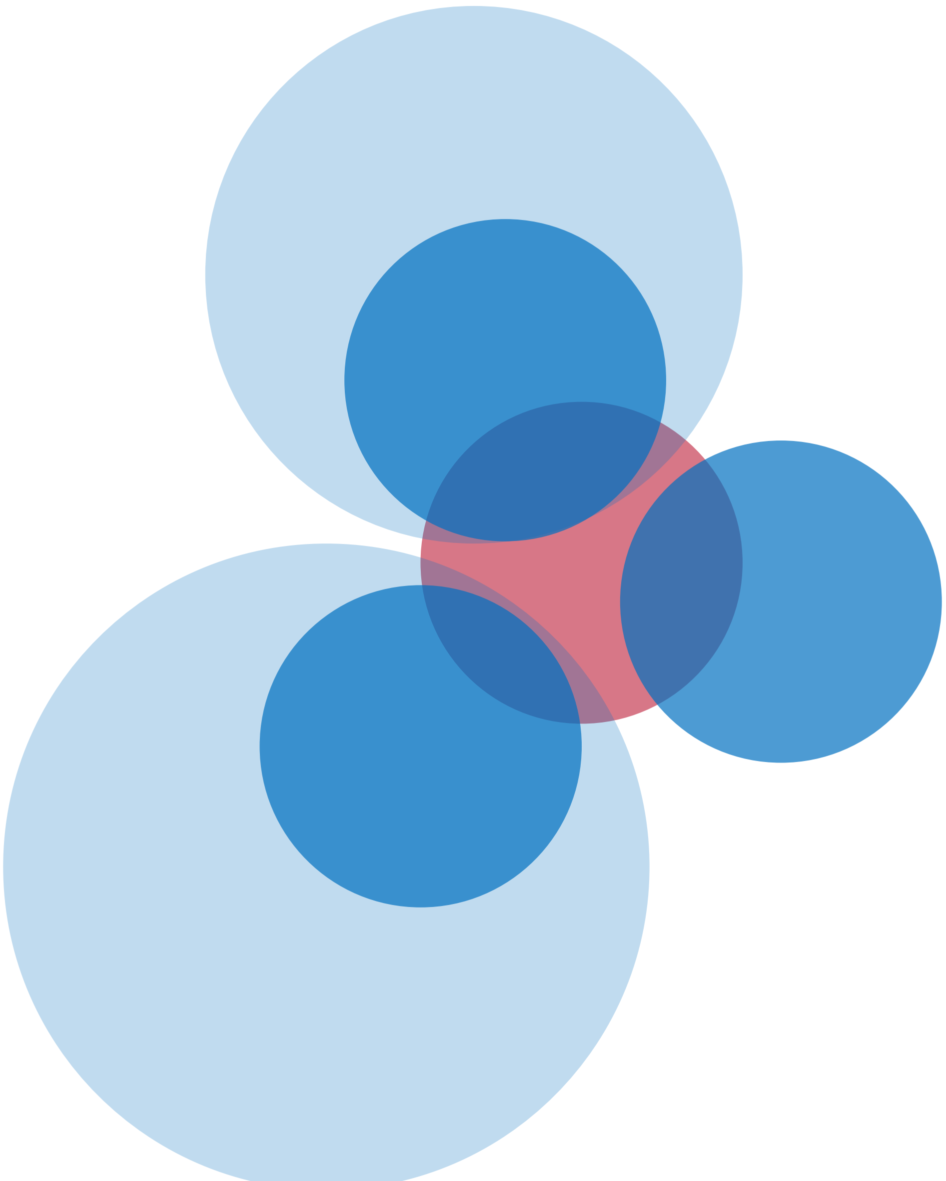

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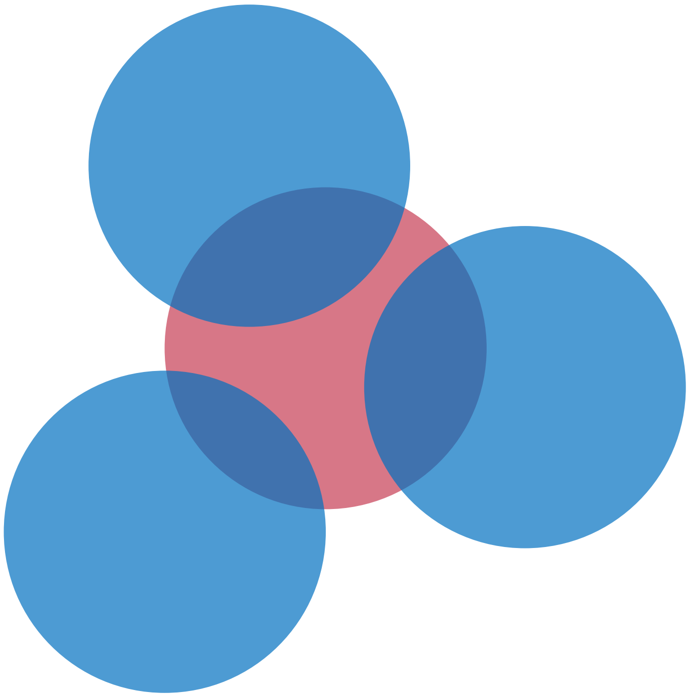

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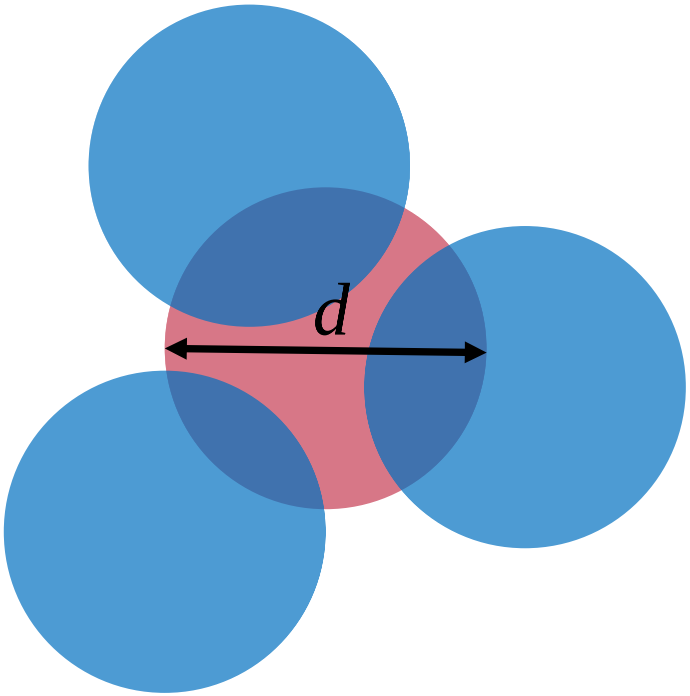

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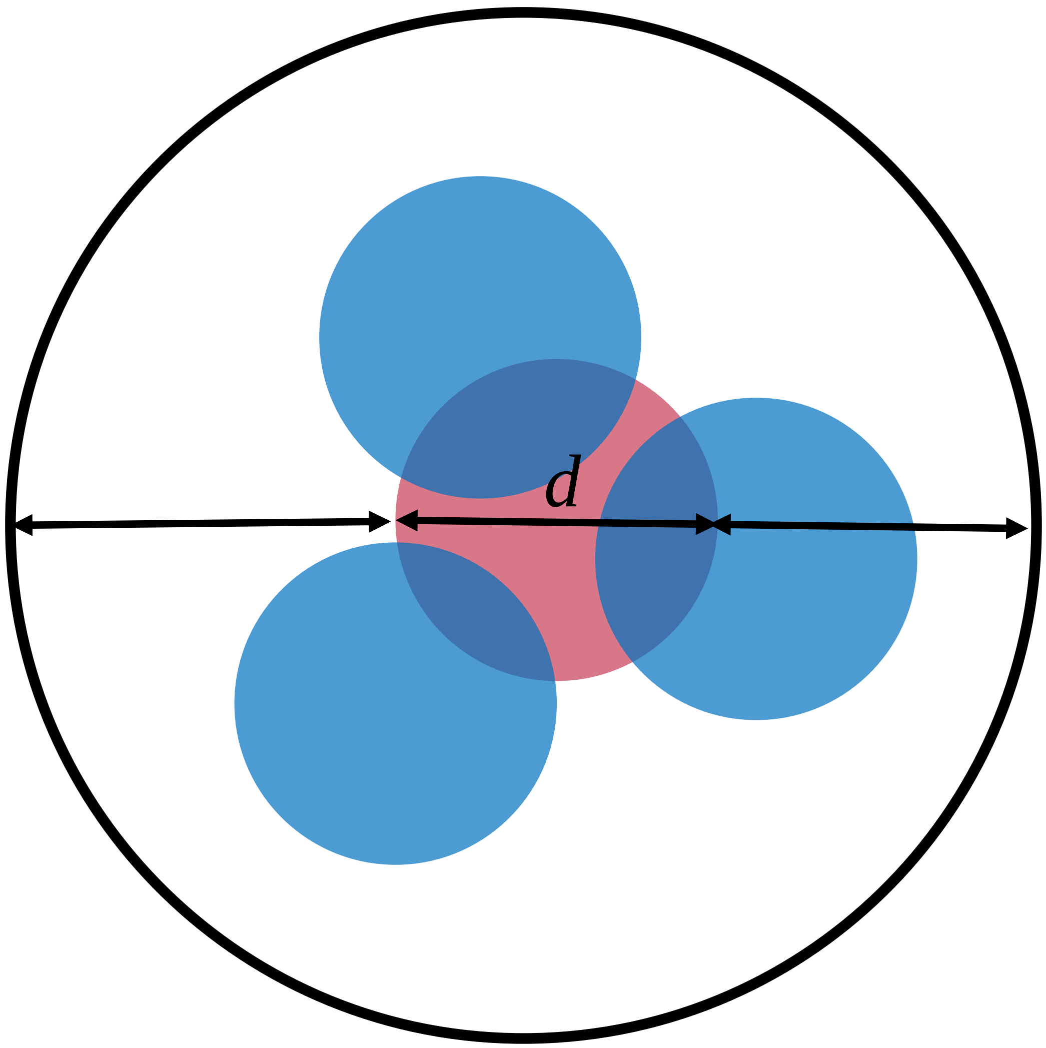

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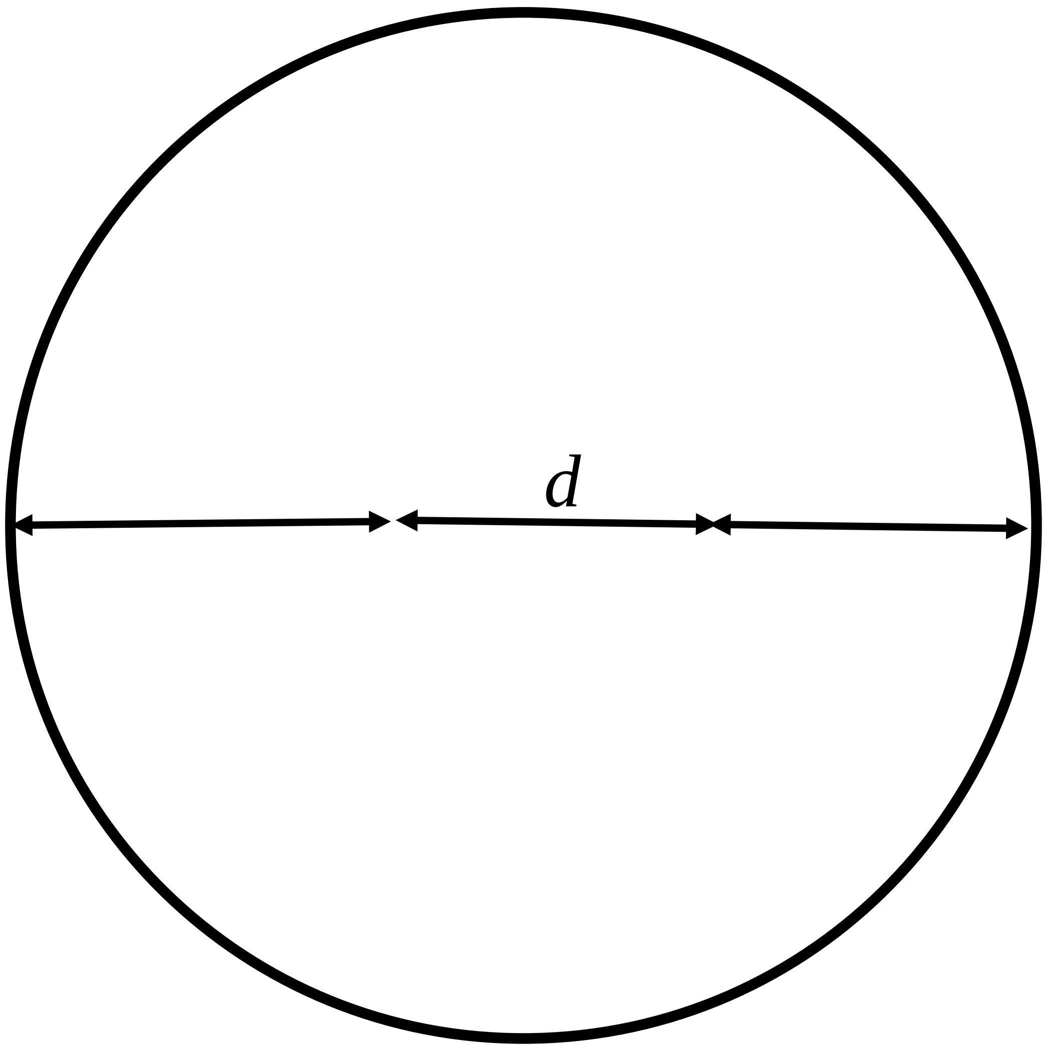

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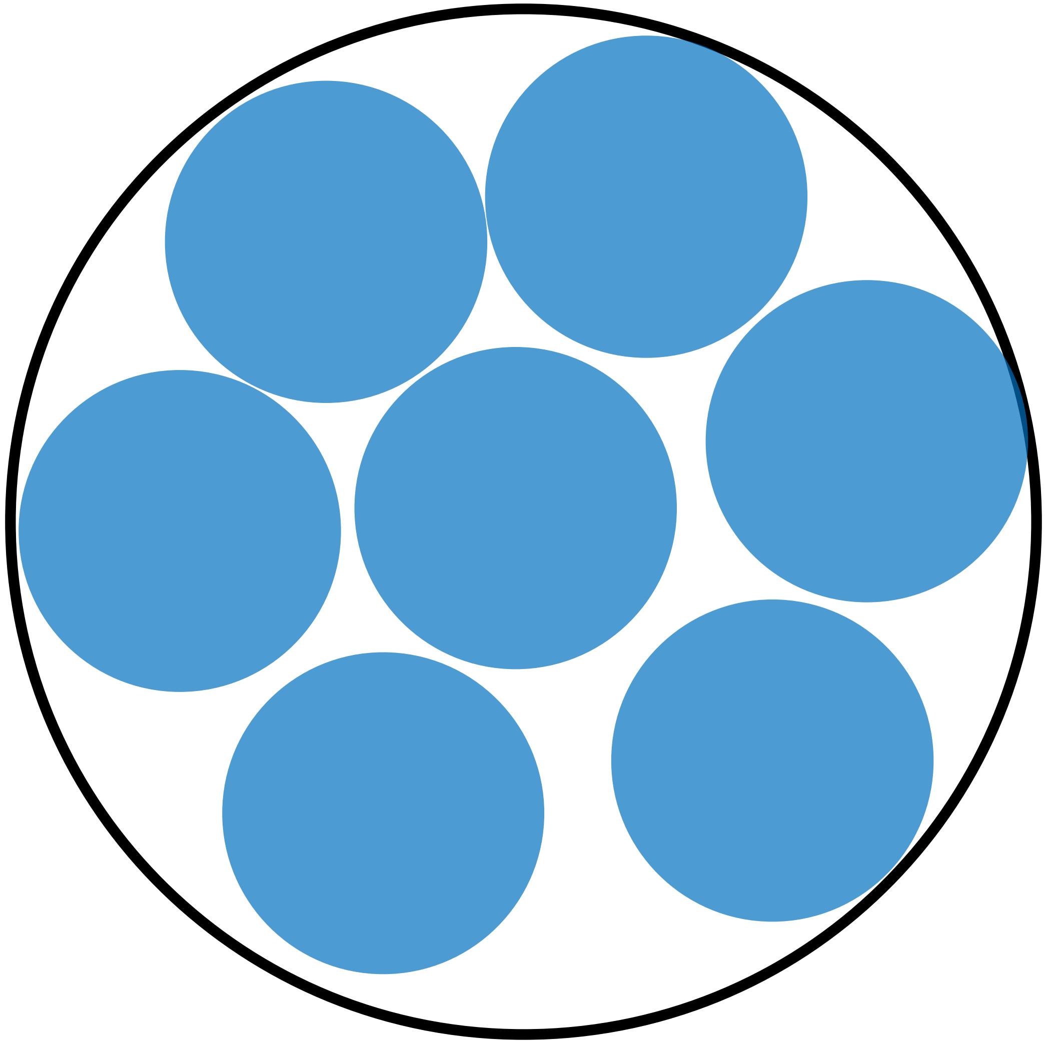

Theoretical Foundation

- Lemma: A single sphere s intersects a constant number of disjoint spheres A with at least the same radius.

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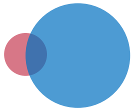
  - Check each sphere $s_i \in A$ against larger spheres in B



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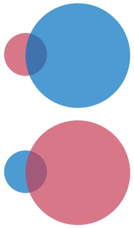

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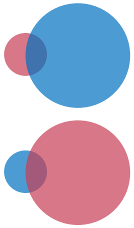
  $\Rightarrow O(1)$ for each $s_i \in A$ and $s_j \in B$



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  - Check each sphere $s_i \in A$ against larger spheres in B
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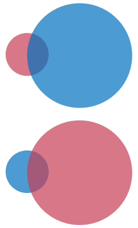
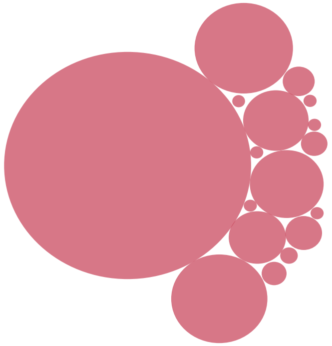
  - $\Rightarrow O(1)$ for each $s_i \in A$ and $s_j \in B$
  - $\Rightarrow O(n)$ in total





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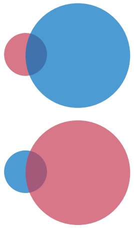

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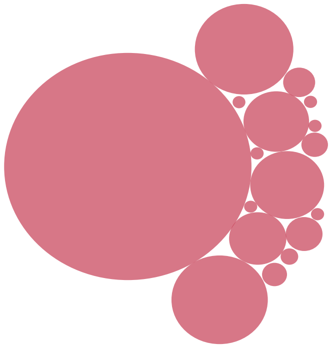
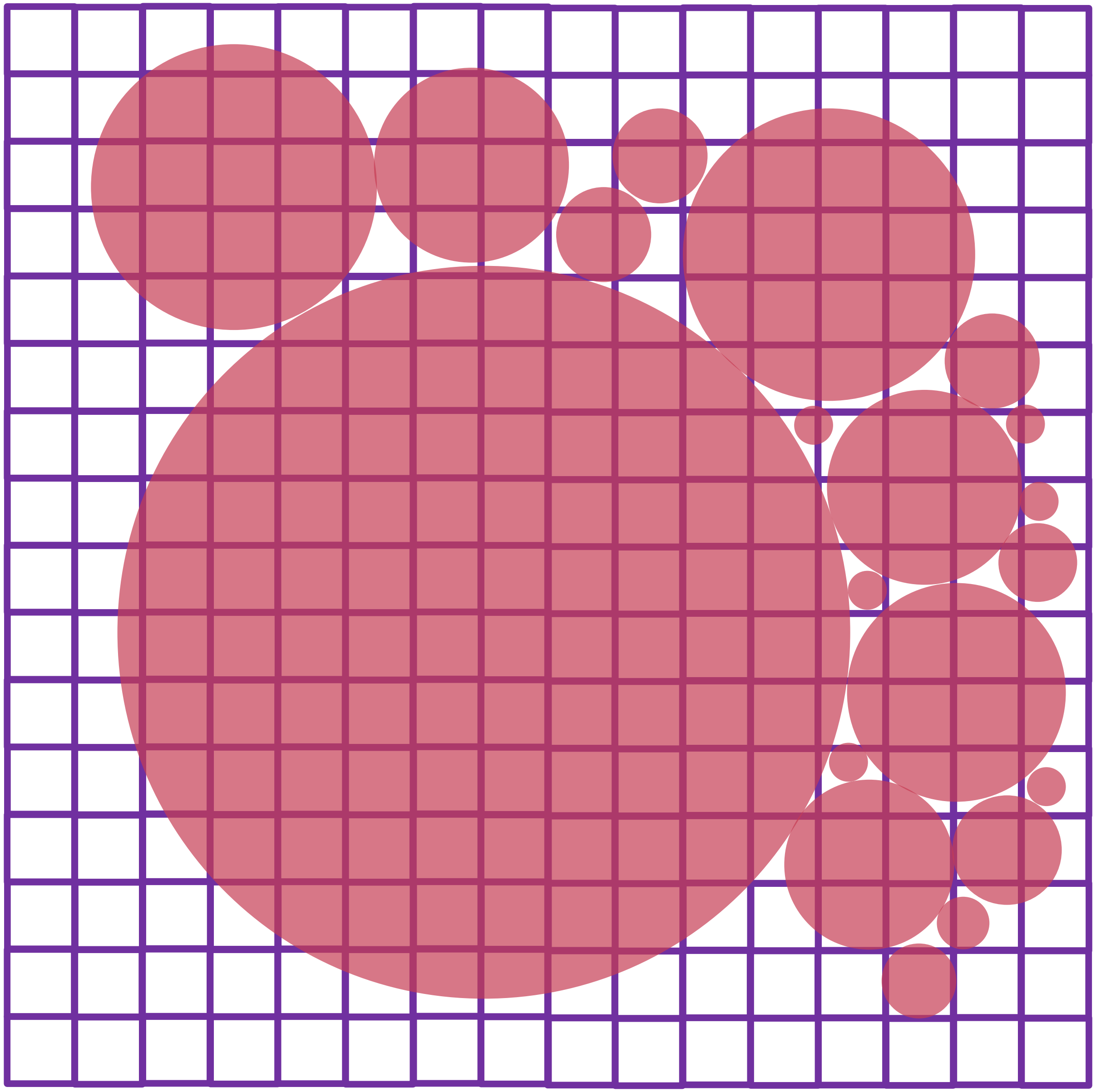
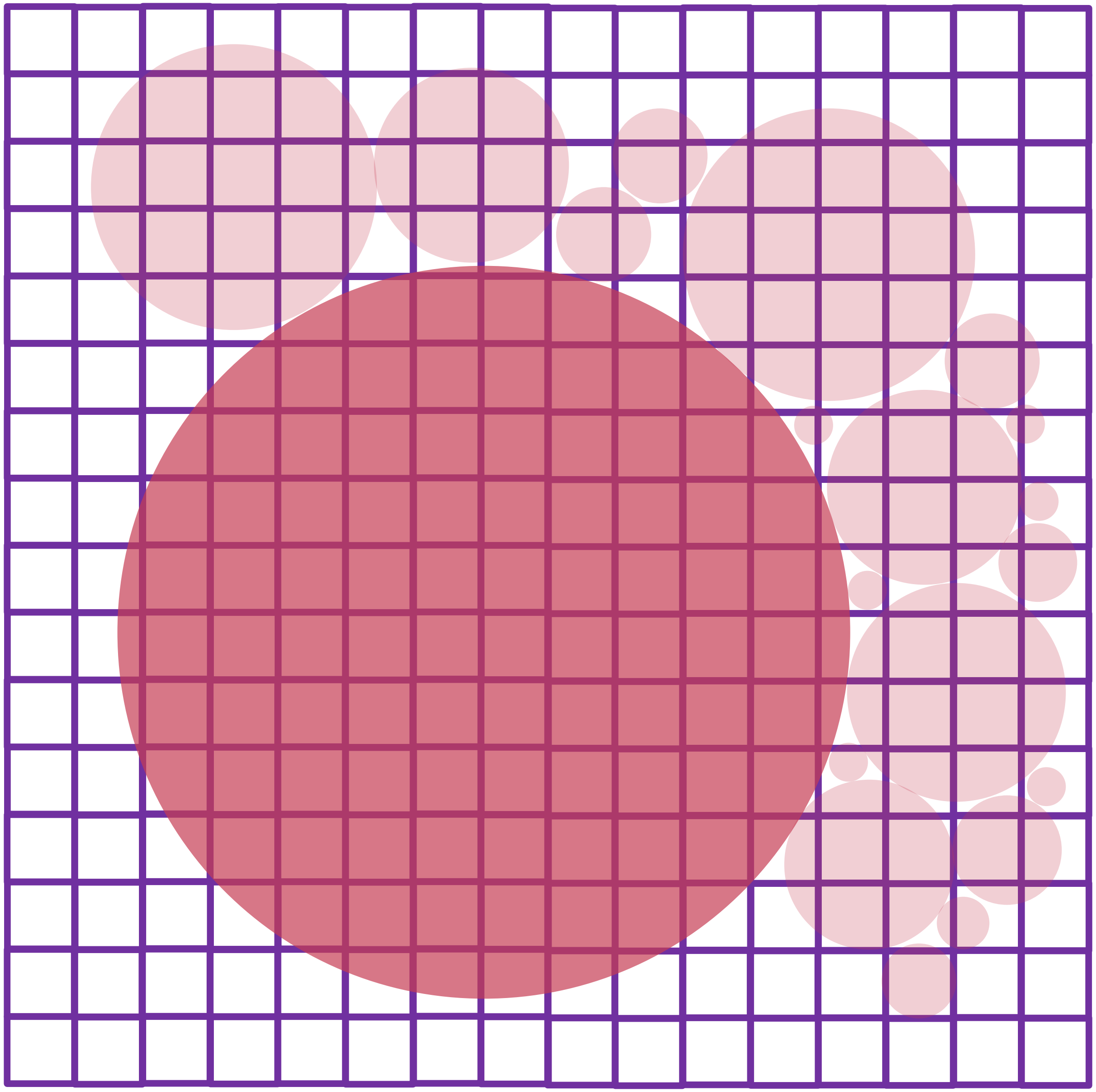


Our Algorithm



Our Algorithm



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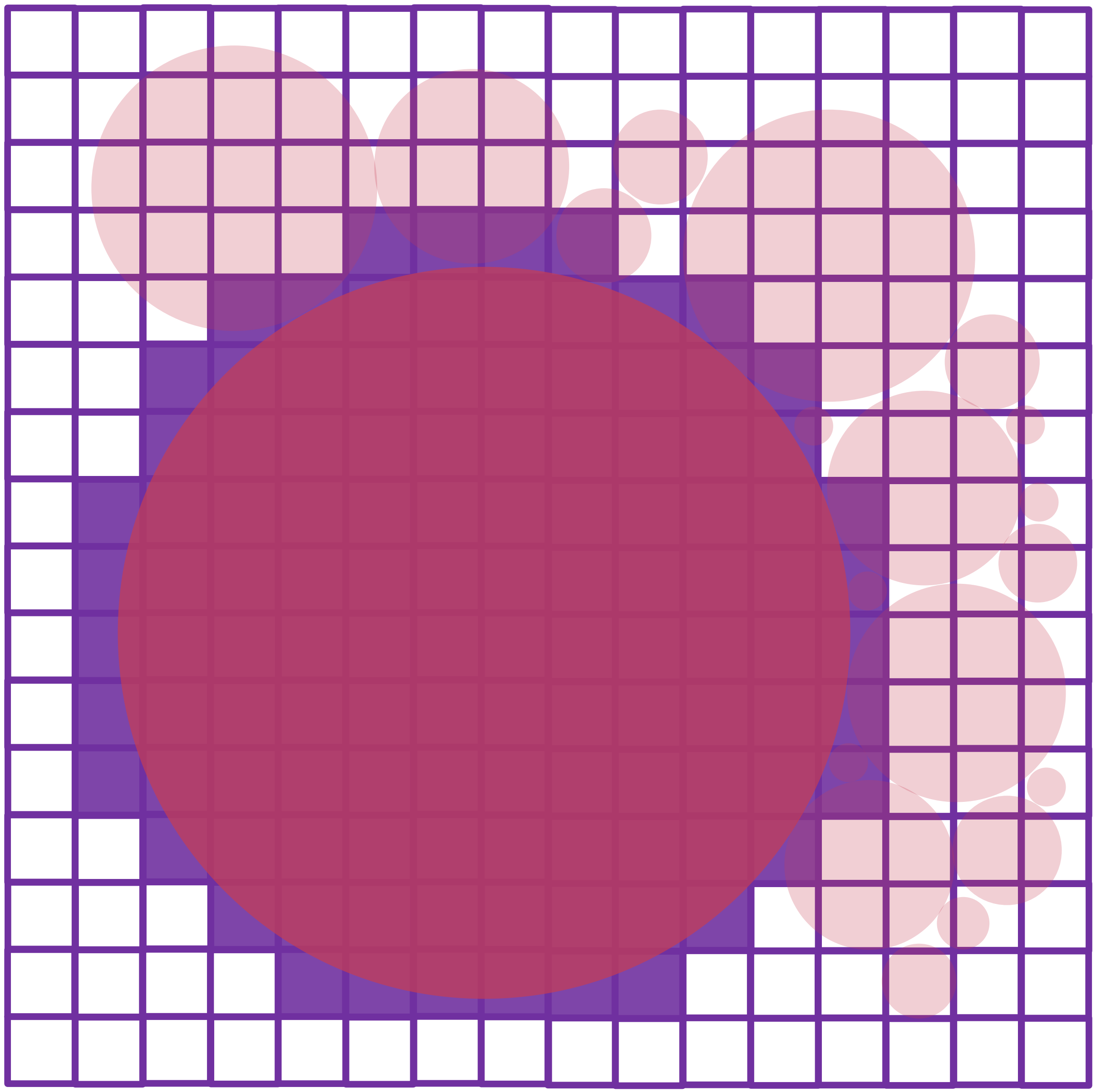

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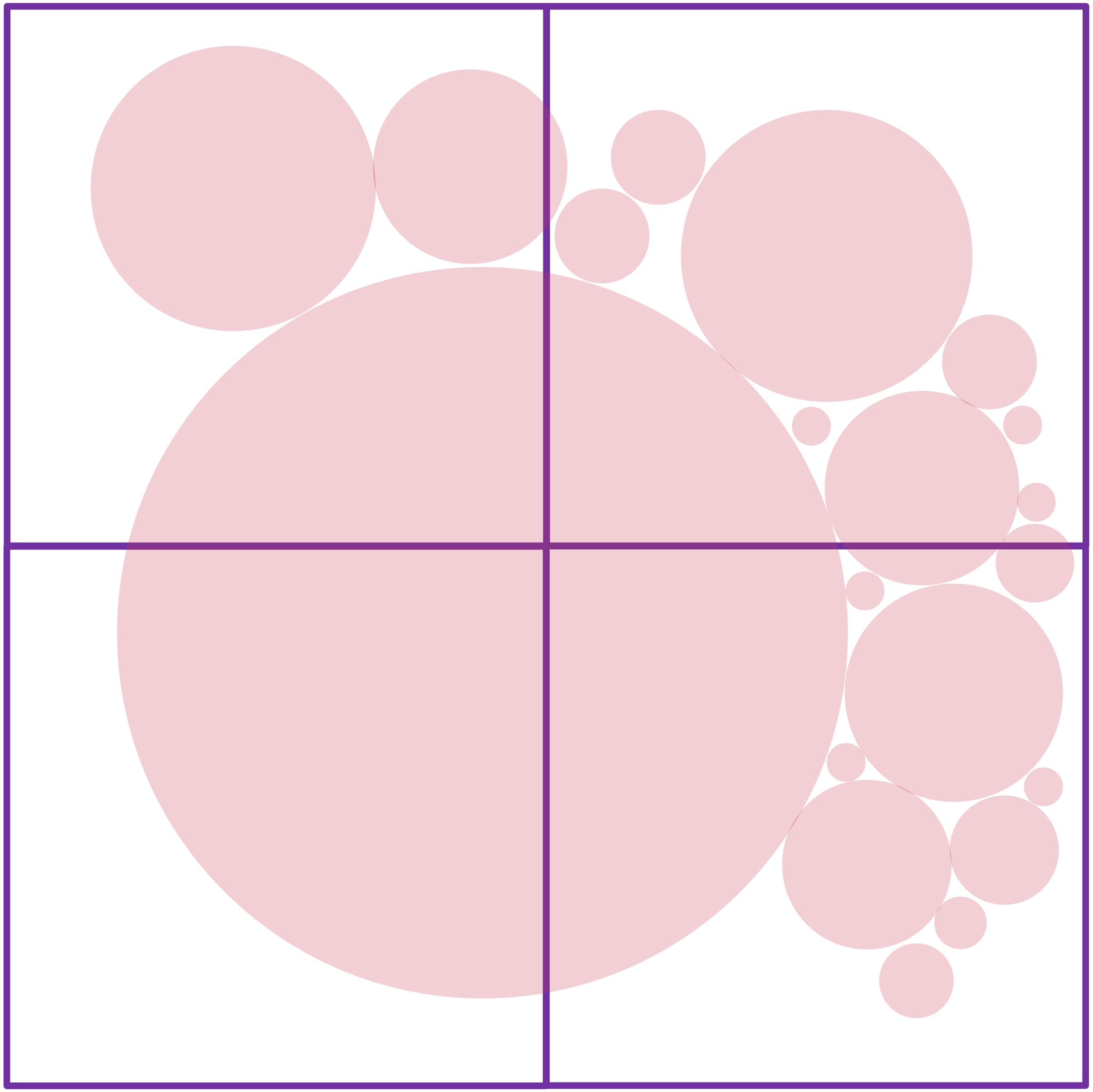

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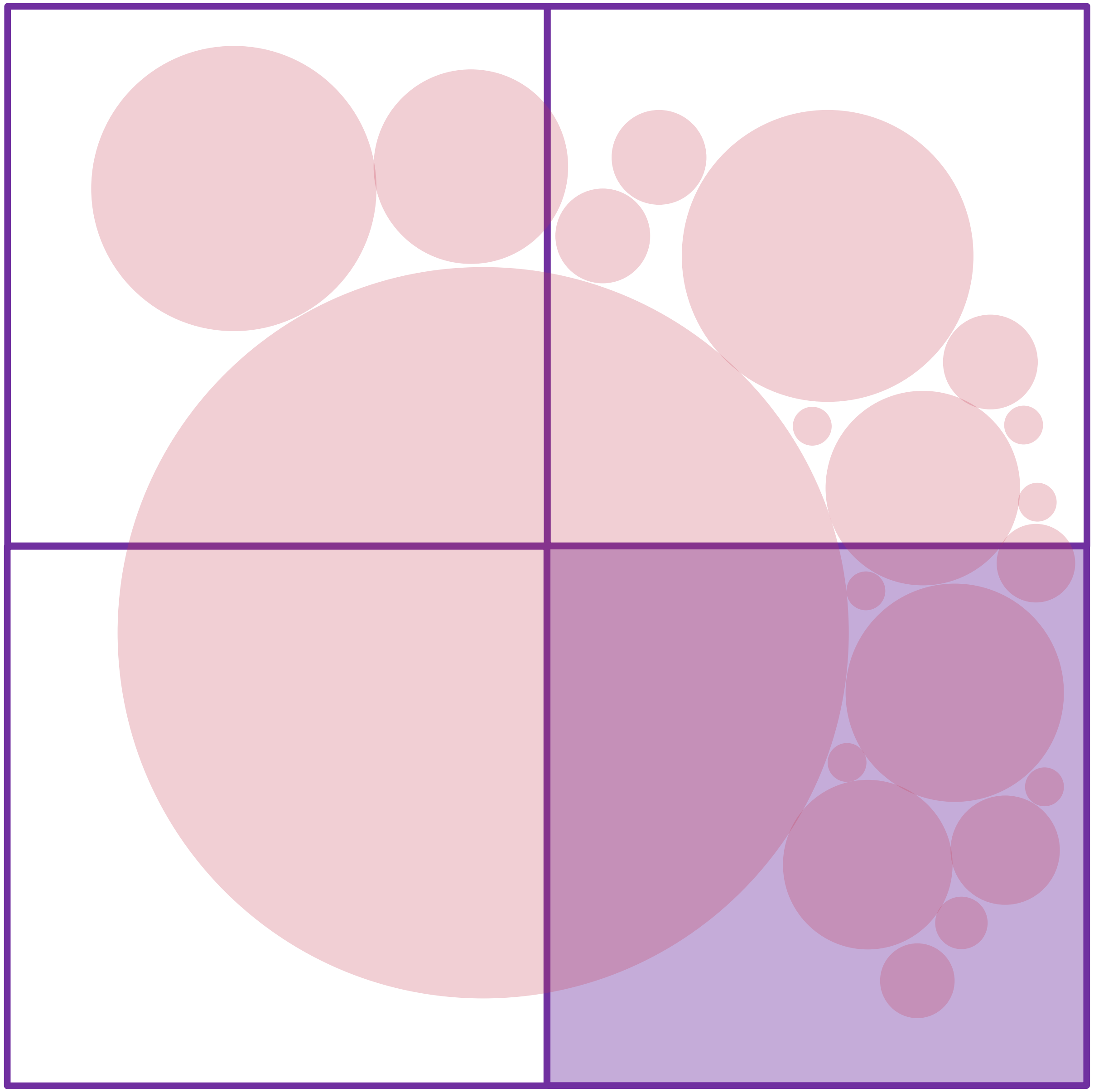

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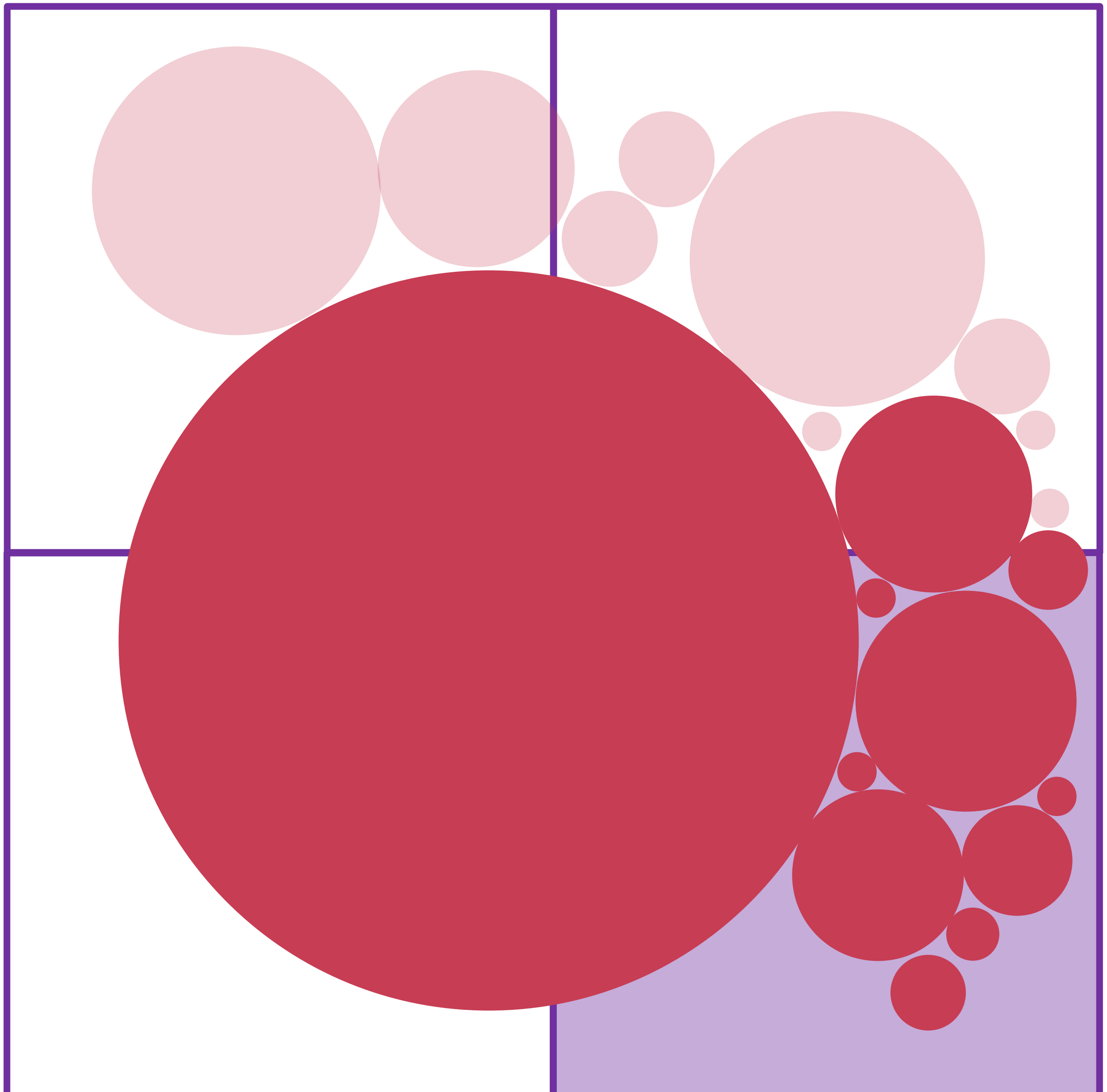

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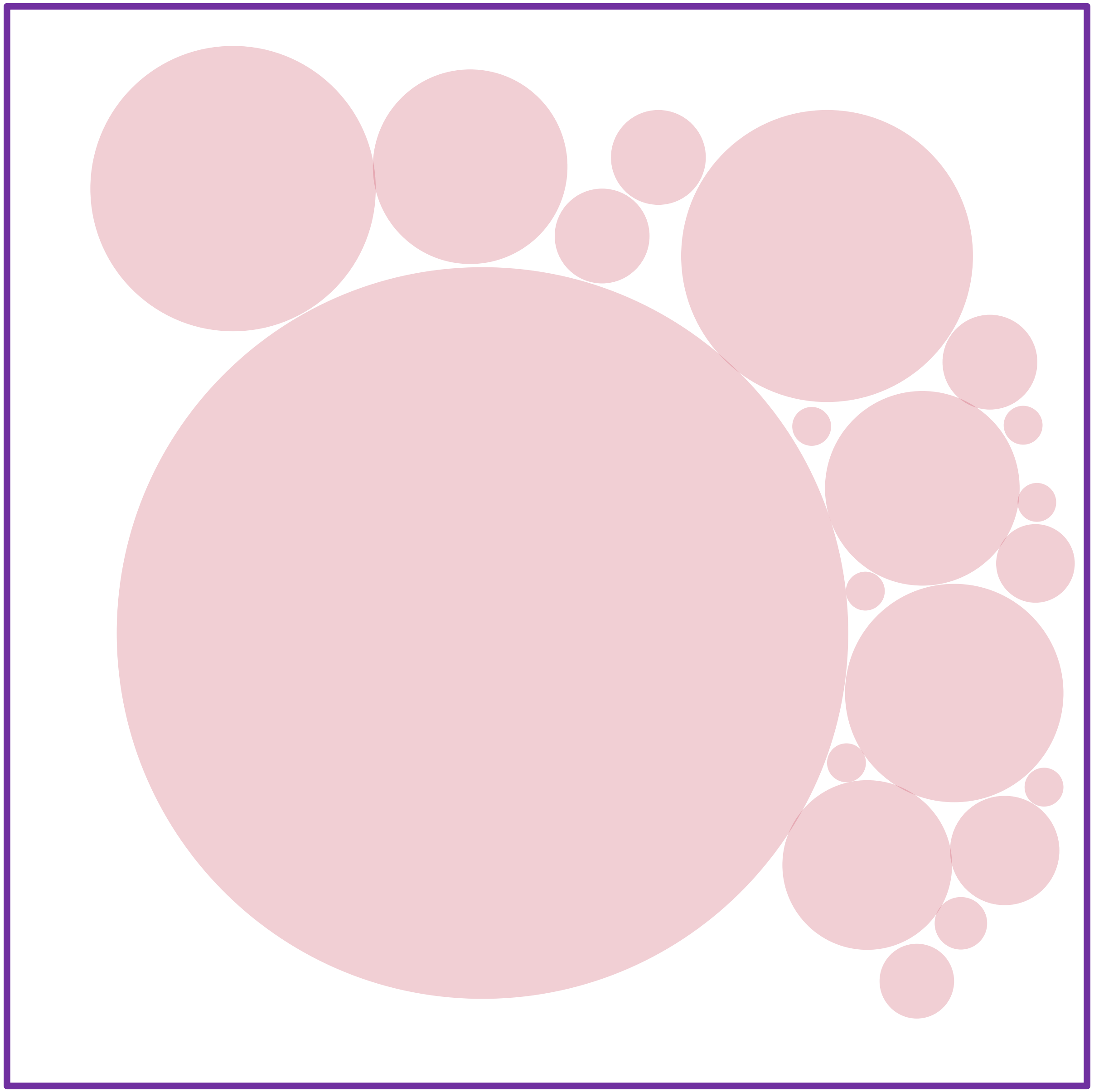
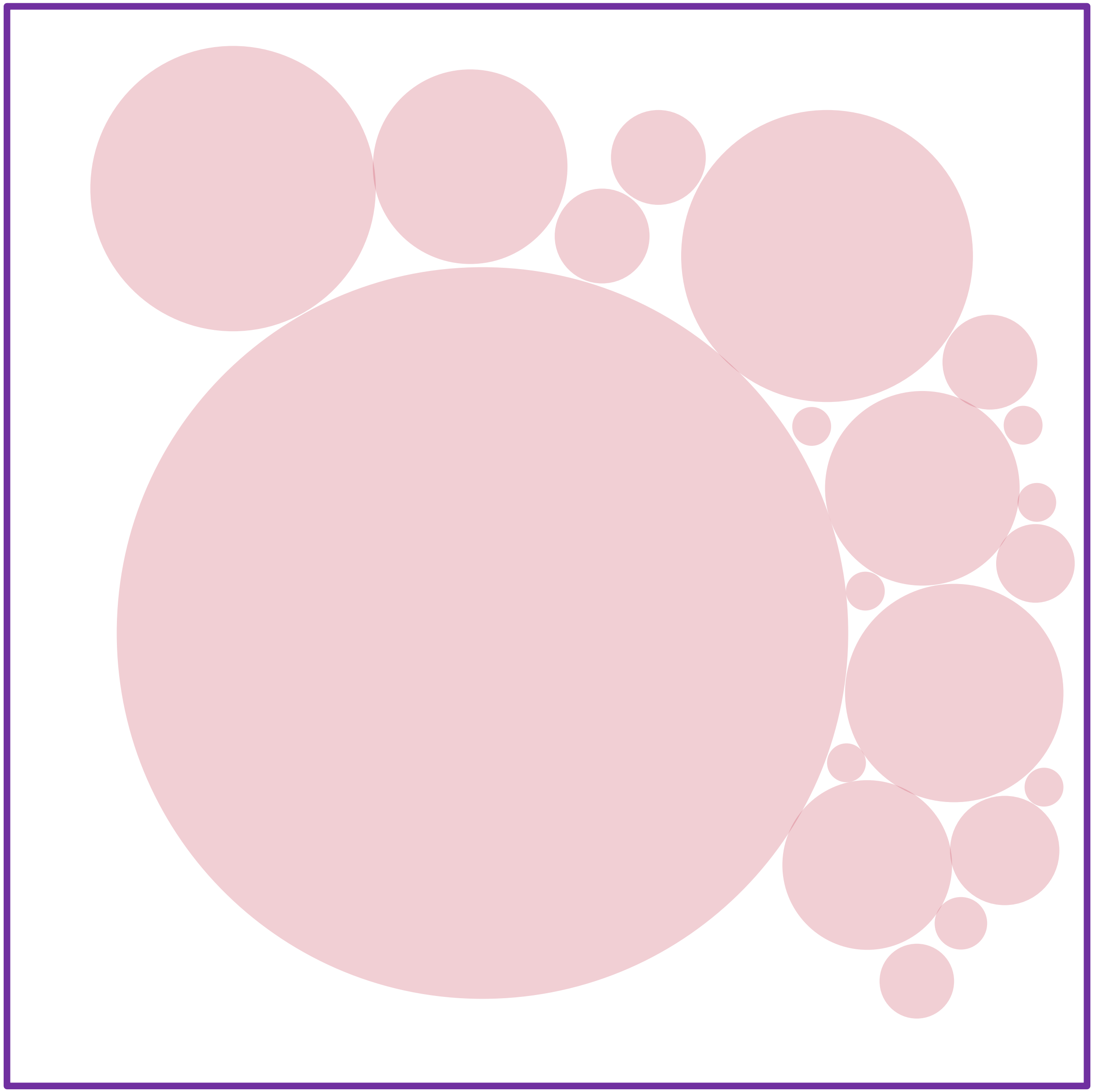
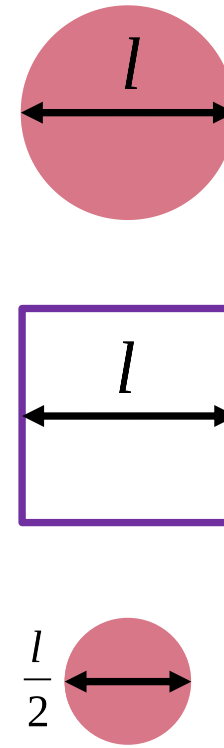


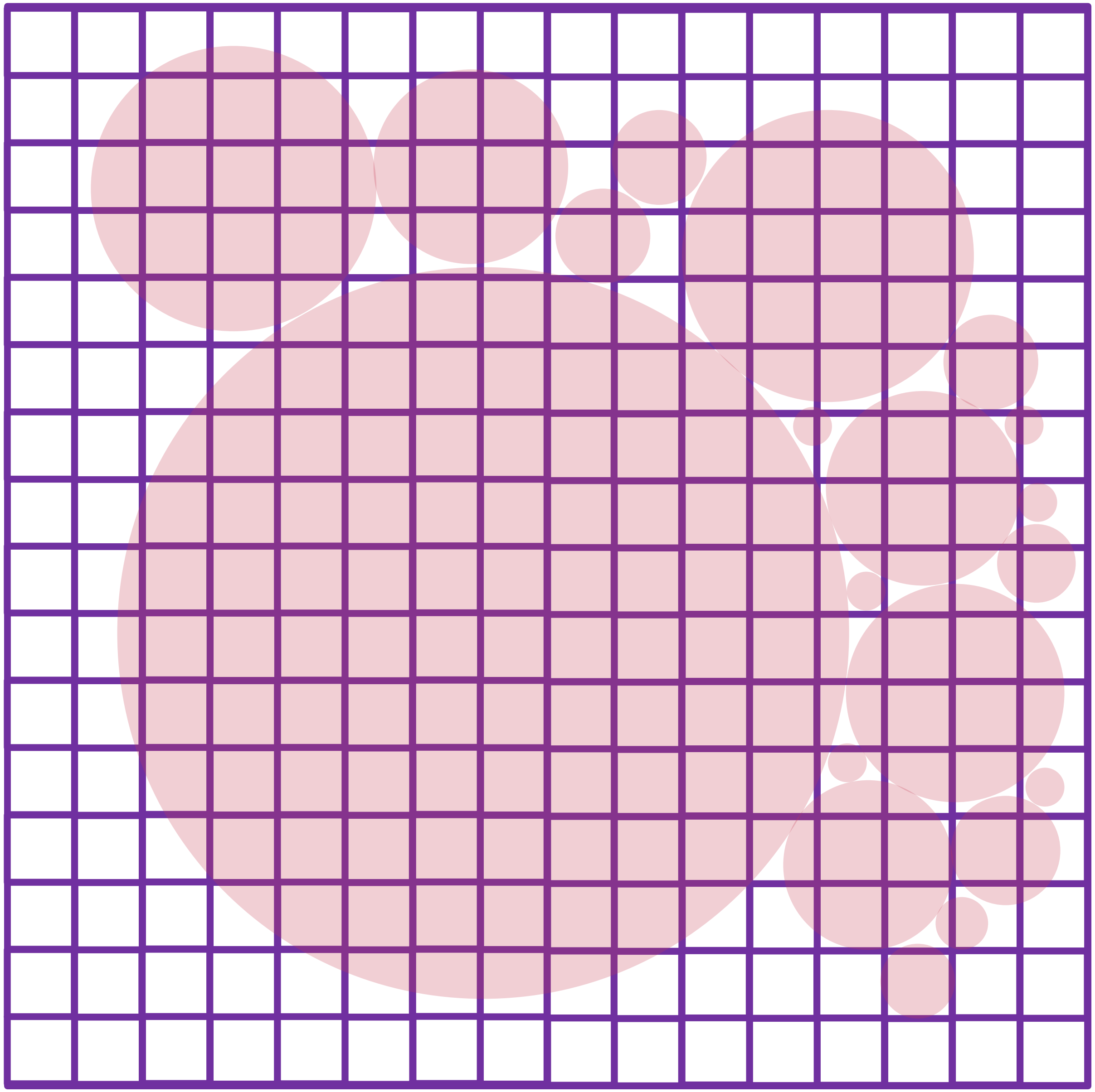
Our Algorithm

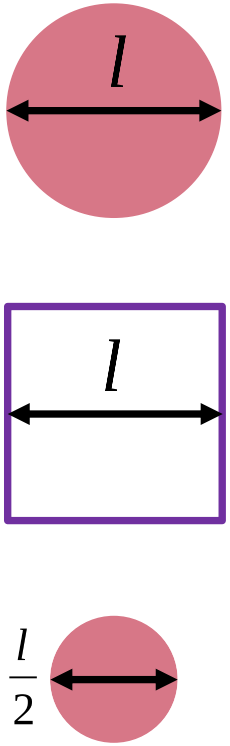




Our Algorithm

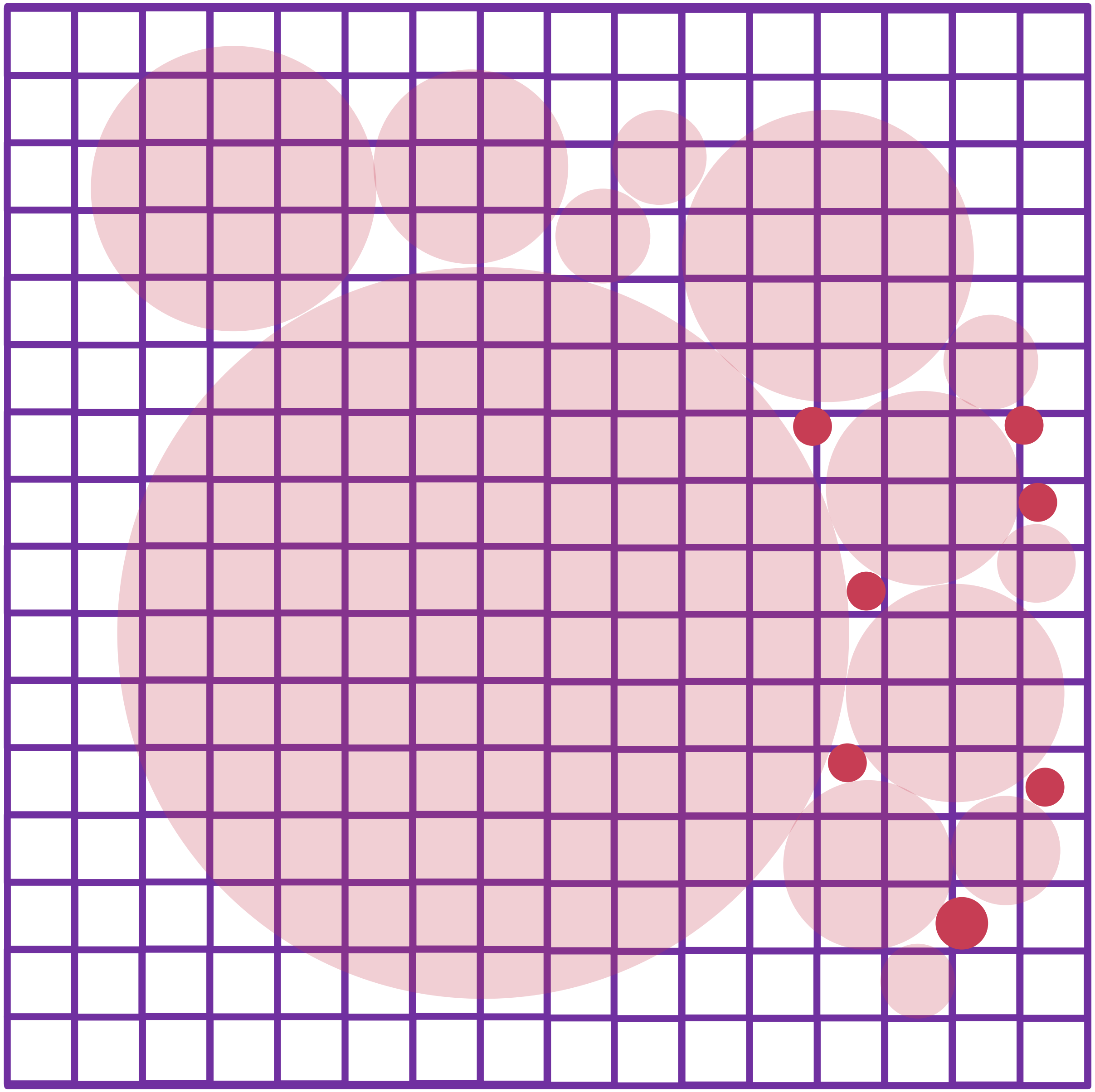
Level 4

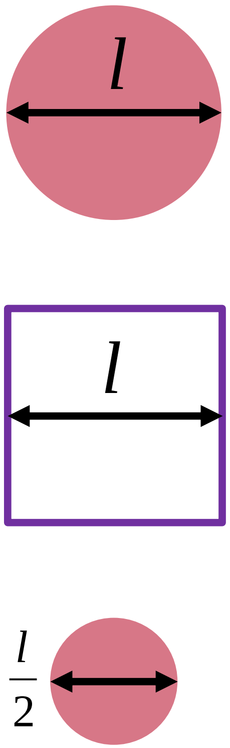




Our Algorithm

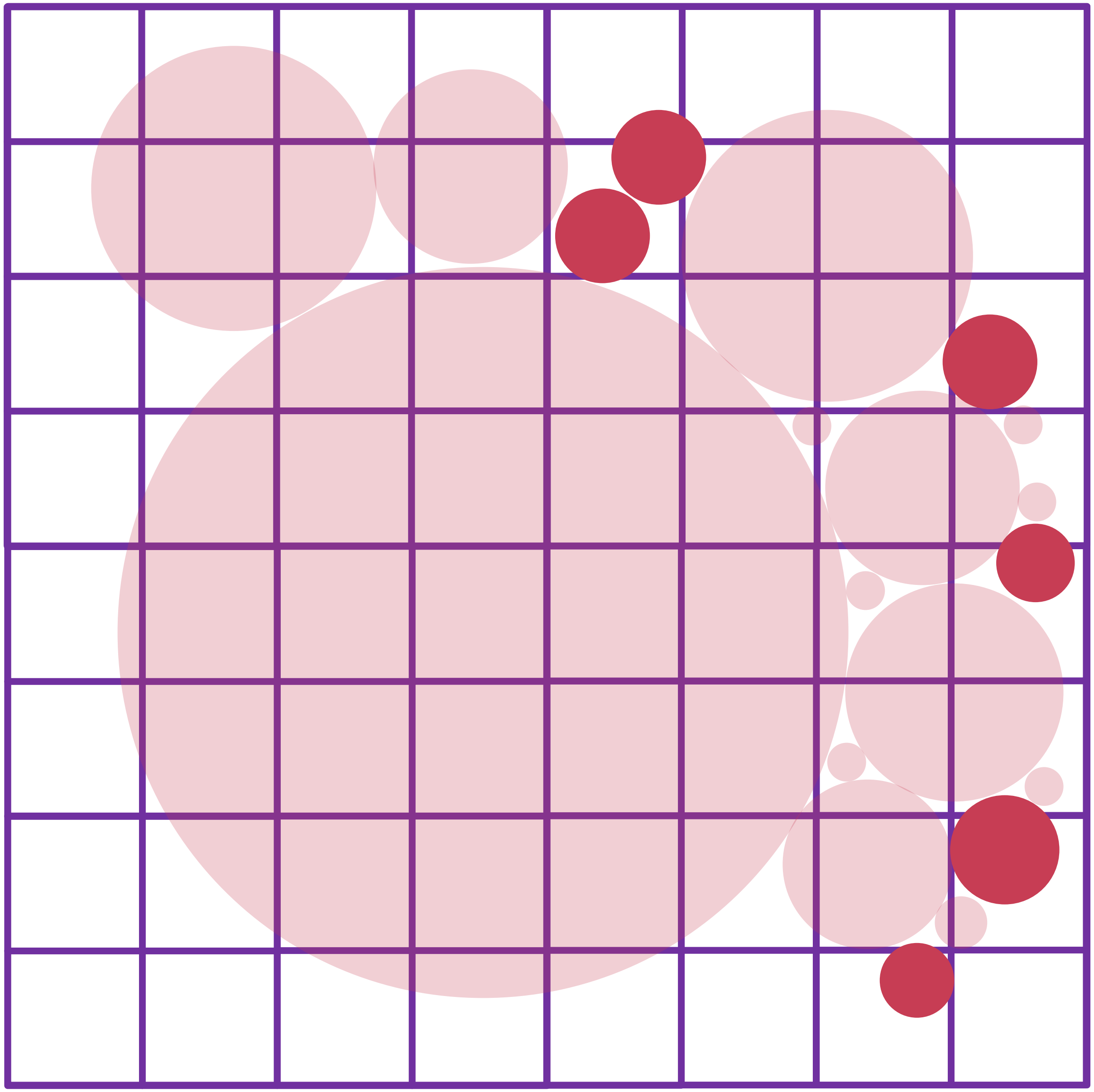
L e v e l 4

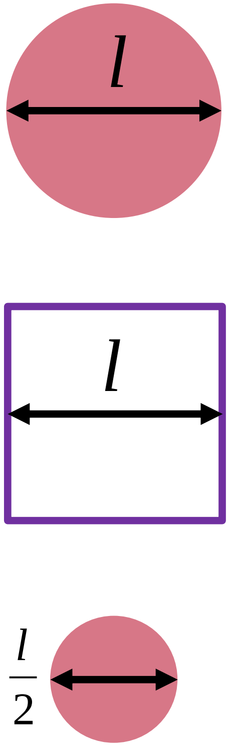




Our Algorithm

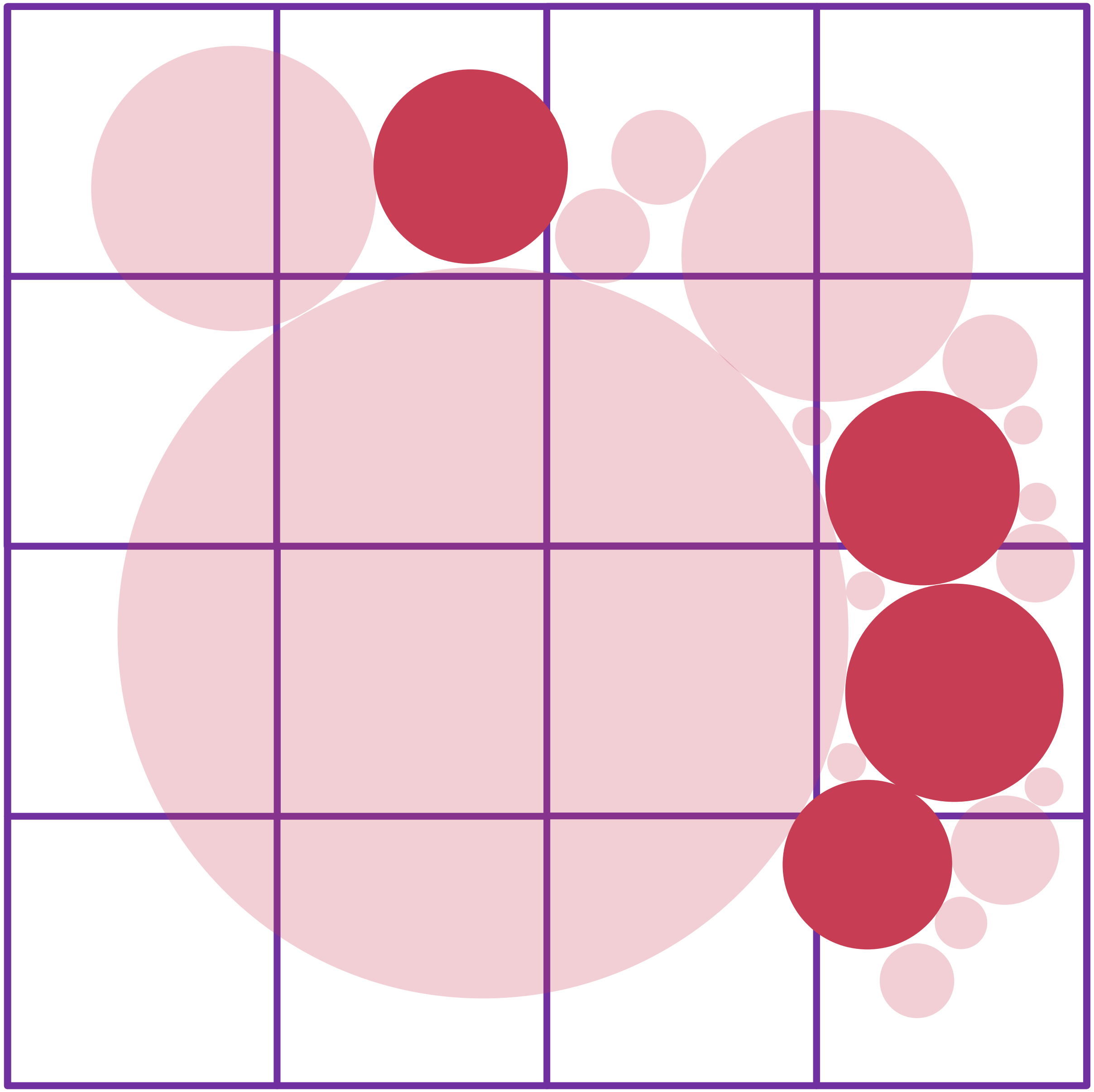
L e v e l 3

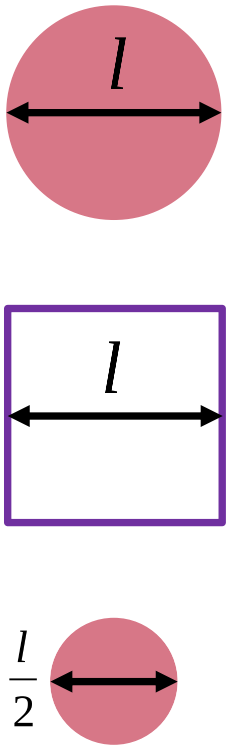




Our Algorithm

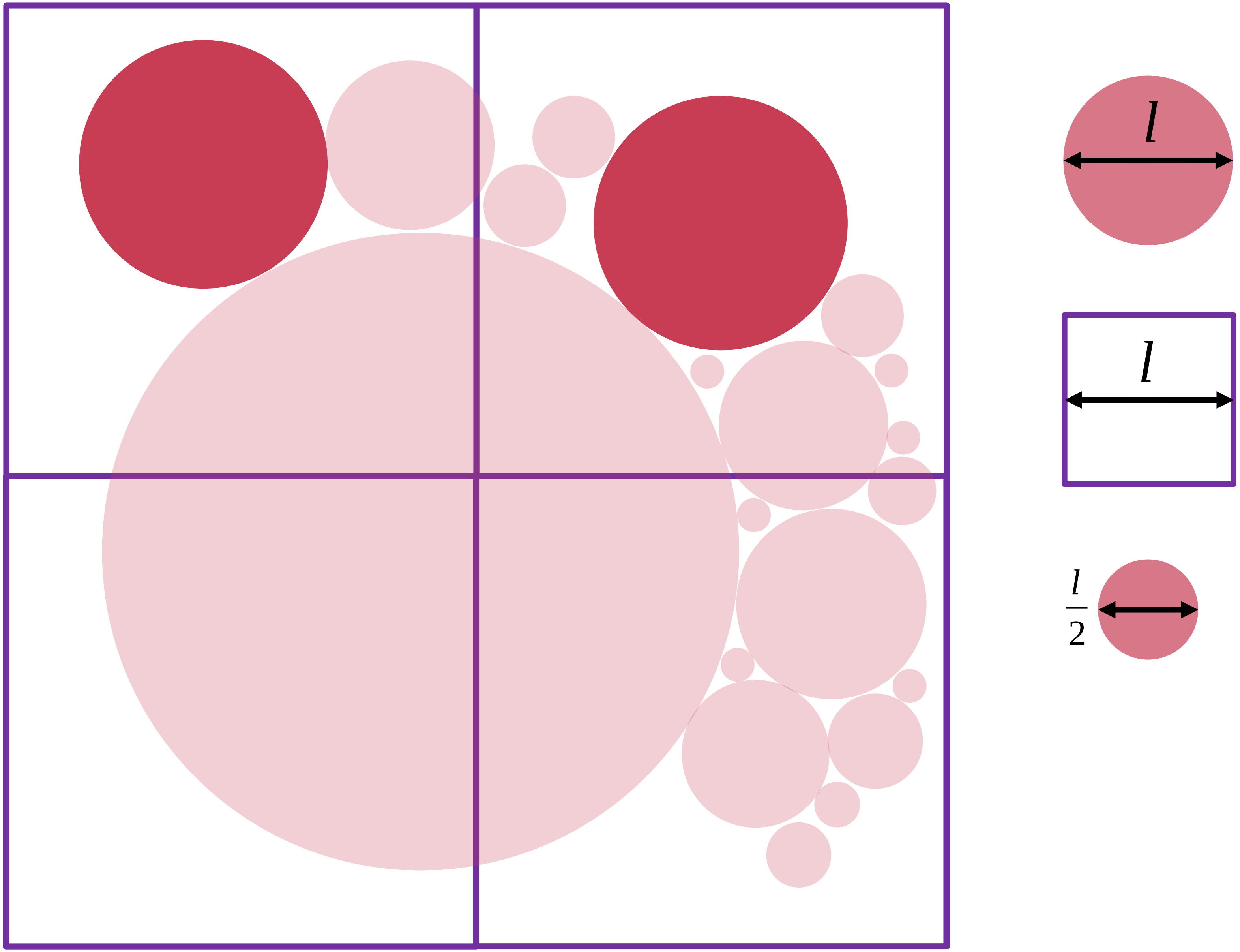
Level 2





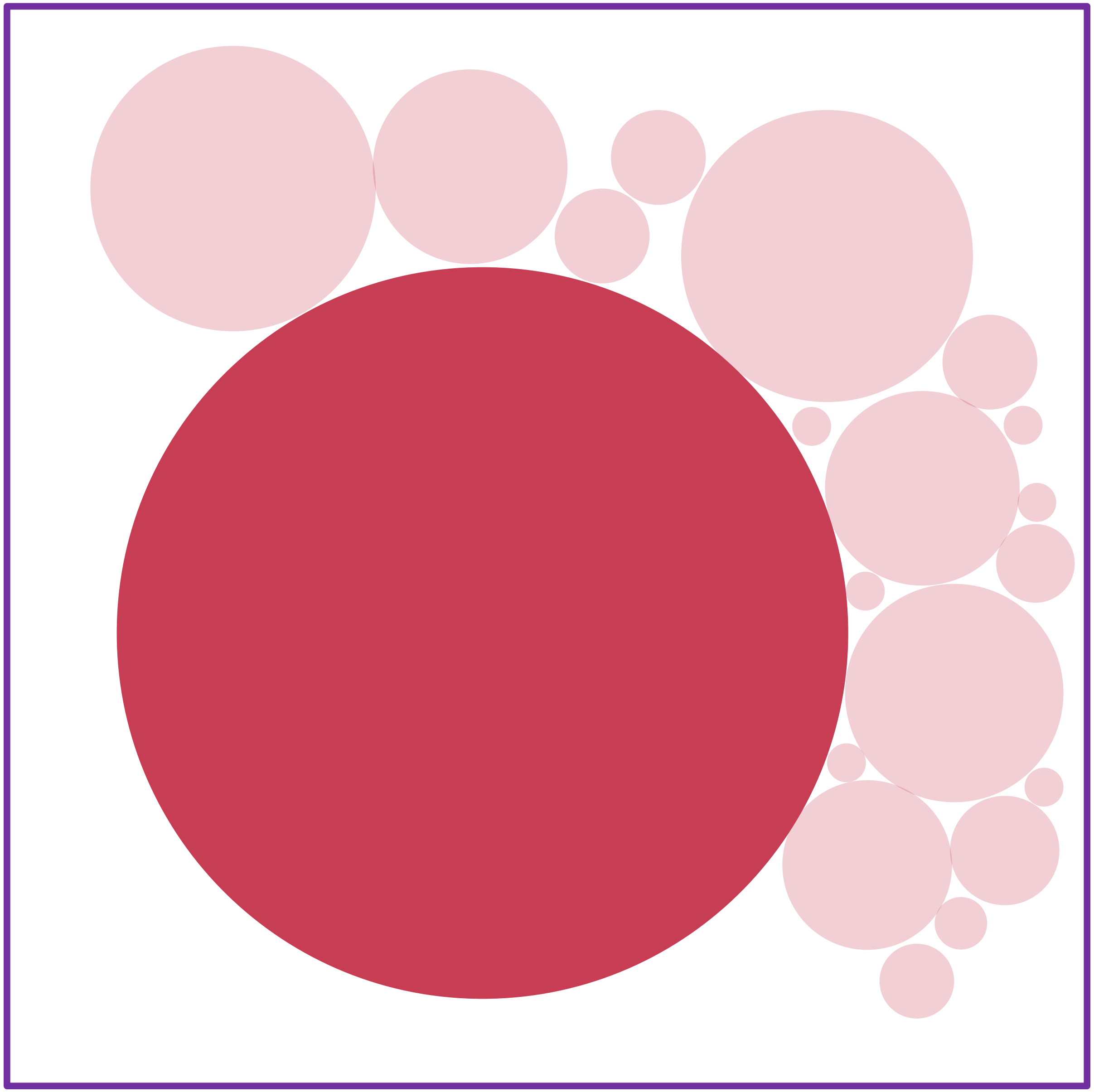
Our Algorithm

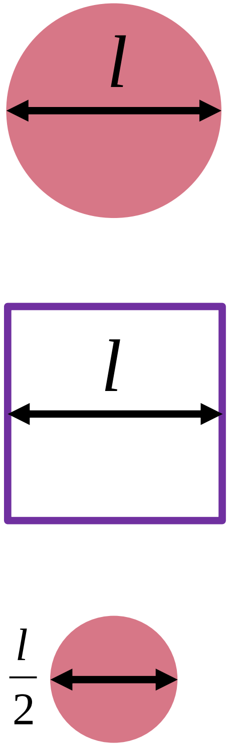
Level 1



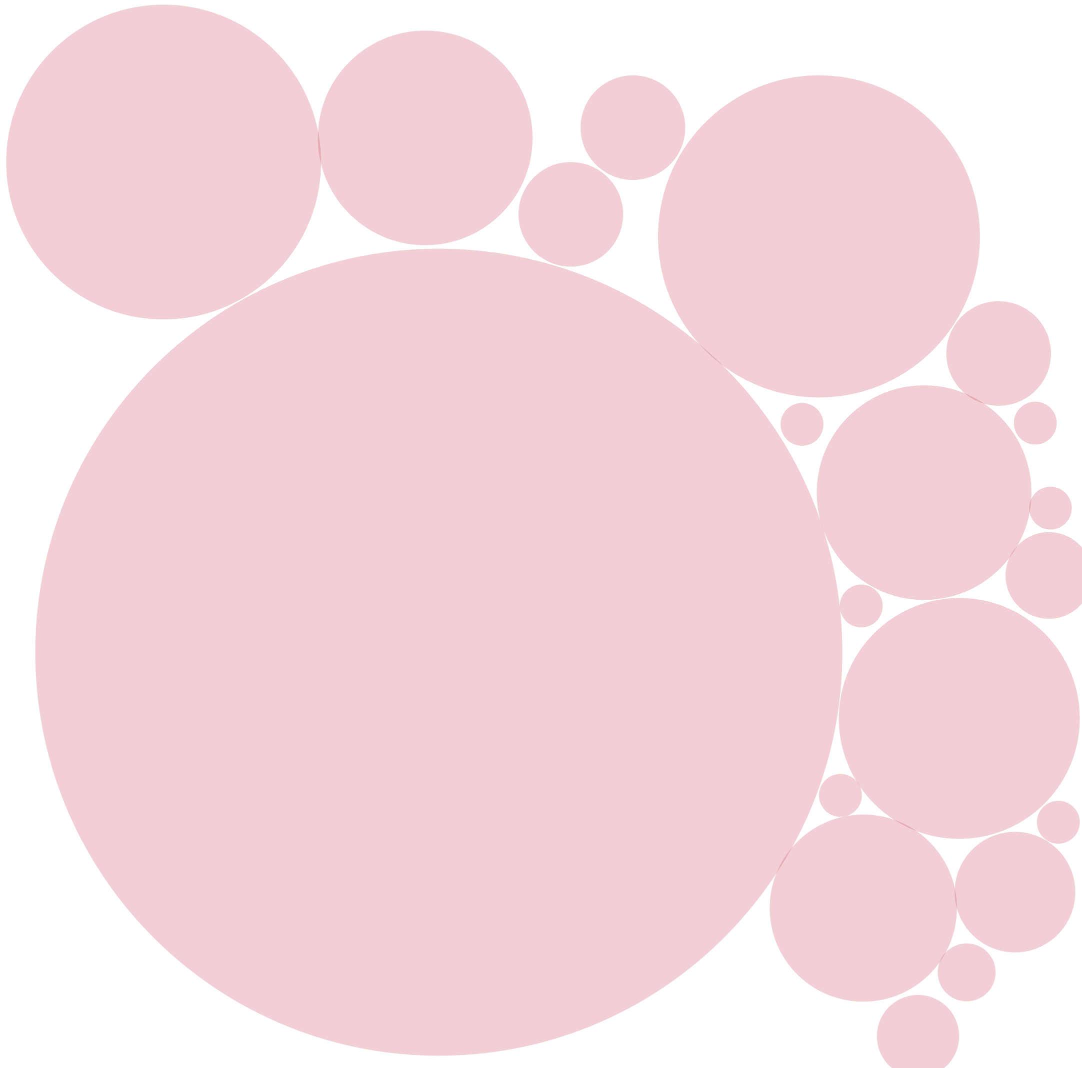
Our Algorithm

Level 0

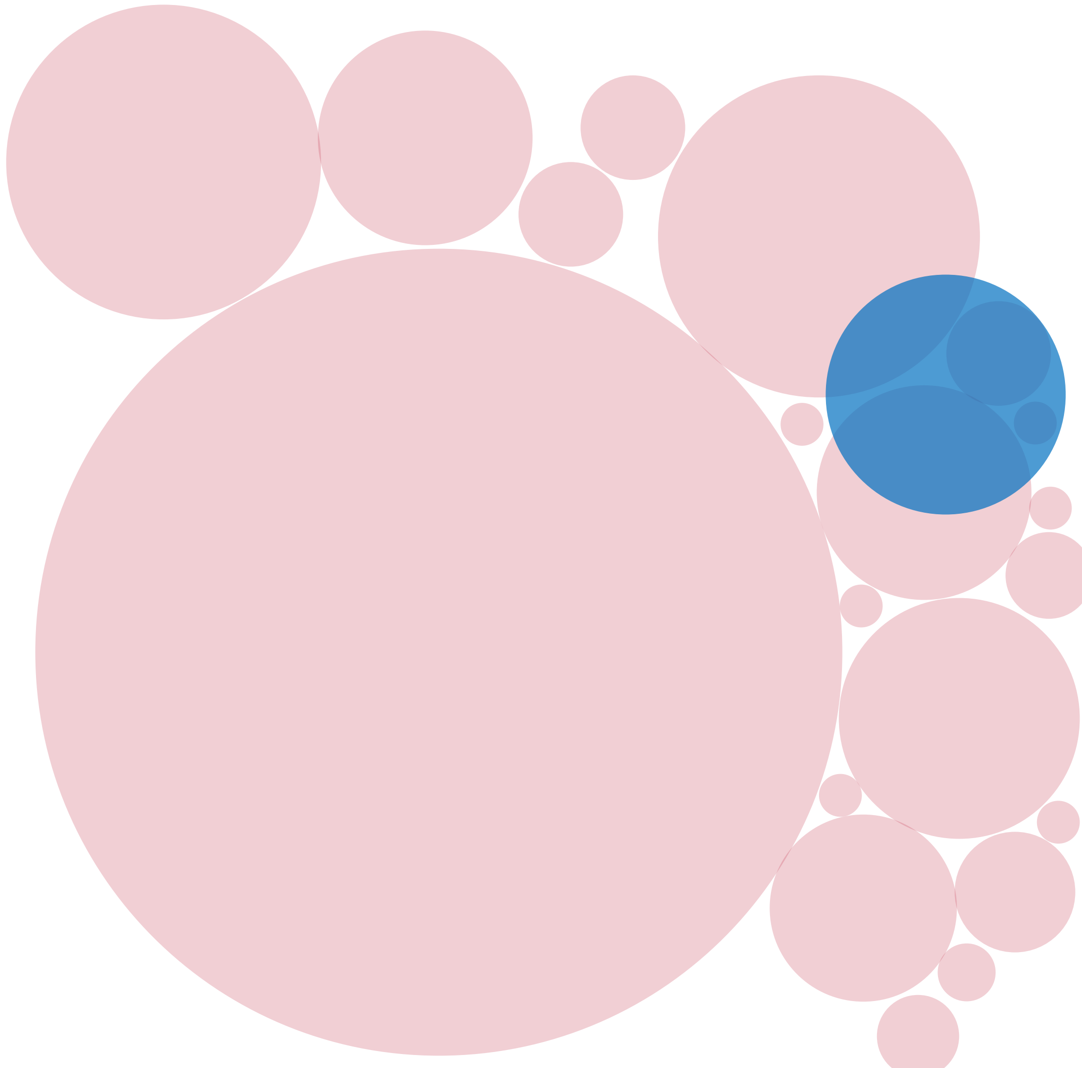




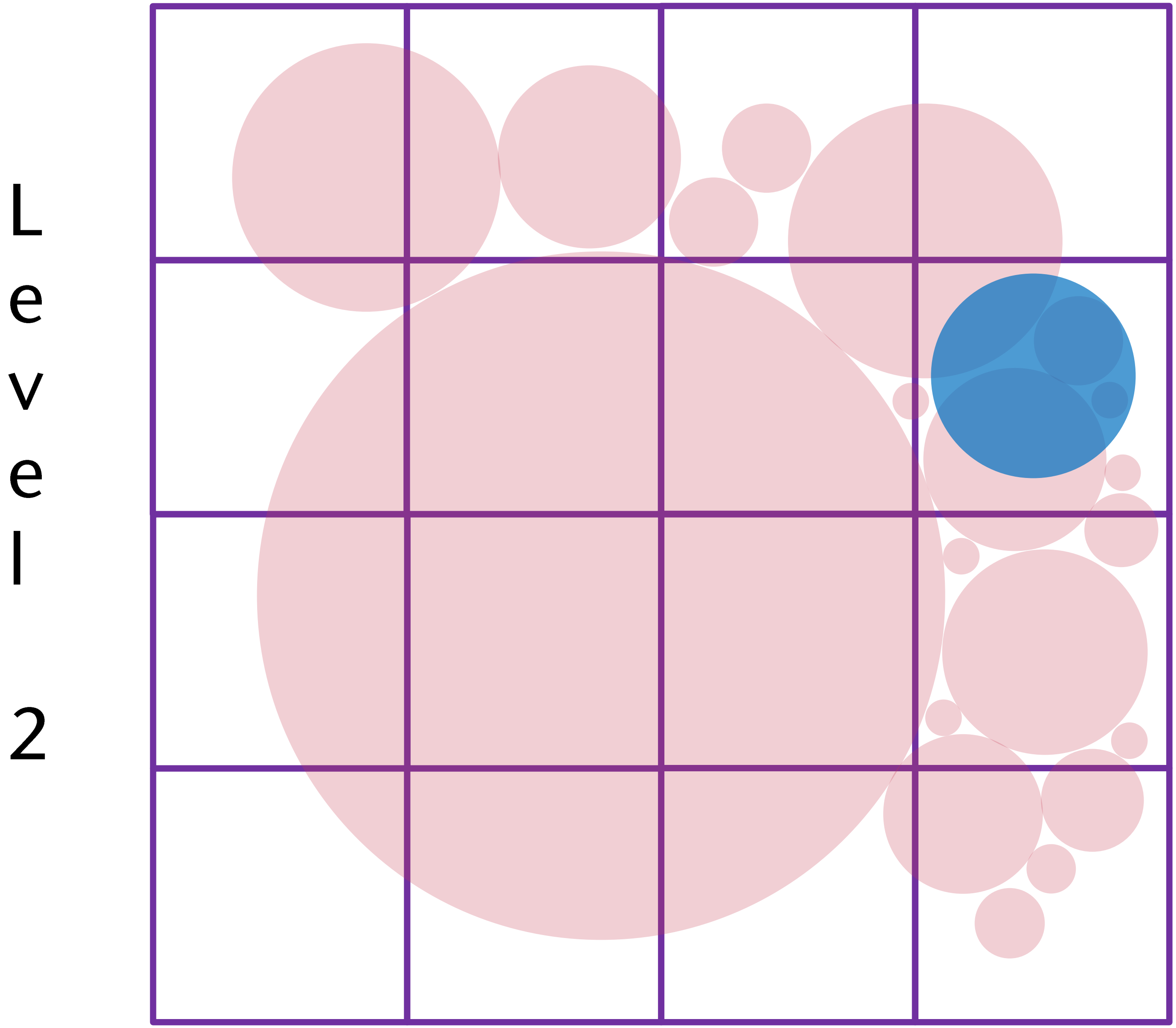
Our Algorithm



Our Algorithm



Our Algorithm



Our Algorithm

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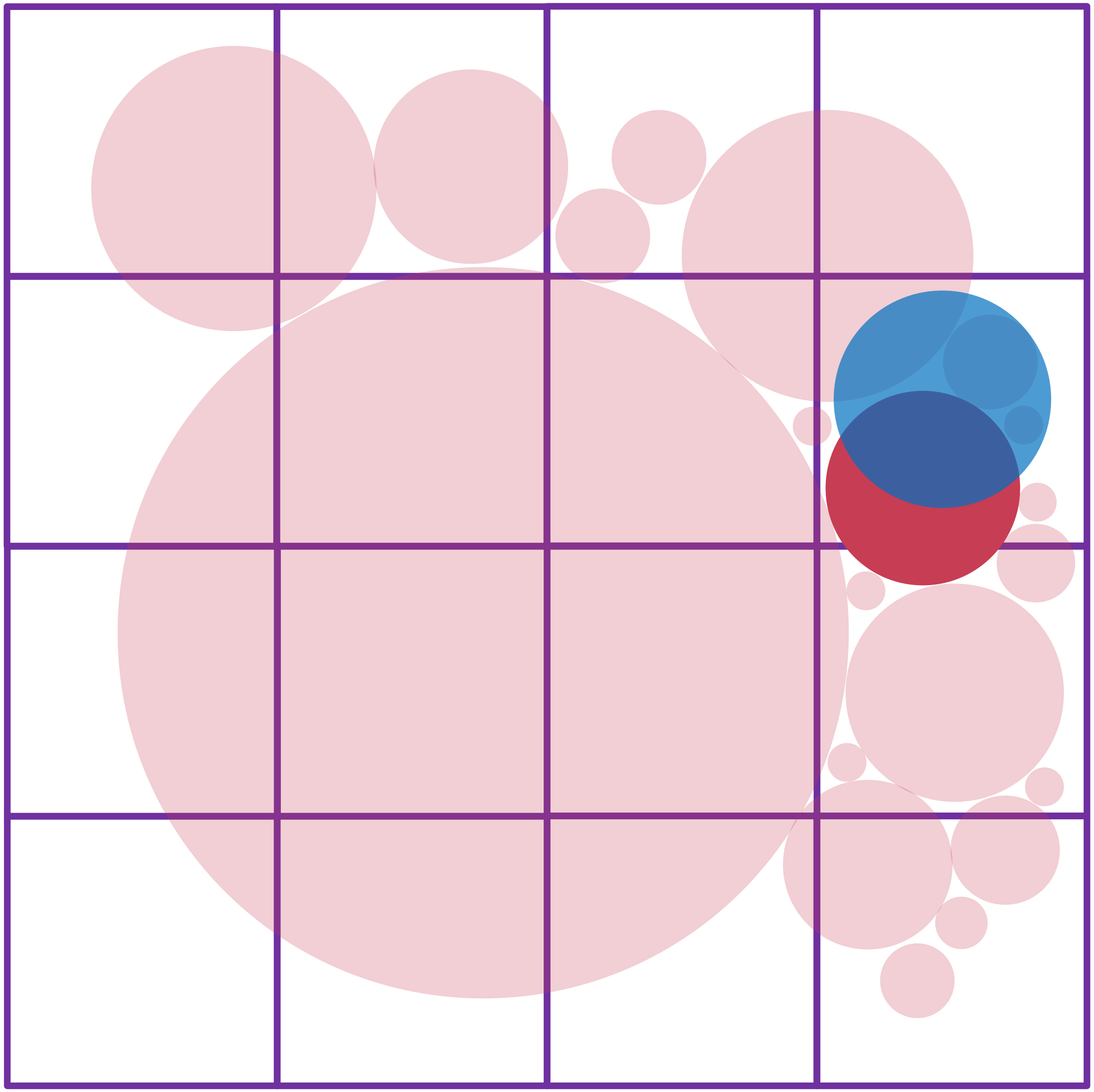
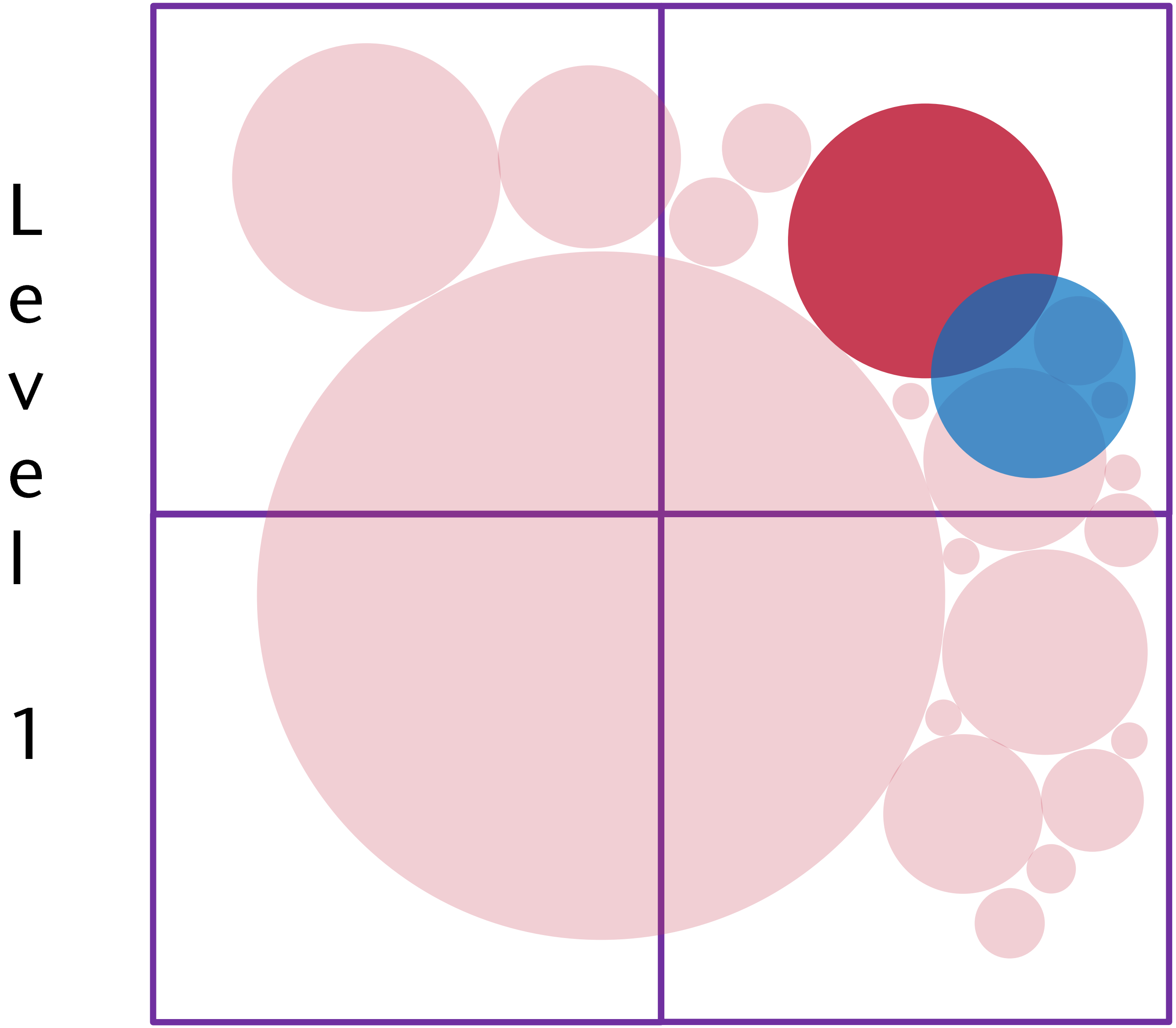
2

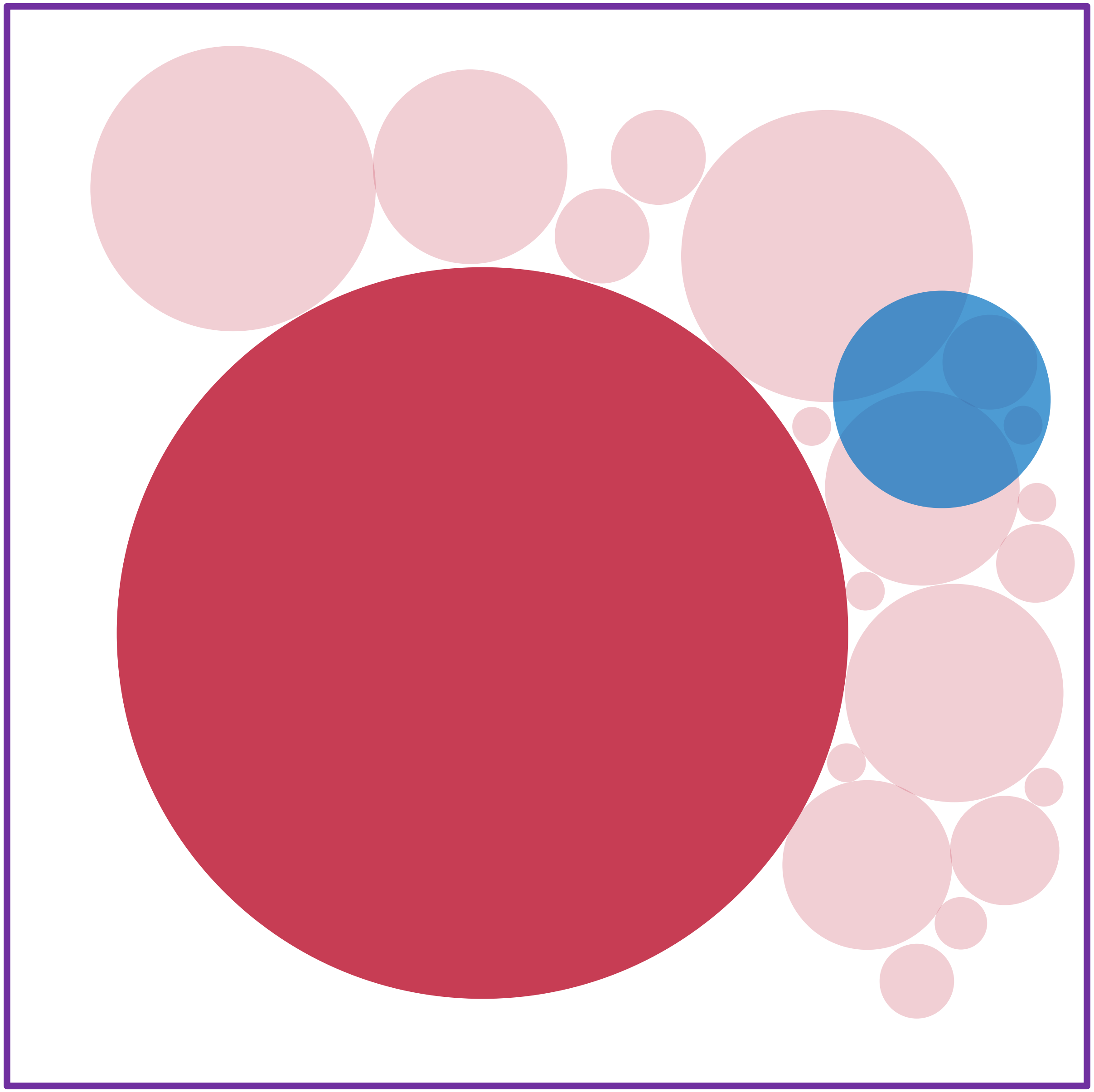


Our Algorithm



Our Algorithm

L e v e l 0



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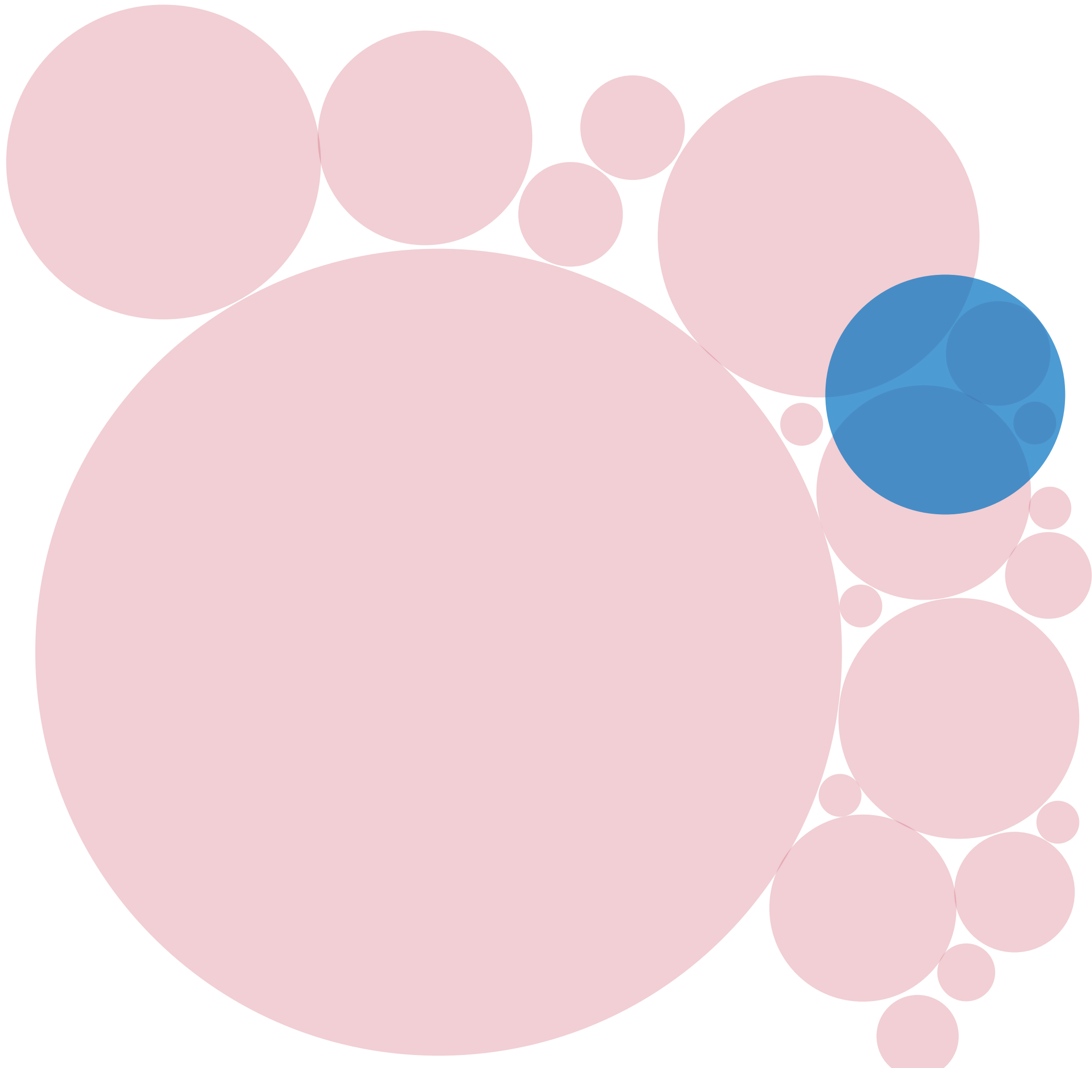

Analysis

For each sphere $s \in \mathbf{B}$:

Analysis

```
For each sphere s ∈ B:
    Compute hierarchy level l
```

Analysis

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For each sphere s ∈ B:
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    For all levels l ≤ l_i ≤ l_max:
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Analysis

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For each sphere s ∈ B:
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    For all levels l ≤ l_i ≤ l_max:
        For all cells c_j in level l_i overlapped by s
```

Analysis

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For each sphere s ∈ B:
    Compute hierarchy level l
    For all levels l ≤ l_i ≤ l_max:
        For all cells c_j in level l_i overlapped by s
            For all spheres s_k ∈ c_j
```

Analysis

```
For each sphere s ∈ B:
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    For all levels l ≤ l_i ≤ l_max:
        For all cells c_j in level l_i overlapped by s
            For all spheres s_k ∈ c_j
                Compute overlap volume for s and s_k
```

Analysis

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Compute overlap volume for s and s_k $\Rightarrow O(1)$

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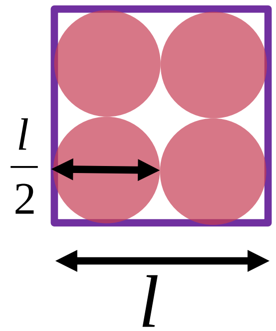

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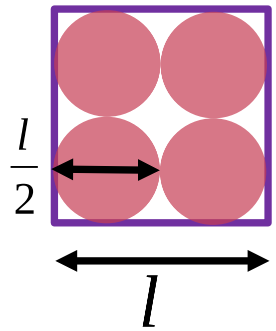

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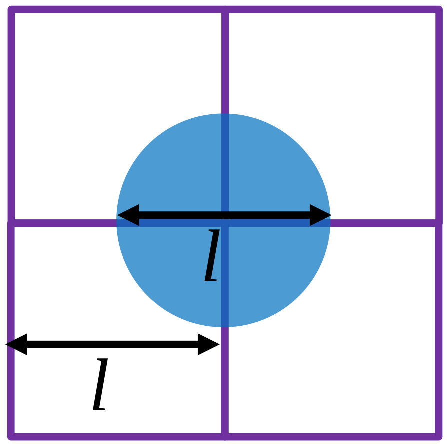

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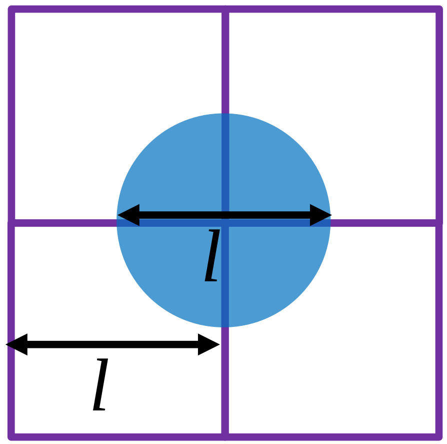

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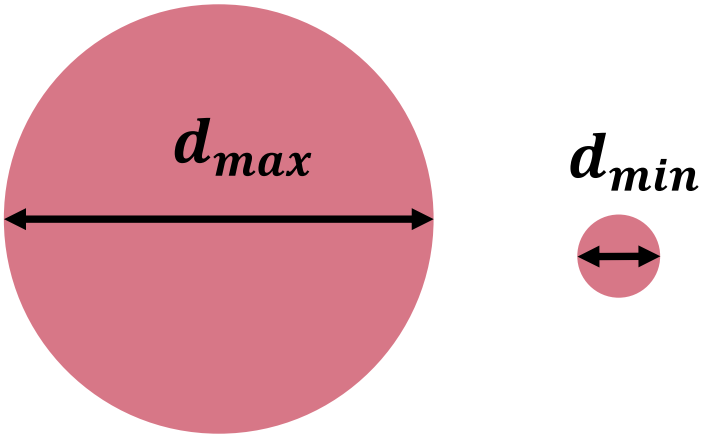

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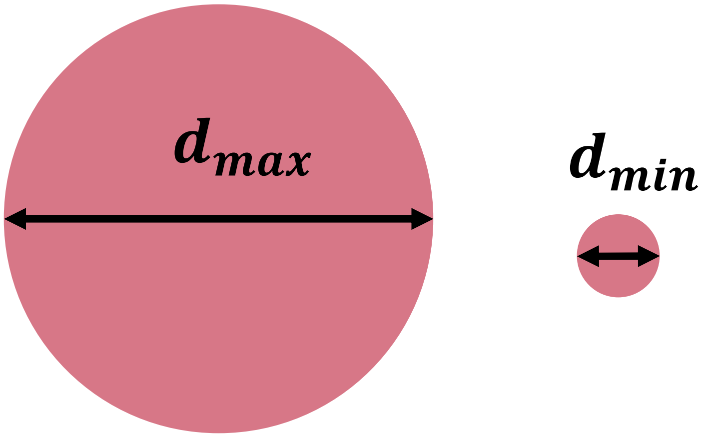

$$\Rightarrow O\left(\log \frac{d_{\max}}{d_{\min}}\right)$$

Analysis

For each sphere $s \in B$:

  Compute hierarchy level l

  For all levels $l \le l_i \le l_{max}$: $\Rightarrow O(1)$

    For all cells c_j in level l_i overlapped by s $\Rightarrow O(1)$

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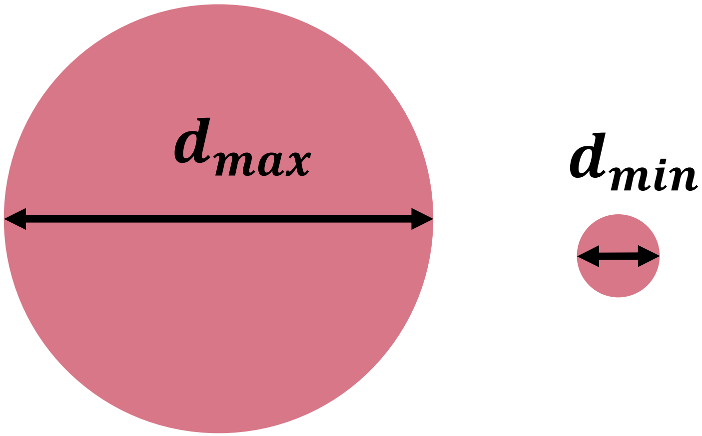

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For each sphere $s \in B$:

- Compute hierarchy level l
- For all levels $l \leq l_i \leq l_{max}$: $\Rightarrow O(1)$
  - For all cells c_j in level l_i overlapped by s $\Rightarrow O(1)$
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Analysis

```
For each sphere s ∈ B:                                        ⇒ O(n)
    Compute hierarchy level l
    For all levels l ≤ l_i ≤ l_max:                           ⇒ O(1)
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$$\textit{Total Time}: \quad O(n)$$

Analysis

```
For each sphere s ∈ B:                                    ⇒ O(n)
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    For all levels l ≤ l_i ≤ l_max:                       ⇒ O(1)
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For each sphere $s \in B$: $\Rightarrow O(n)$

Compute hierarchy level l

For all levels $l \leq l_i \leq l_{max}$: $\Rightarrow O(1)$

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Compute overlap volume for s and s_k $\Rightarrow O(1)$

$$\textit{Total Time}: \quad O(n)$$

Analysis

```
In Parallel for all spheres s ∈ B:                          ⇒ O(n)
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In Parallel for all spheres s ∈ B:                                  ⇒ O(n)
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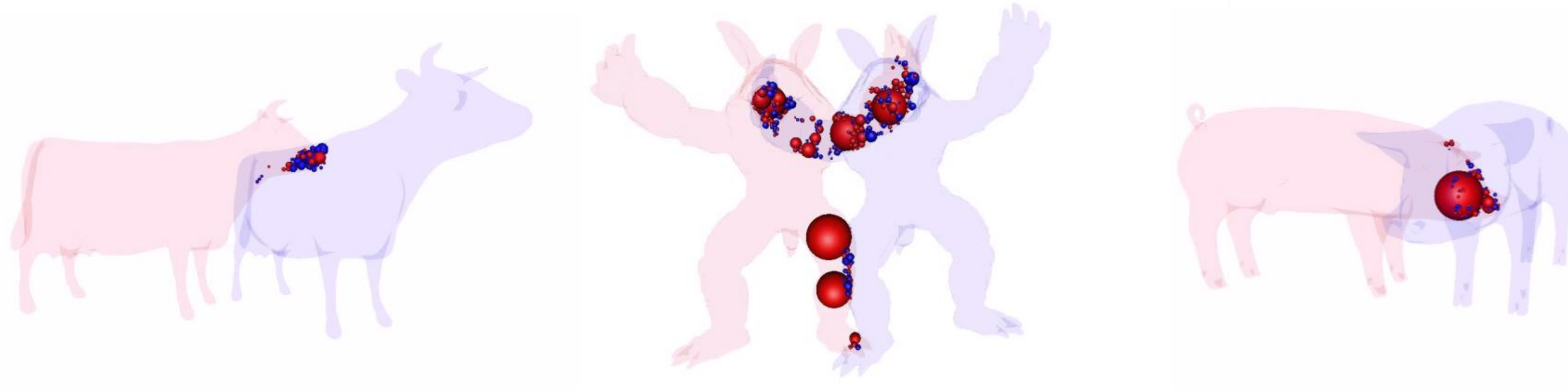
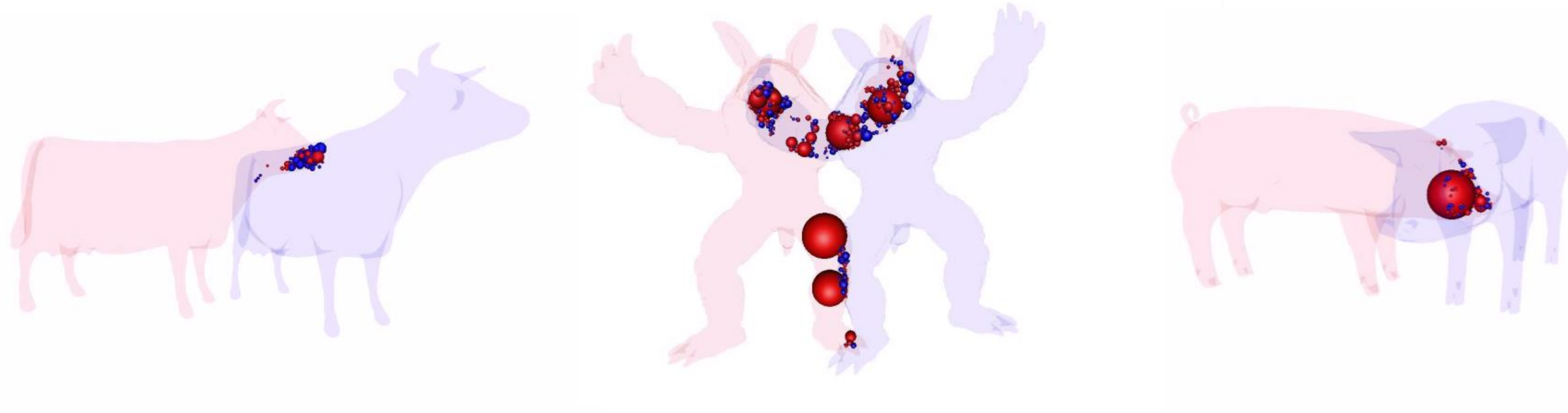
Total Time: $O(n)$

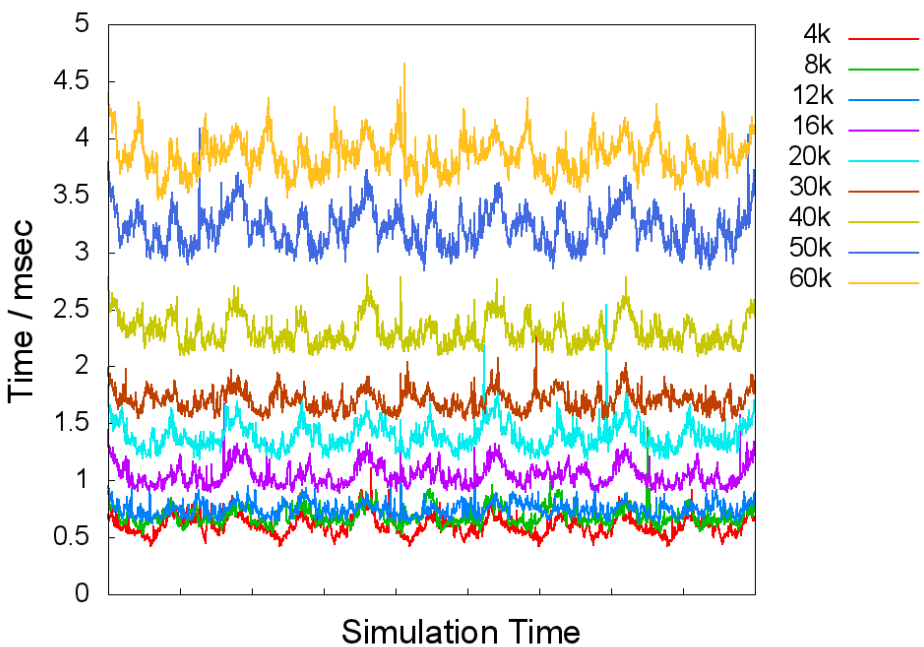
Total Parallel Time: $O(1)$

Results

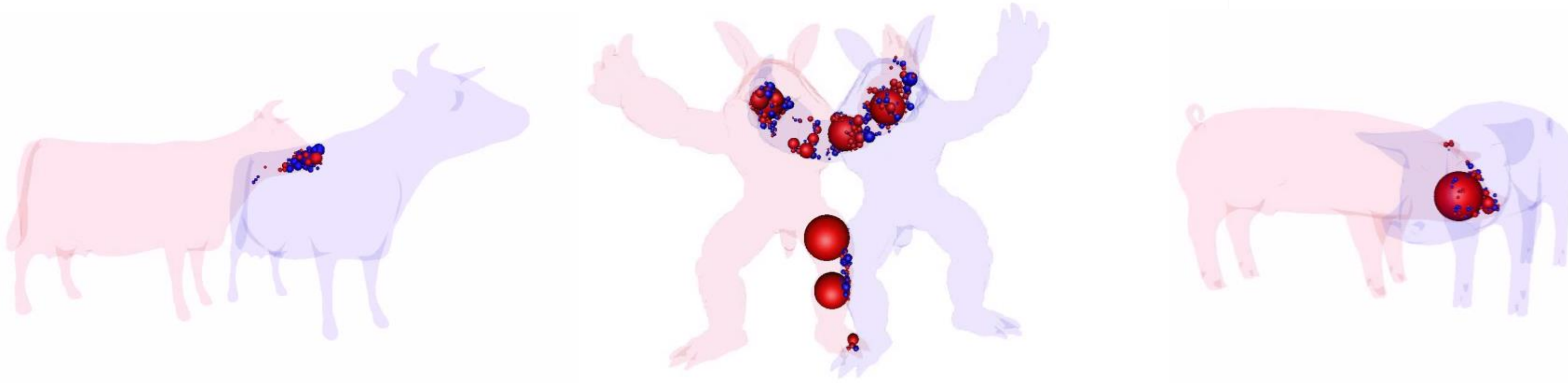


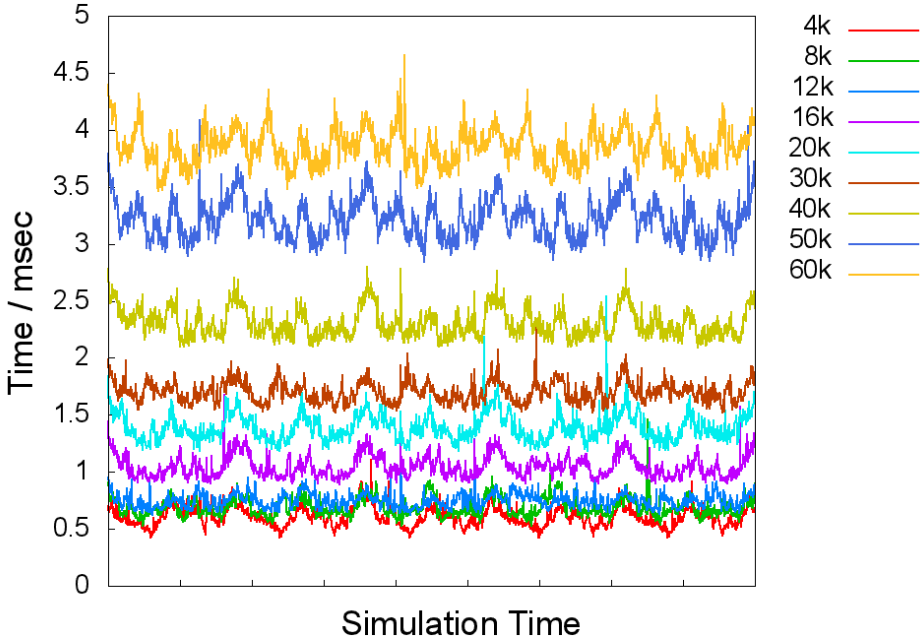
Results

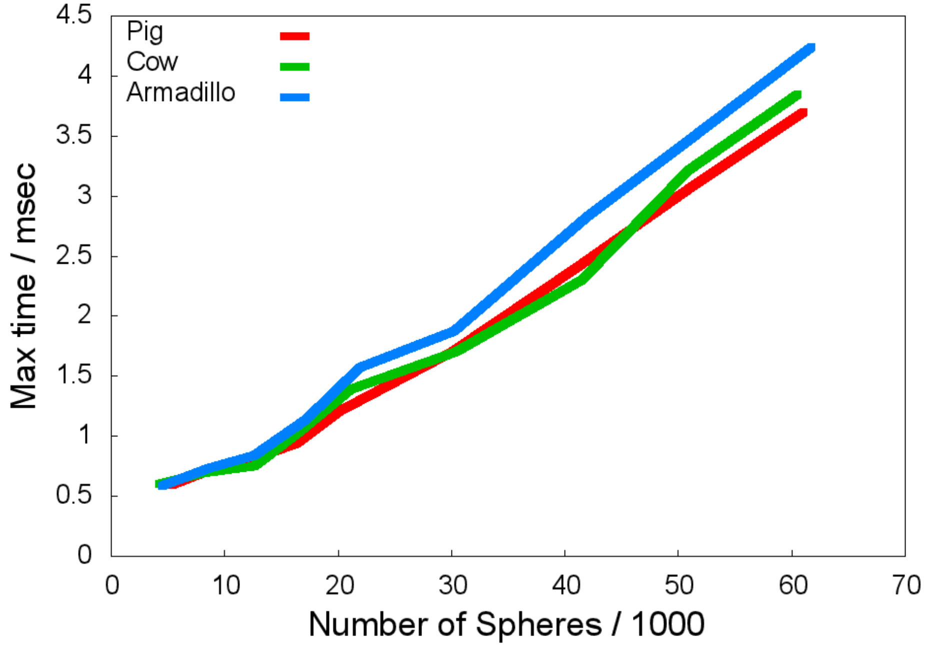



Time / msec
0
0.5
1
1.5
2
2.5
3
3.5
4
4.5
5
Simulation Time
4k
8k
12k
16k
20k
30k
40k
50k
60k

Results

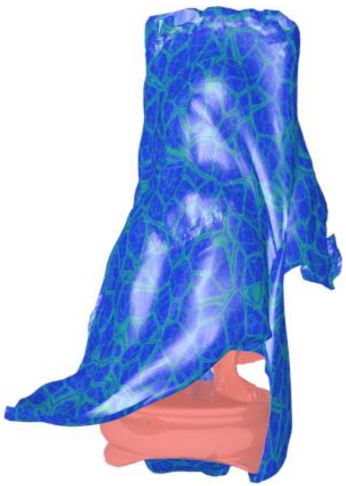


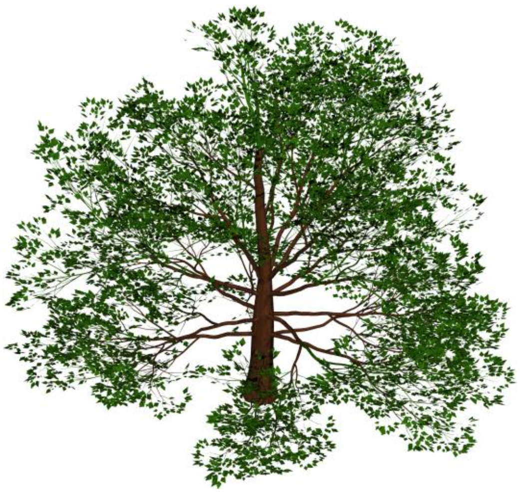

5
4.5
4
3.5
3
2.5
2
1.5
1
0.5
0
Time / msec
Simulation Time
4k
8k
12k
16k
20k
30k
40k
50k
60k


4.5
4
3.5
3
2.5
2
1.5
1
0.5
0
Max time / msec
0
10
20
30
40
50
60
70
Number of Spheres / 1000
Pig
Cow
Armadillo

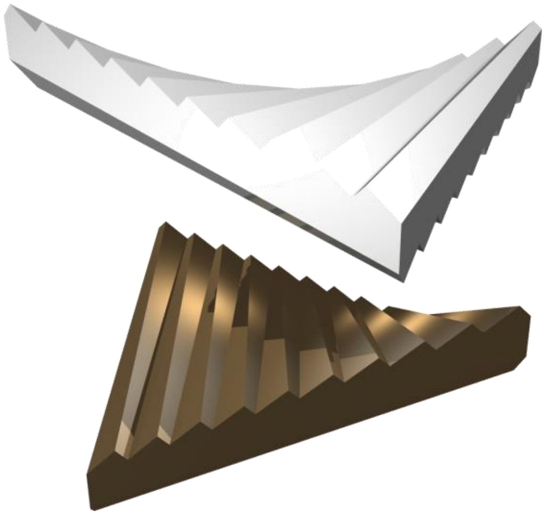
Challenges

- Precondition for ISTs:
  - Watertight
  - Rigid (?)



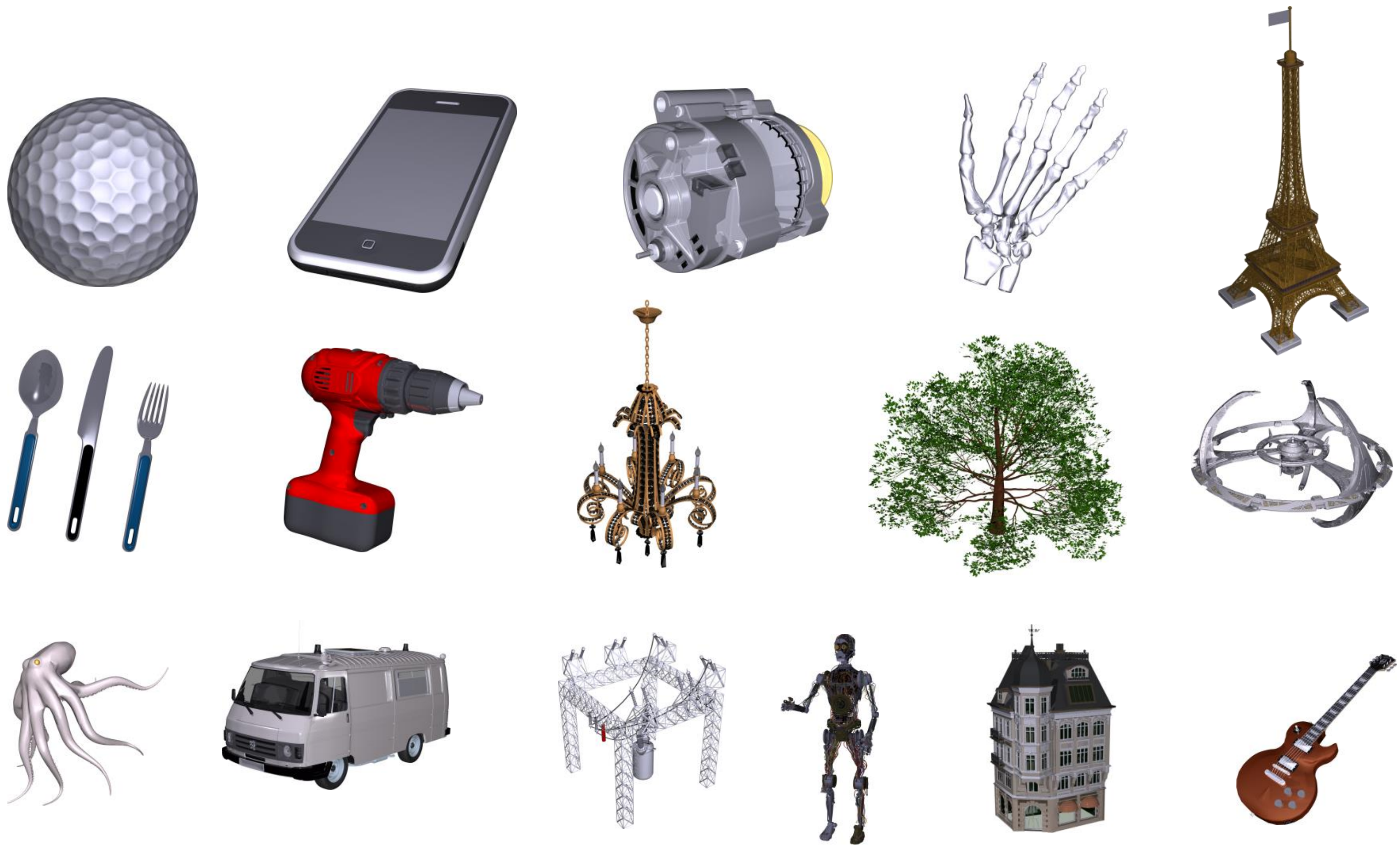


Collision Detection Worst Case



$$O(n^2)$$

What about these objects?



Previous Works

- Distance of convex polytopes [Lin & Canny, 1991]:

  Worst case: $O(\sqrt{n})$, n = # faces

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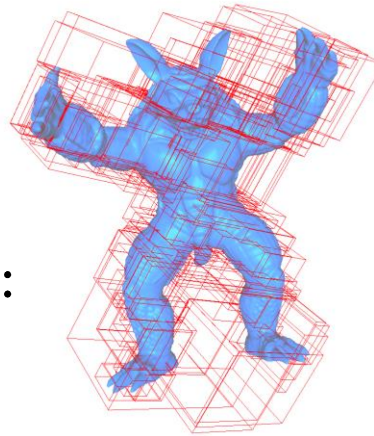

- Distance of convex polytopes [Lin & Canny, 1991]:

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- All intersections of n convex polytopes [Suri et al., 1998]:

  $O((n+k)\log^2 n)$, k = # intersecting pairs

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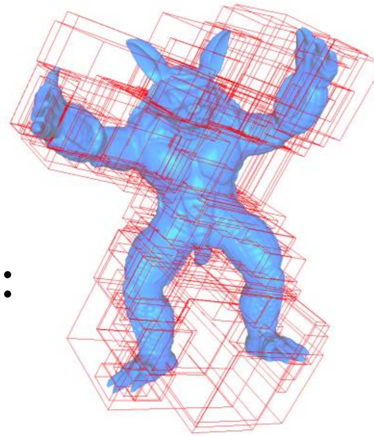

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- Expected $O(\log n)$ depending on bounding volume and object configuration [Weller et al., 2006]

Our Contribution

- A novel geometric predicate that defines a class of „well-shaped“ objects

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  - Predicate needs to consider only a single object
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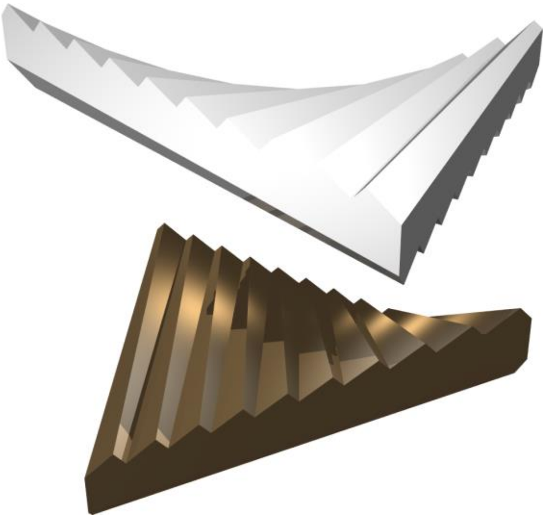
  - Contains (almost) all practically relevant objects

- Proof to show that objects in that class have $O(n)$ intersections
- New algorithm with (almost) linear running time for objects fullfilling our predicate

Basic Idea

- What makes objects like the Chazelle polyhedron so complex to check for collision?



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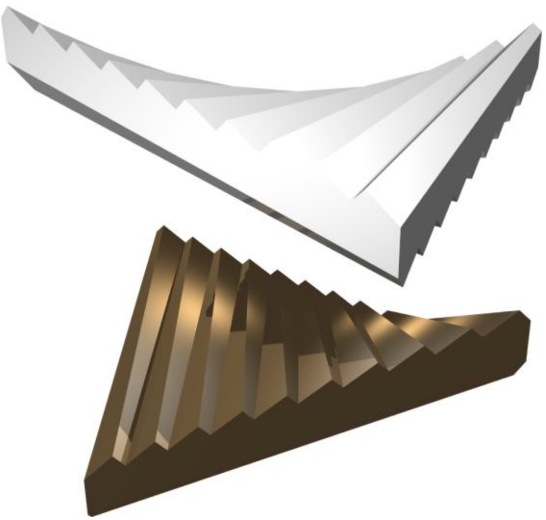
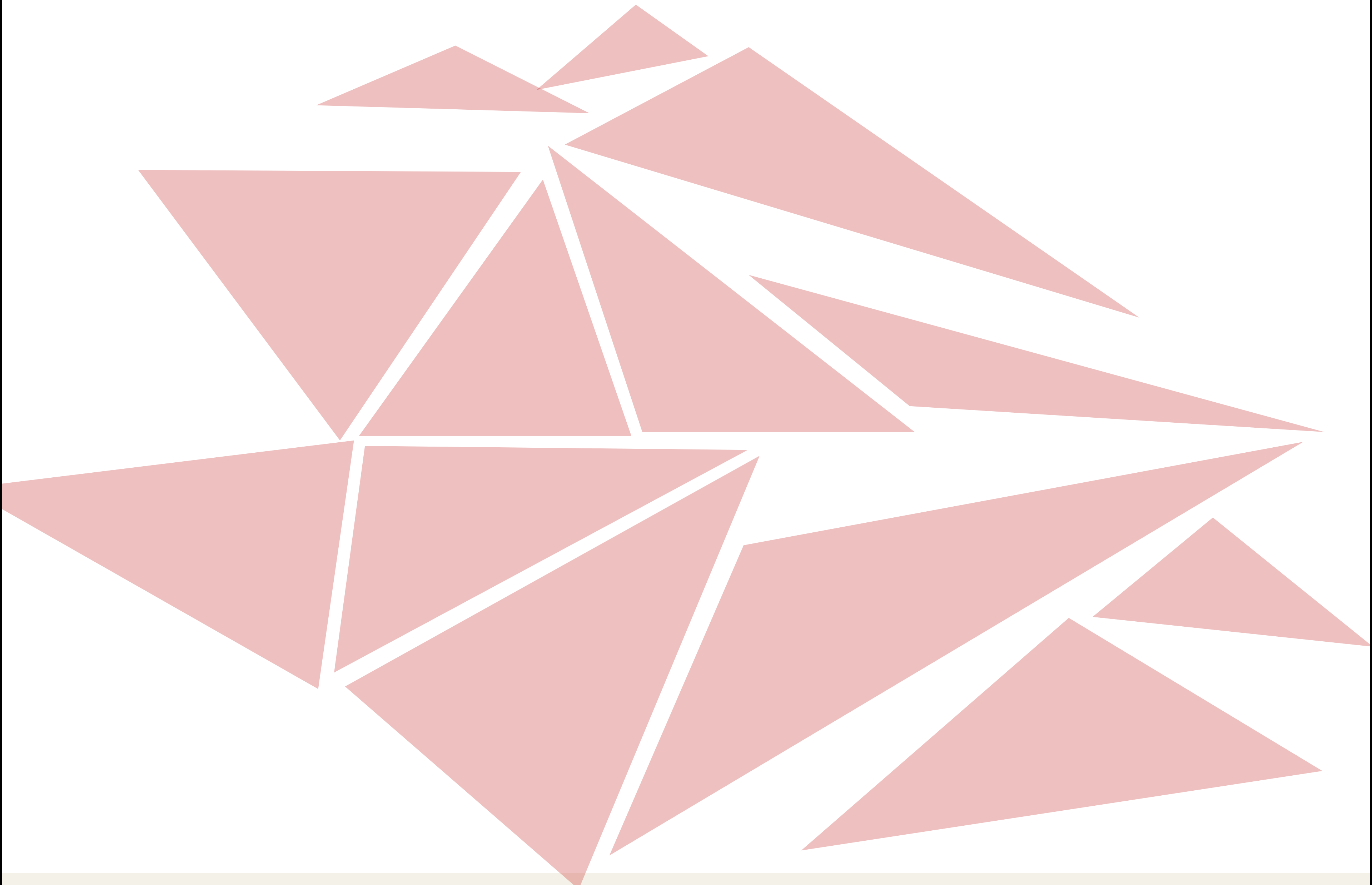


- There can be $O(n)$ polygons in the neighbourhood of each polygon

Our Geometric Predicate



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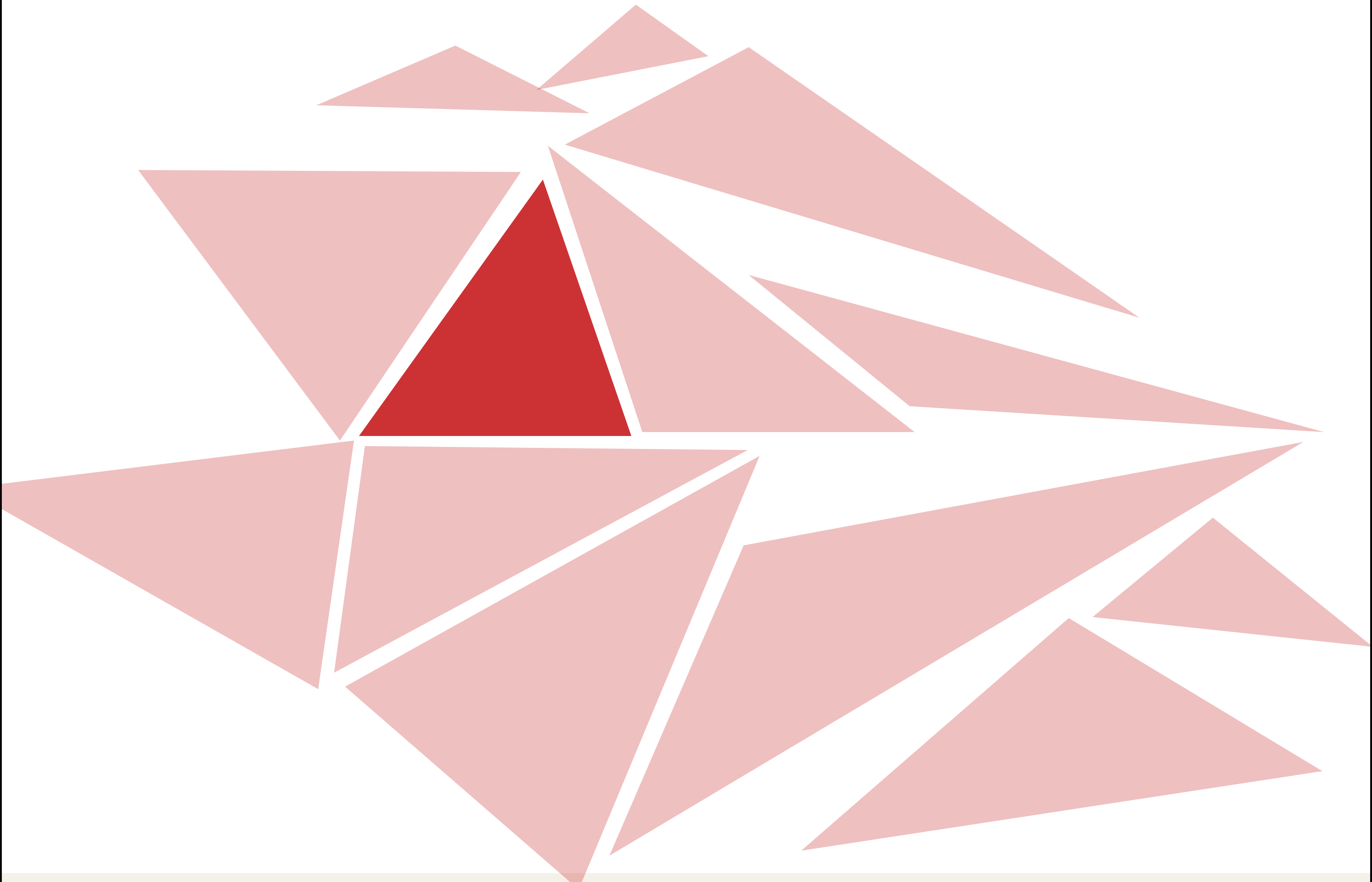

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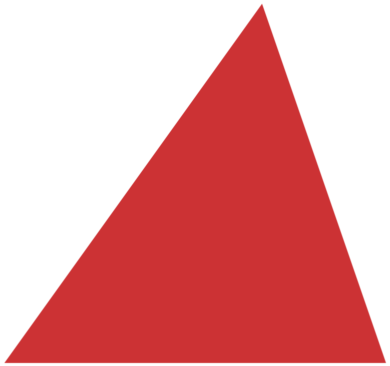

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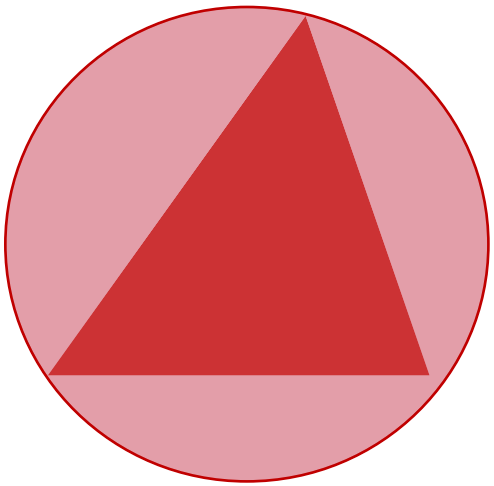

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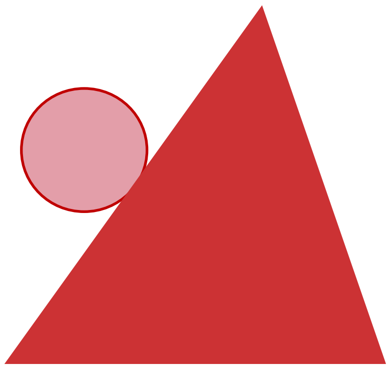

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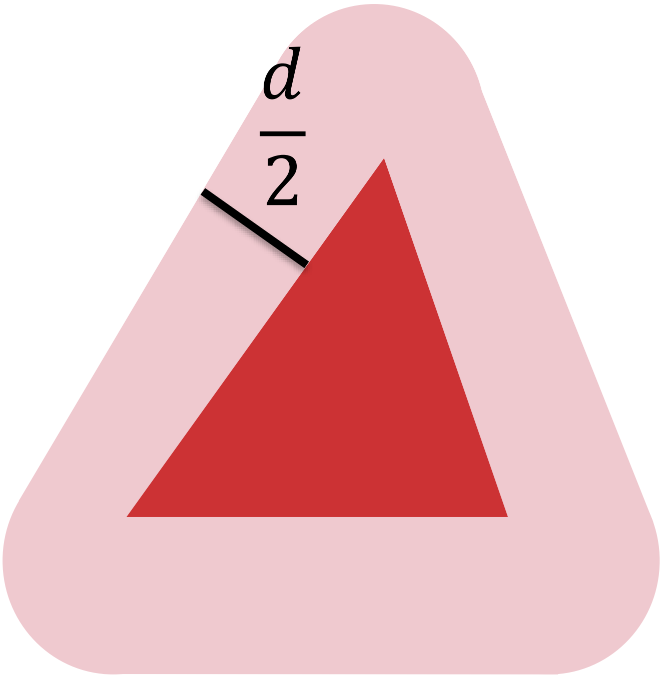

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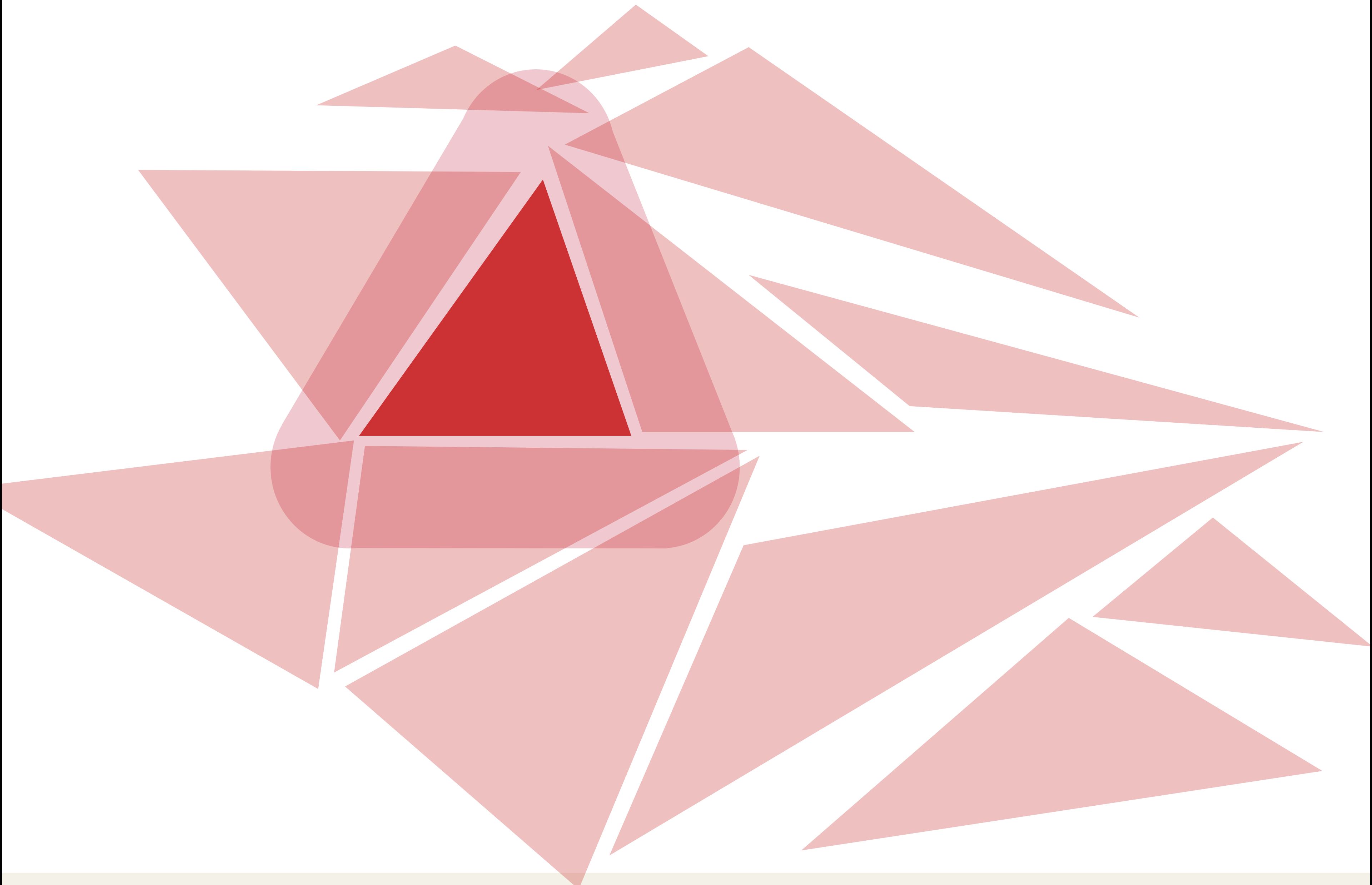

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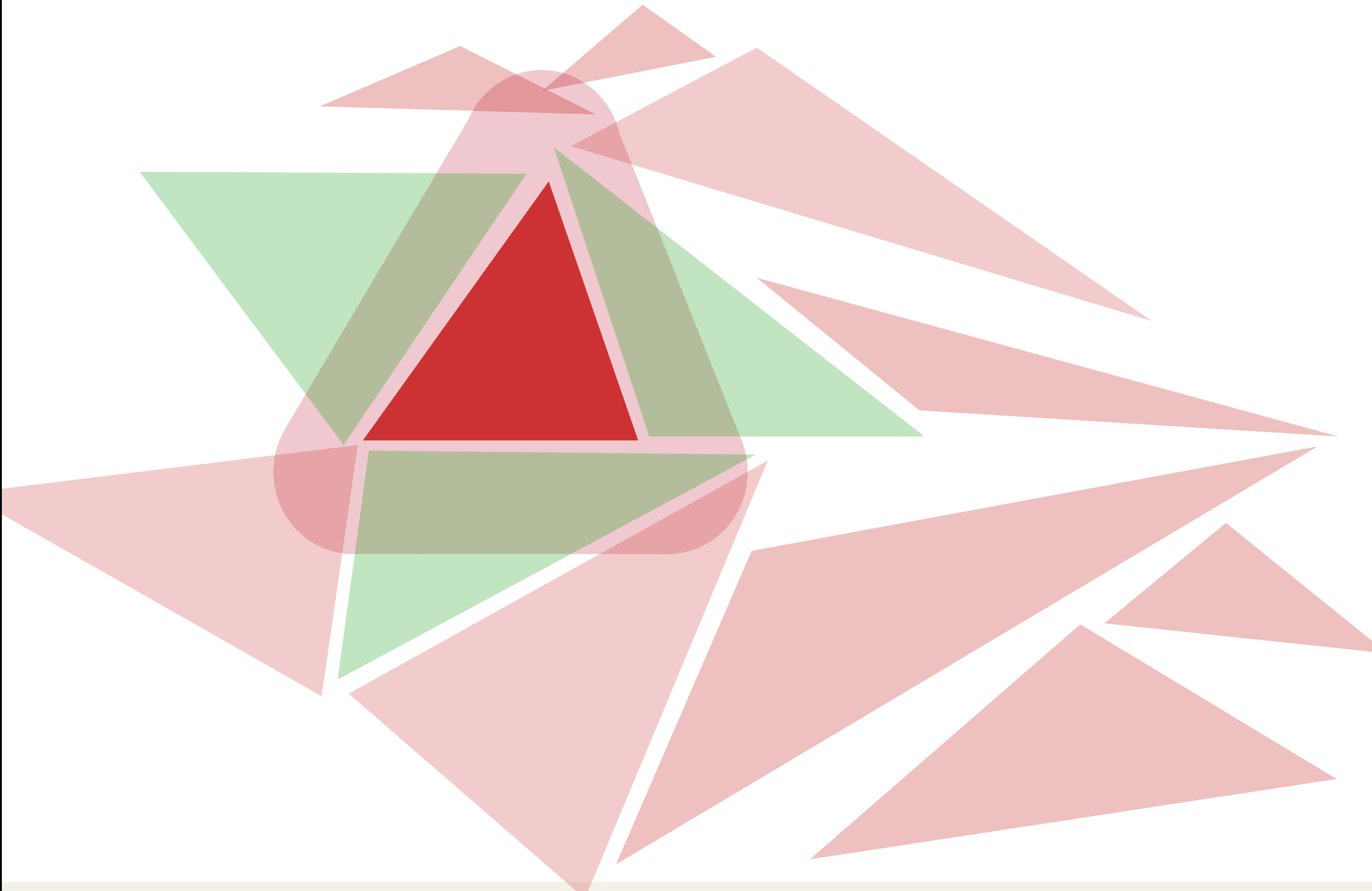

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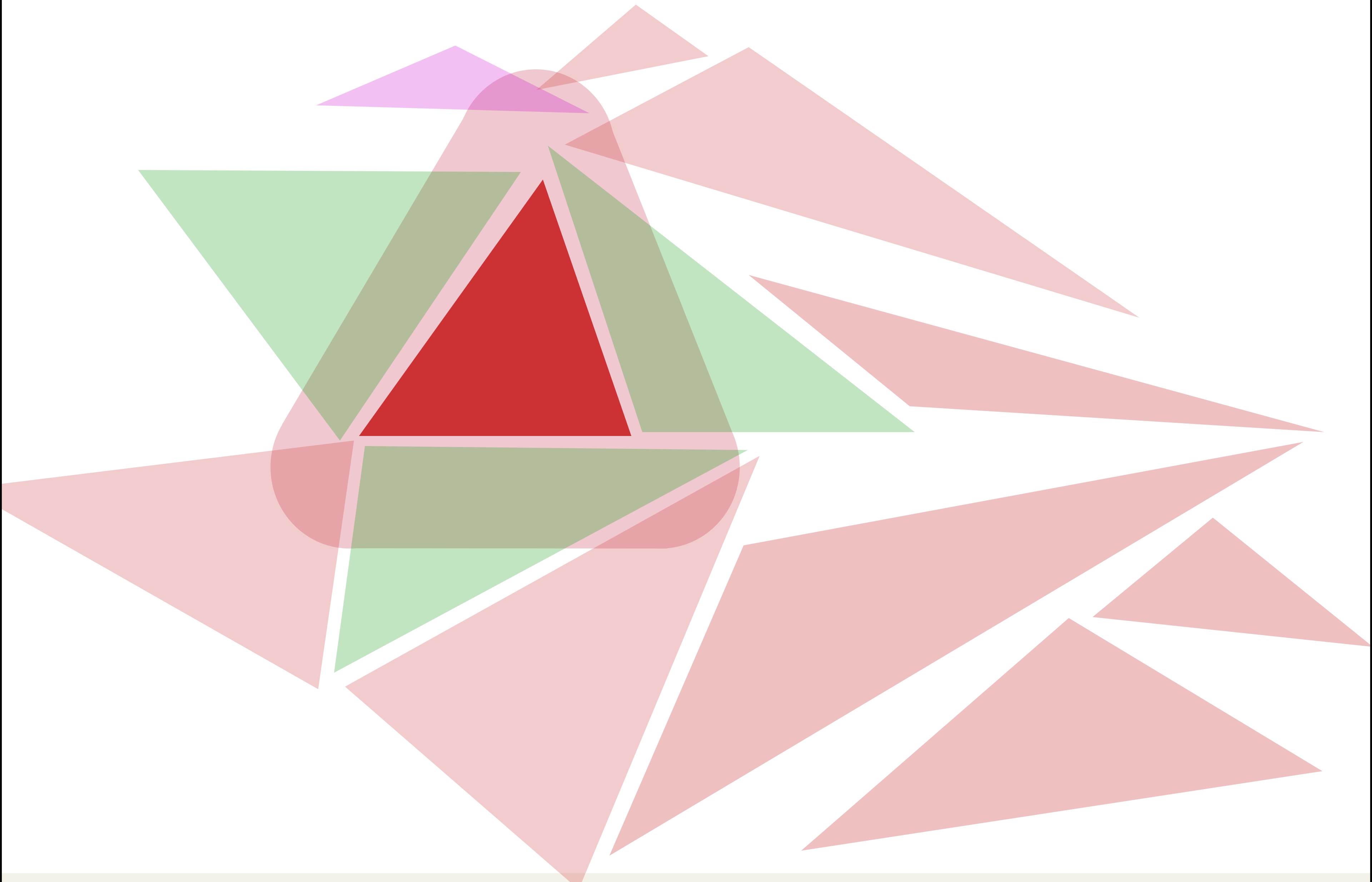
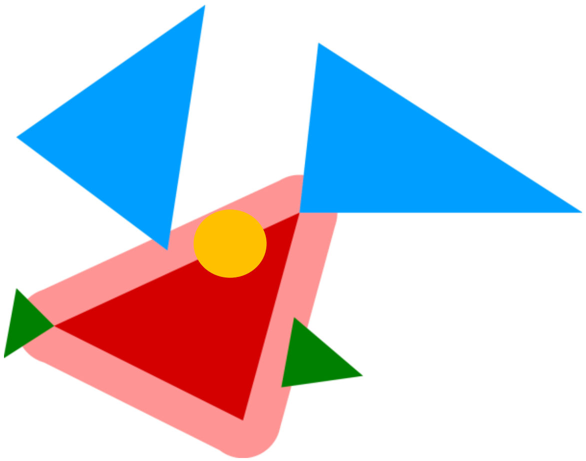


Definition: k-free

- Let t $\in A$ be a triangle in a triangle set A and $k > 0$ some constant. Let s be a sphere with diameter $\frac{d}{2}$, where d is the diameter of the smallest enclosing sphere of t. We call t k-free if $\left|\{t_j \in A; r \leq r_j \; and \; t_j \cap (t \oplus s) \neq \emptyset\}\right| < k$ where d_j is diameter of the smallest enclosing sphere of triangle t_j and $t \oplus s$ is the Minkowski sum of s and t.
- Accordingly, we call the whole set of triangles A k-free, if all triangles $t_j \in A$ are k-free.



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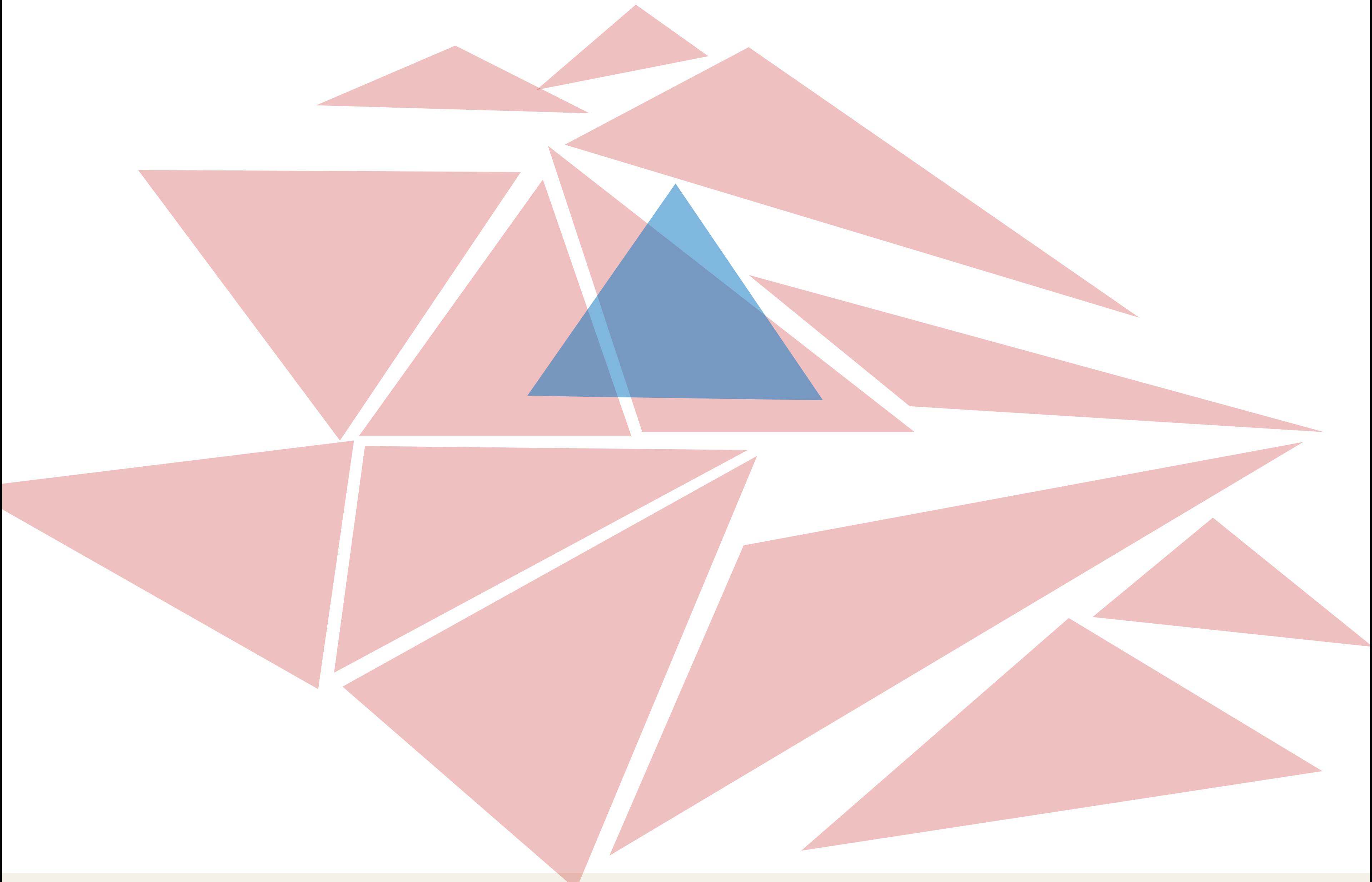

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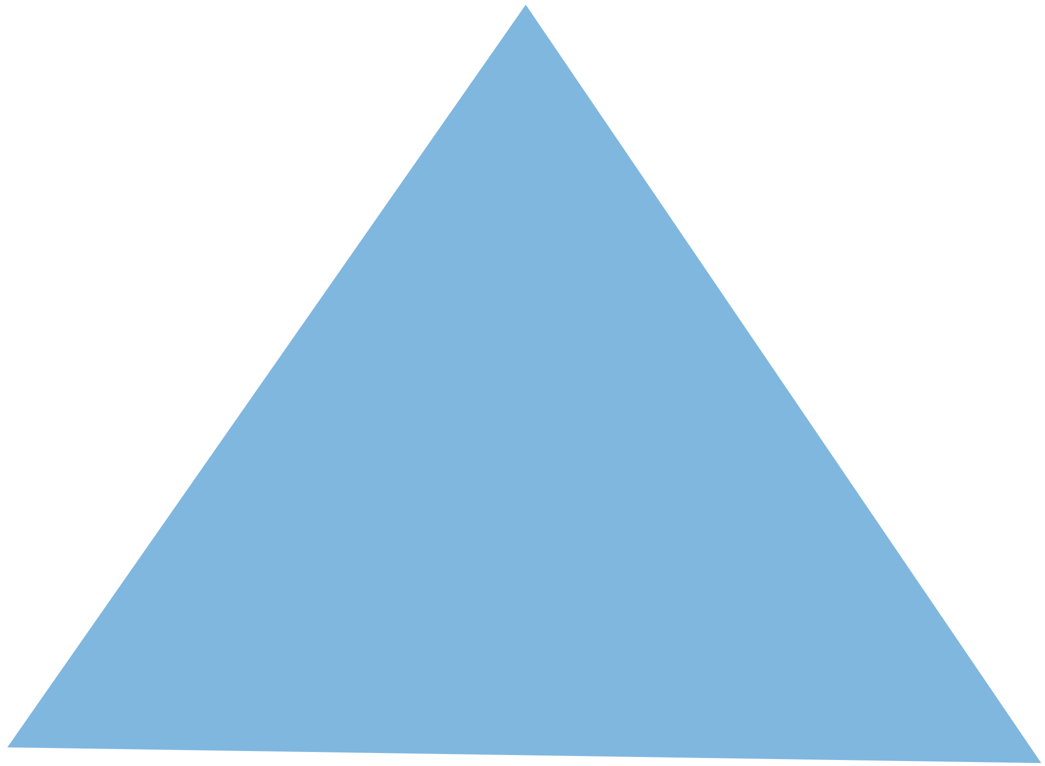

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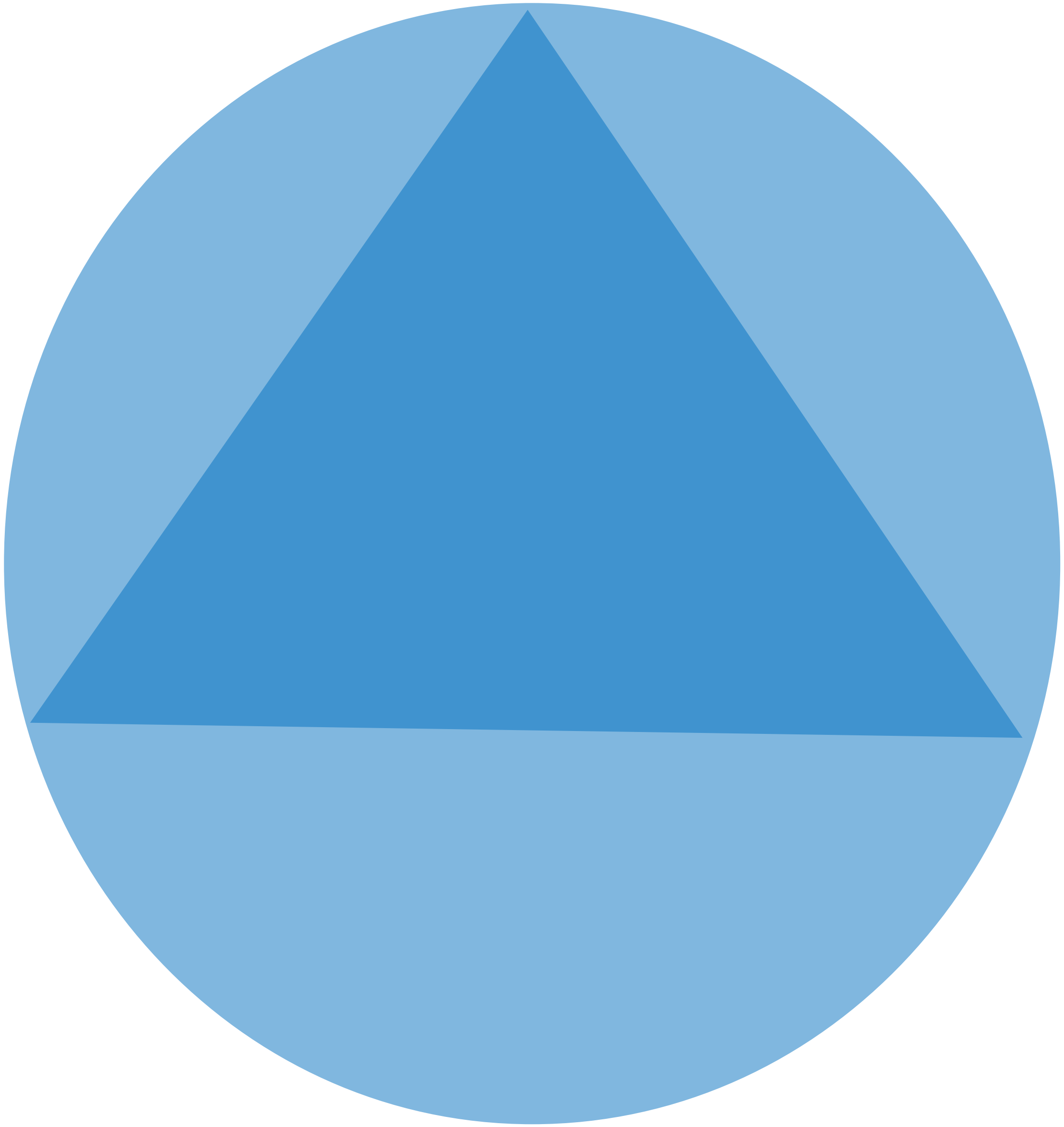

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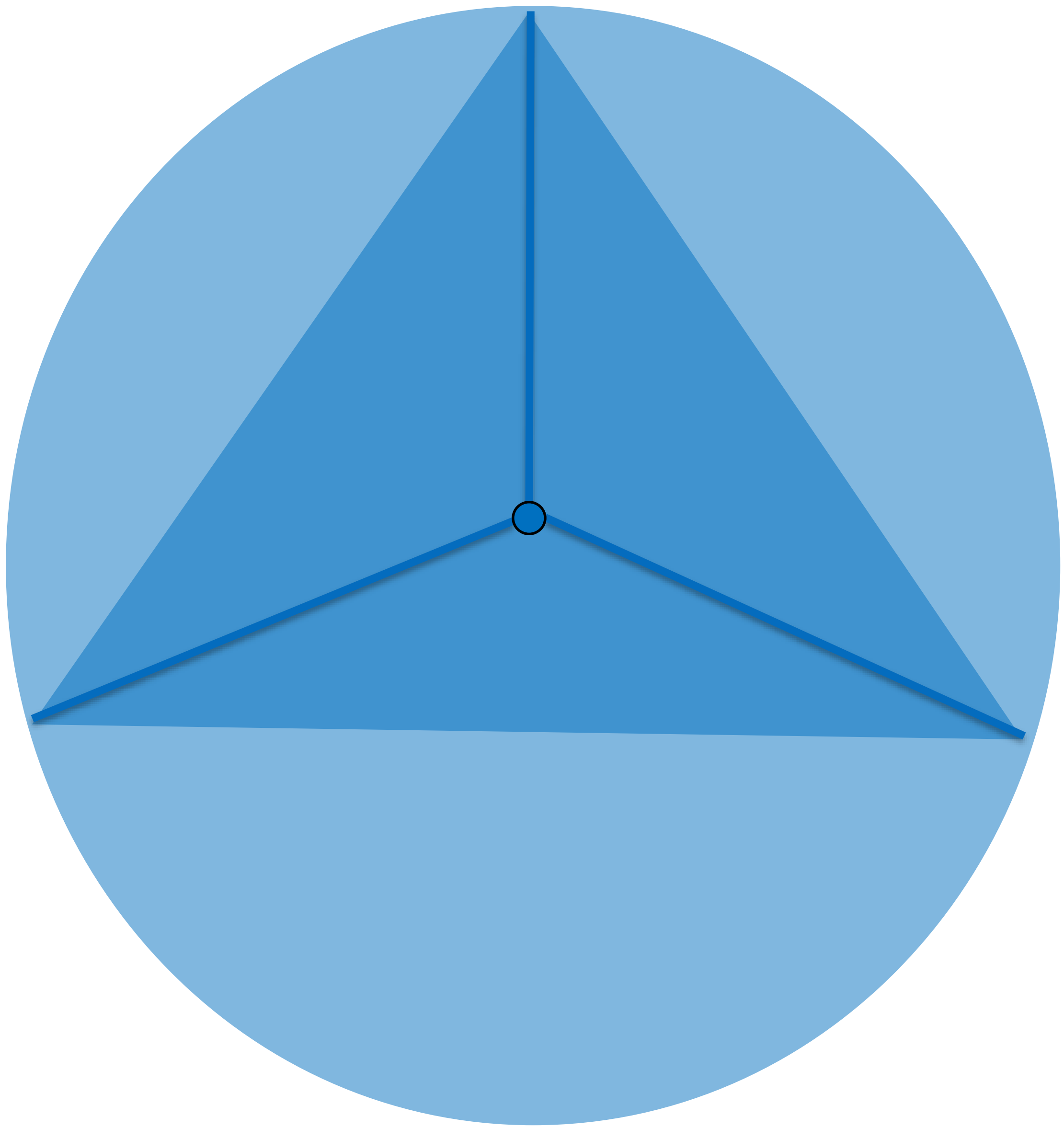

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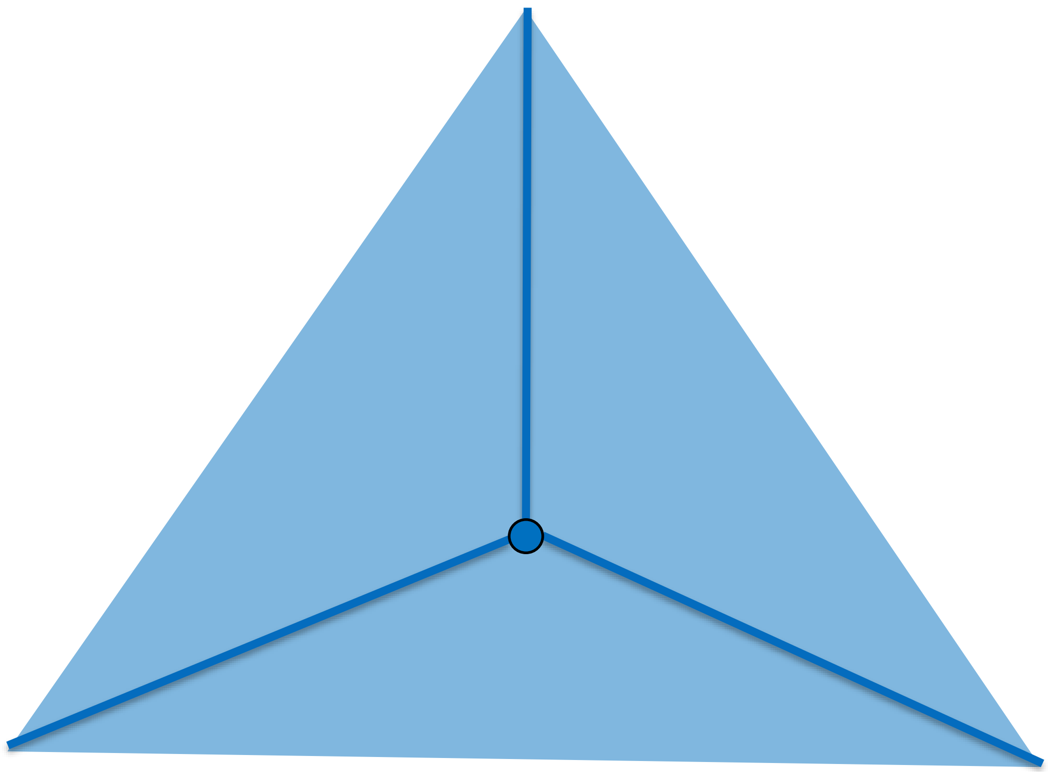

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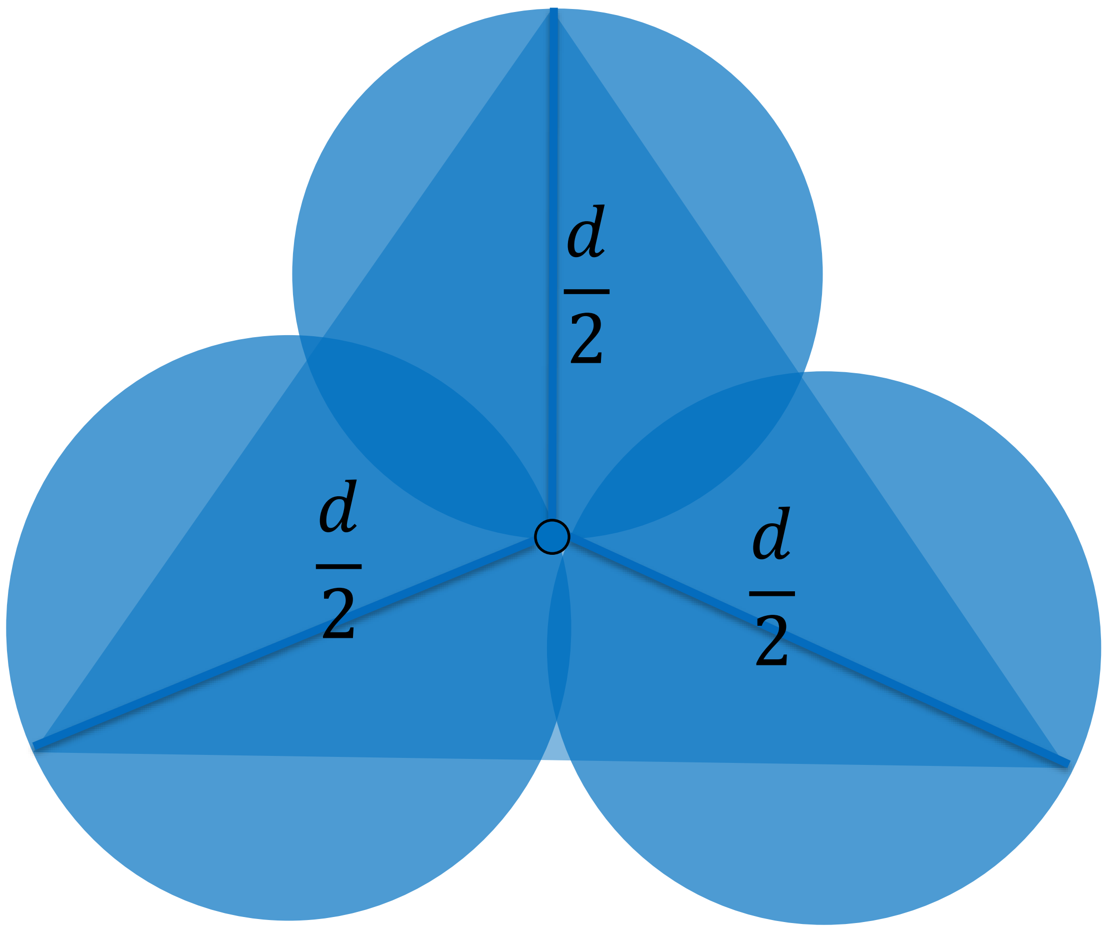

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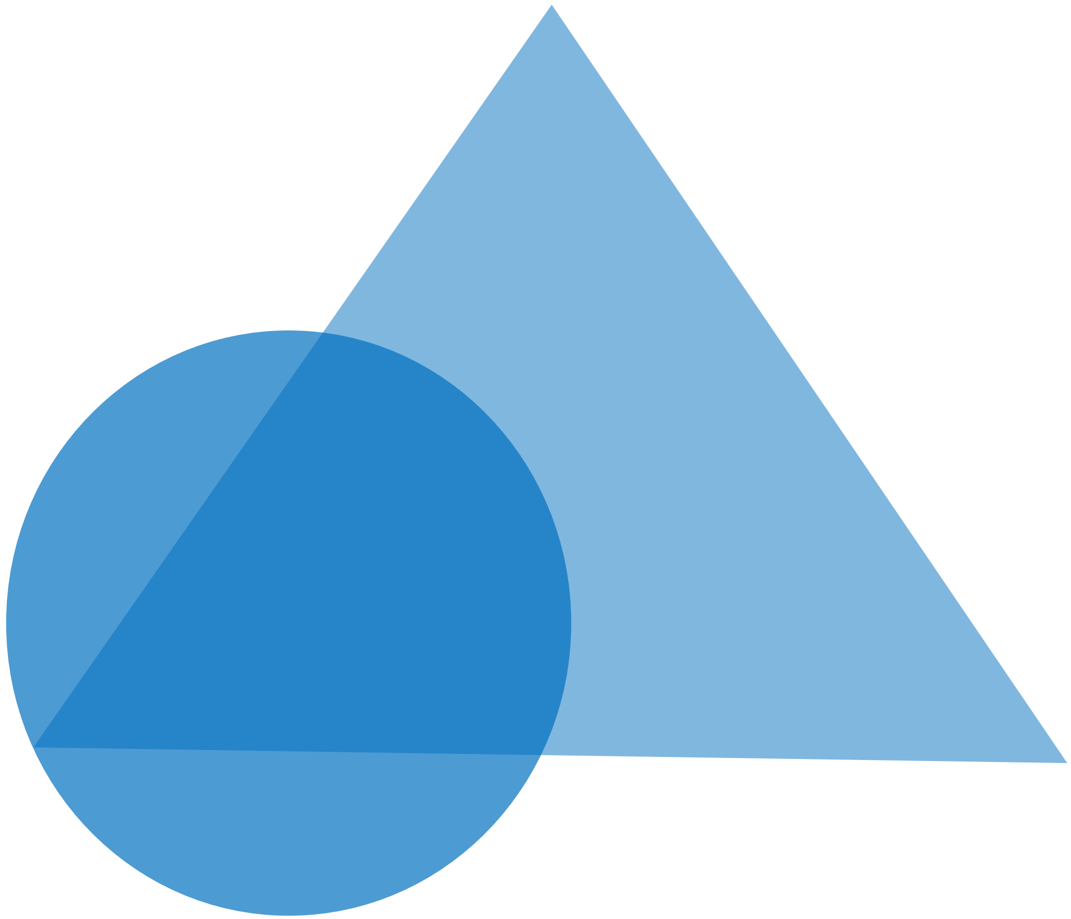

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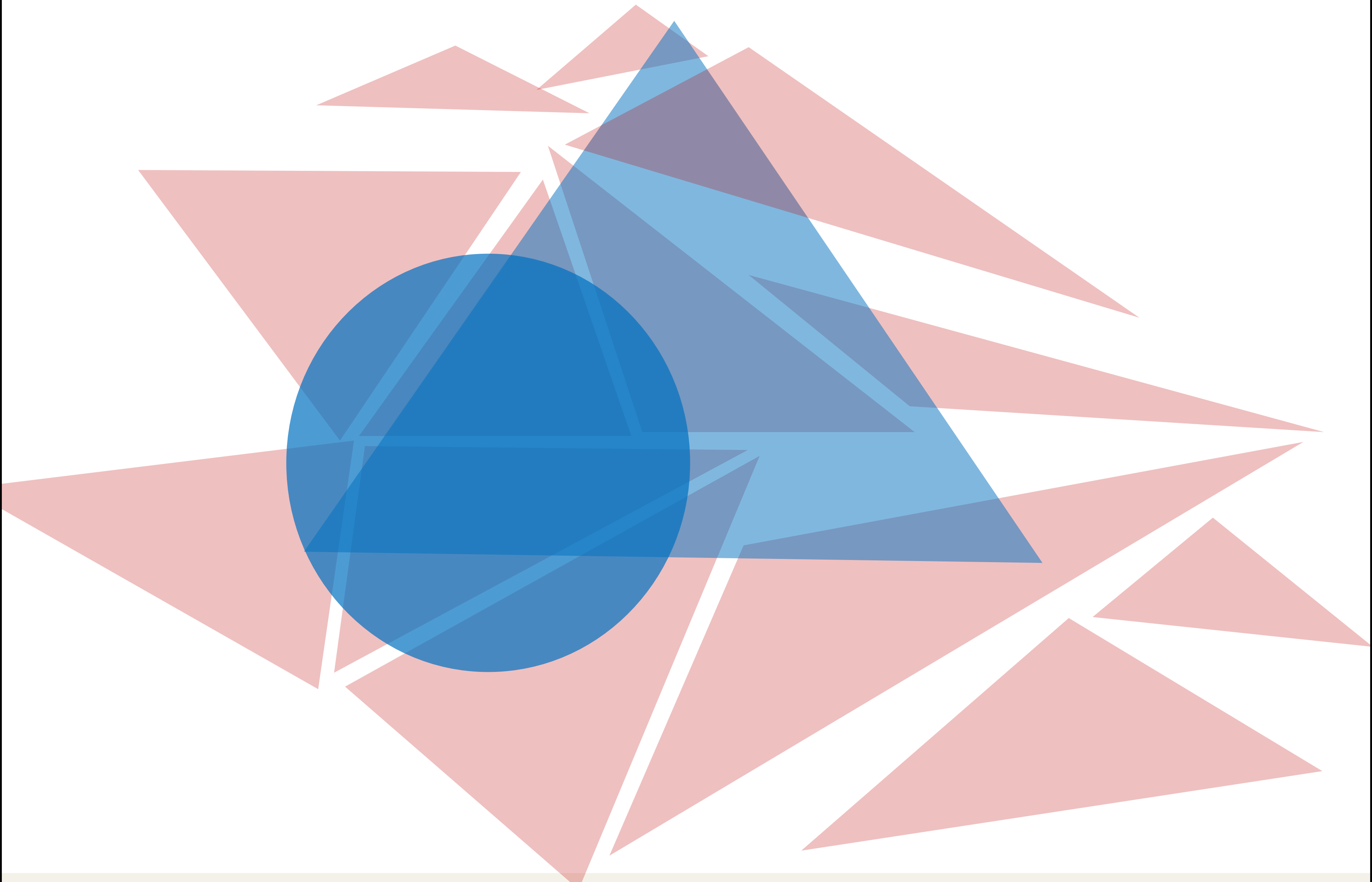

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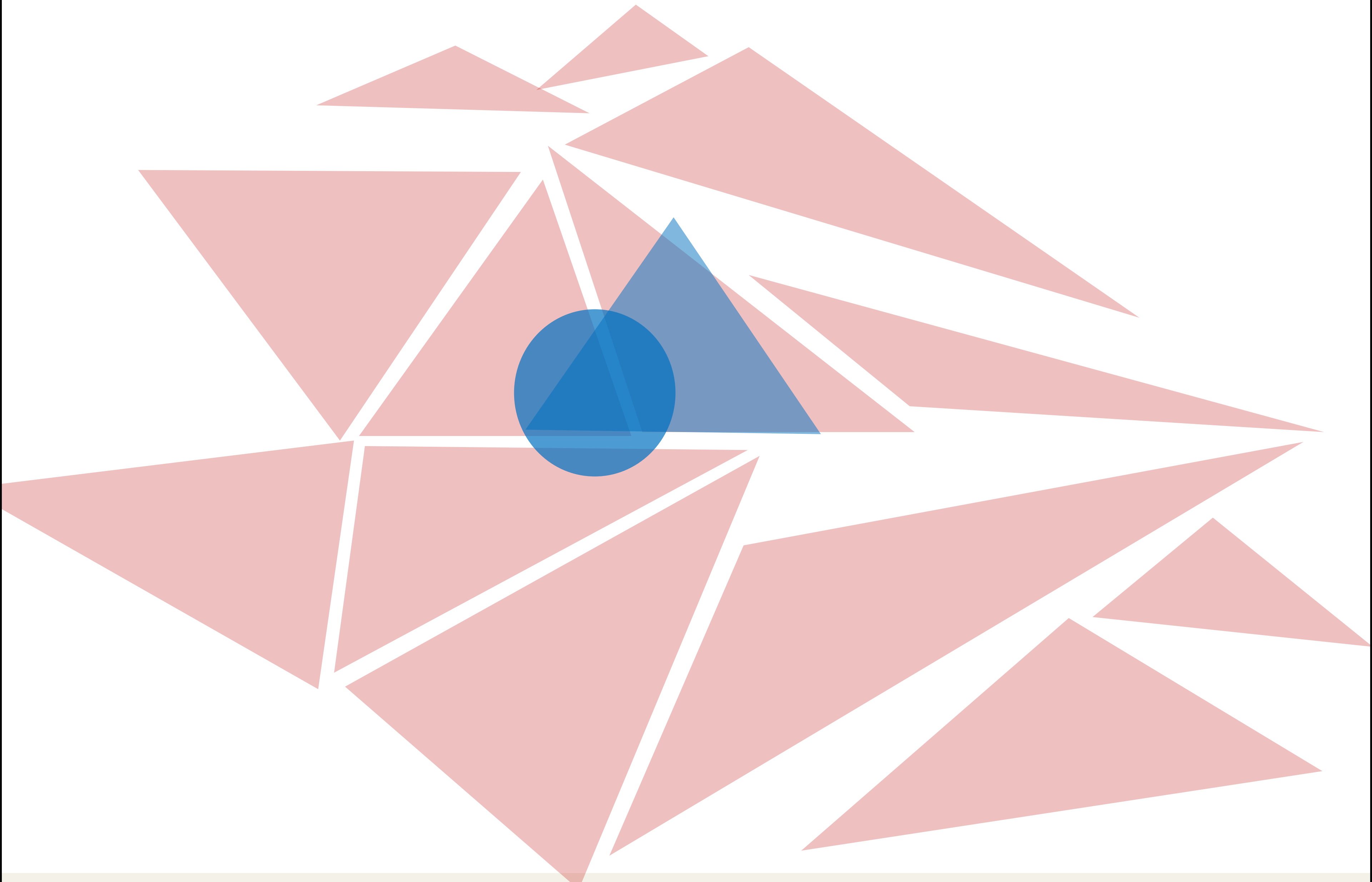

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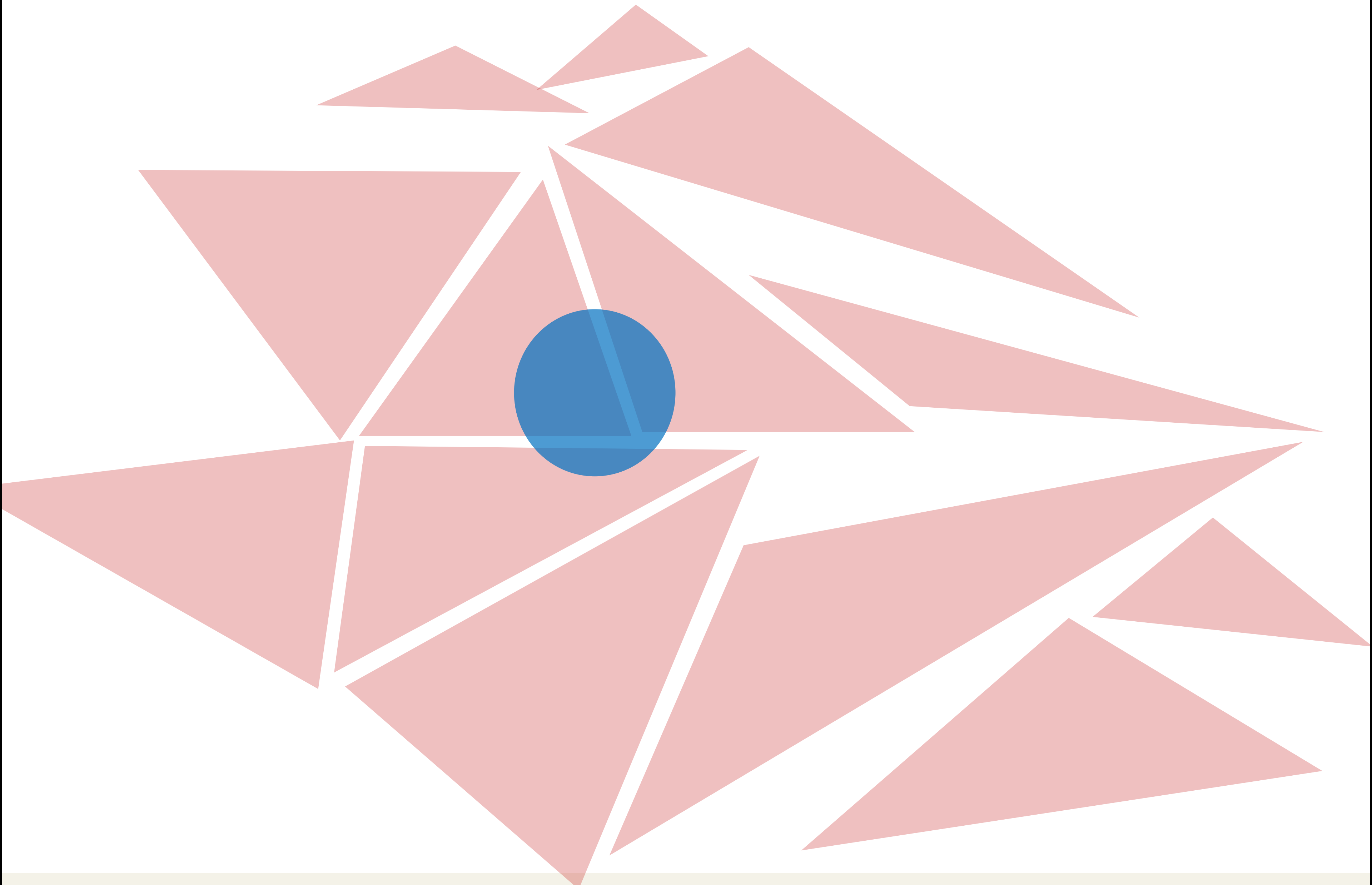

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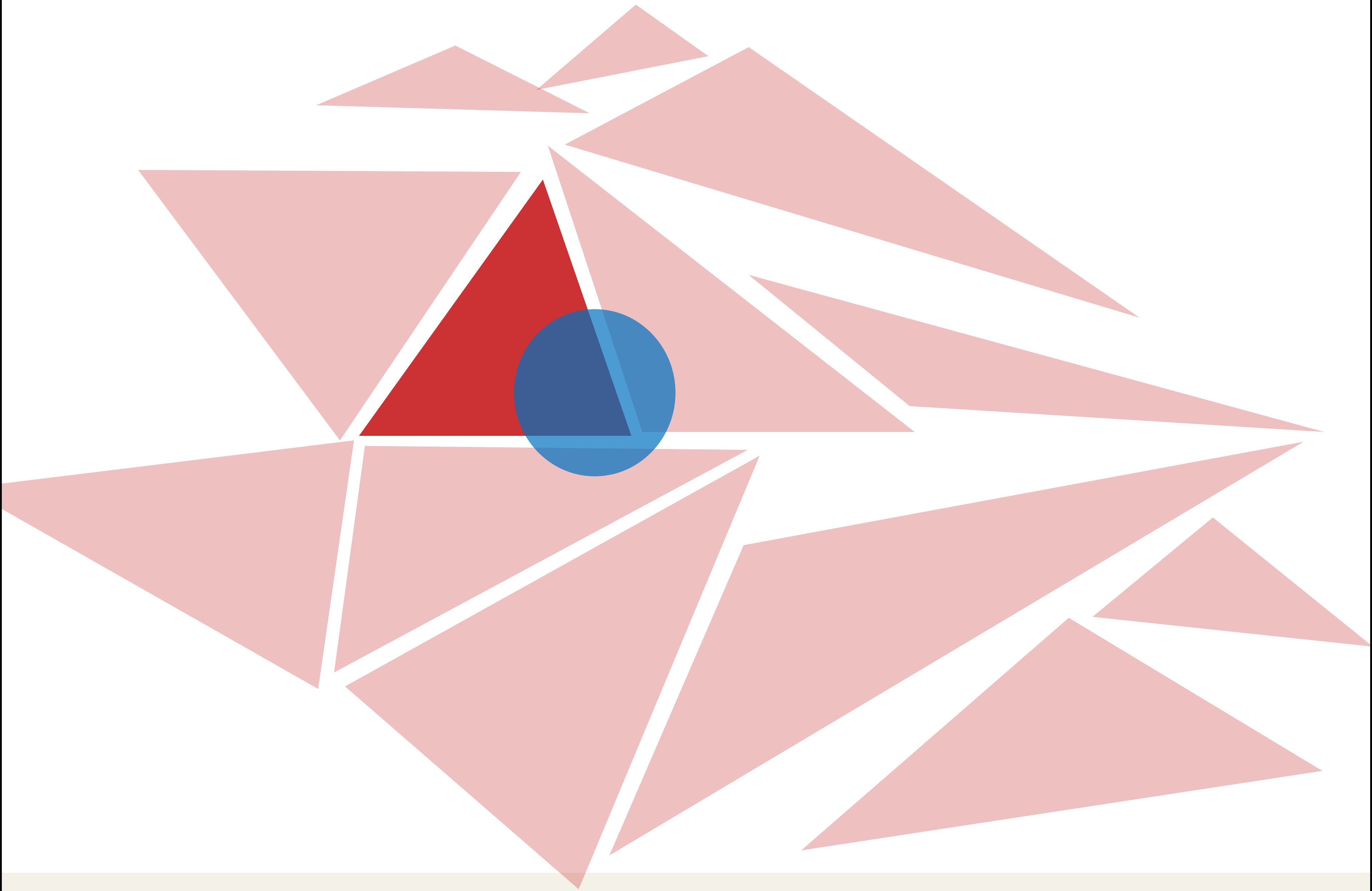

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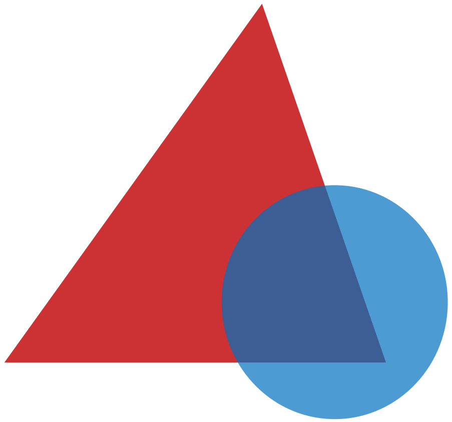

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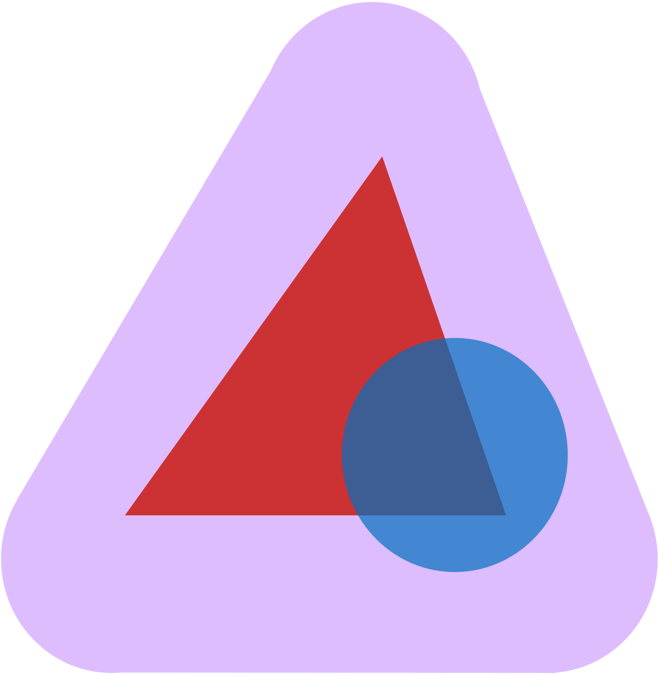

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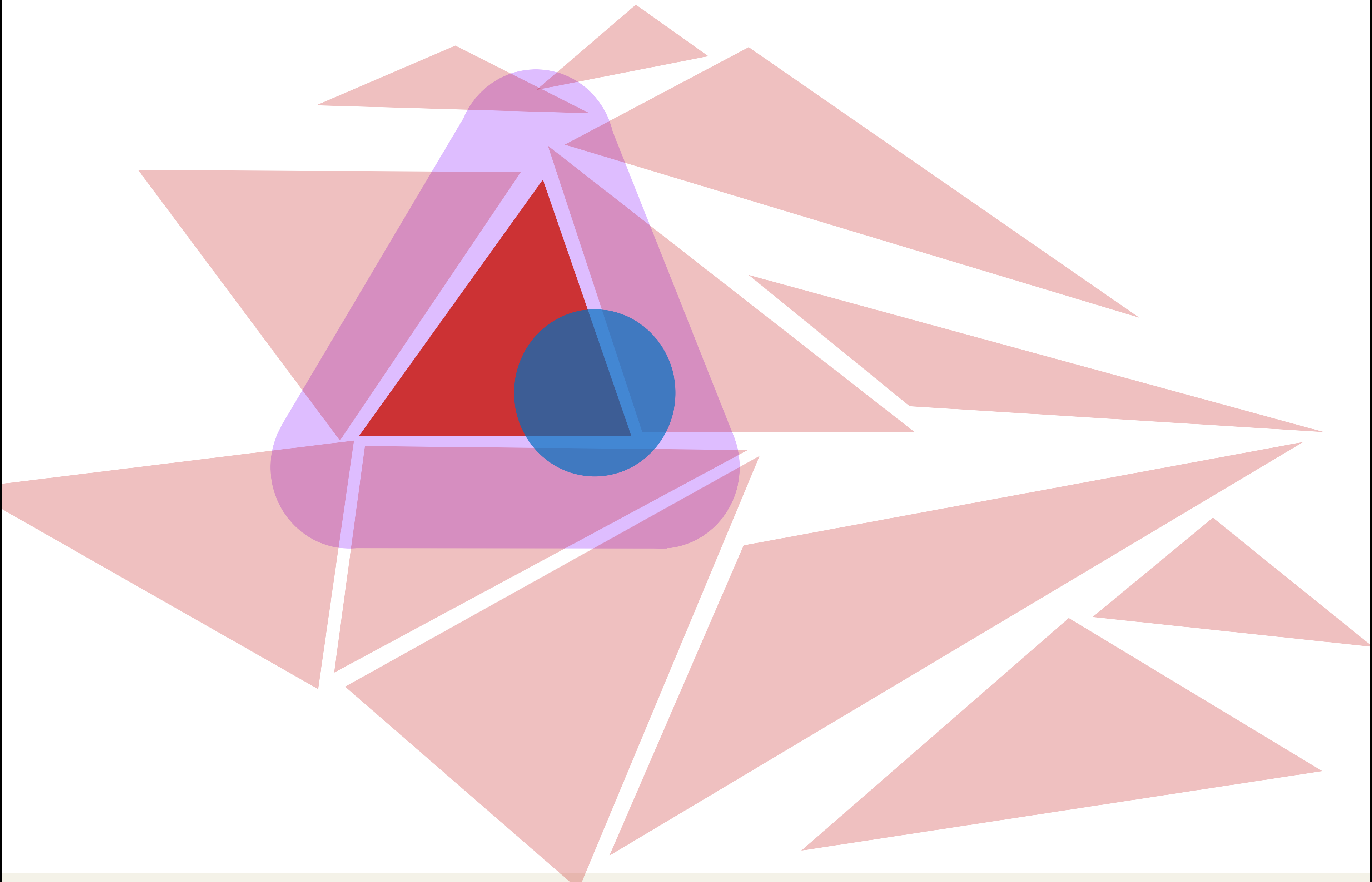

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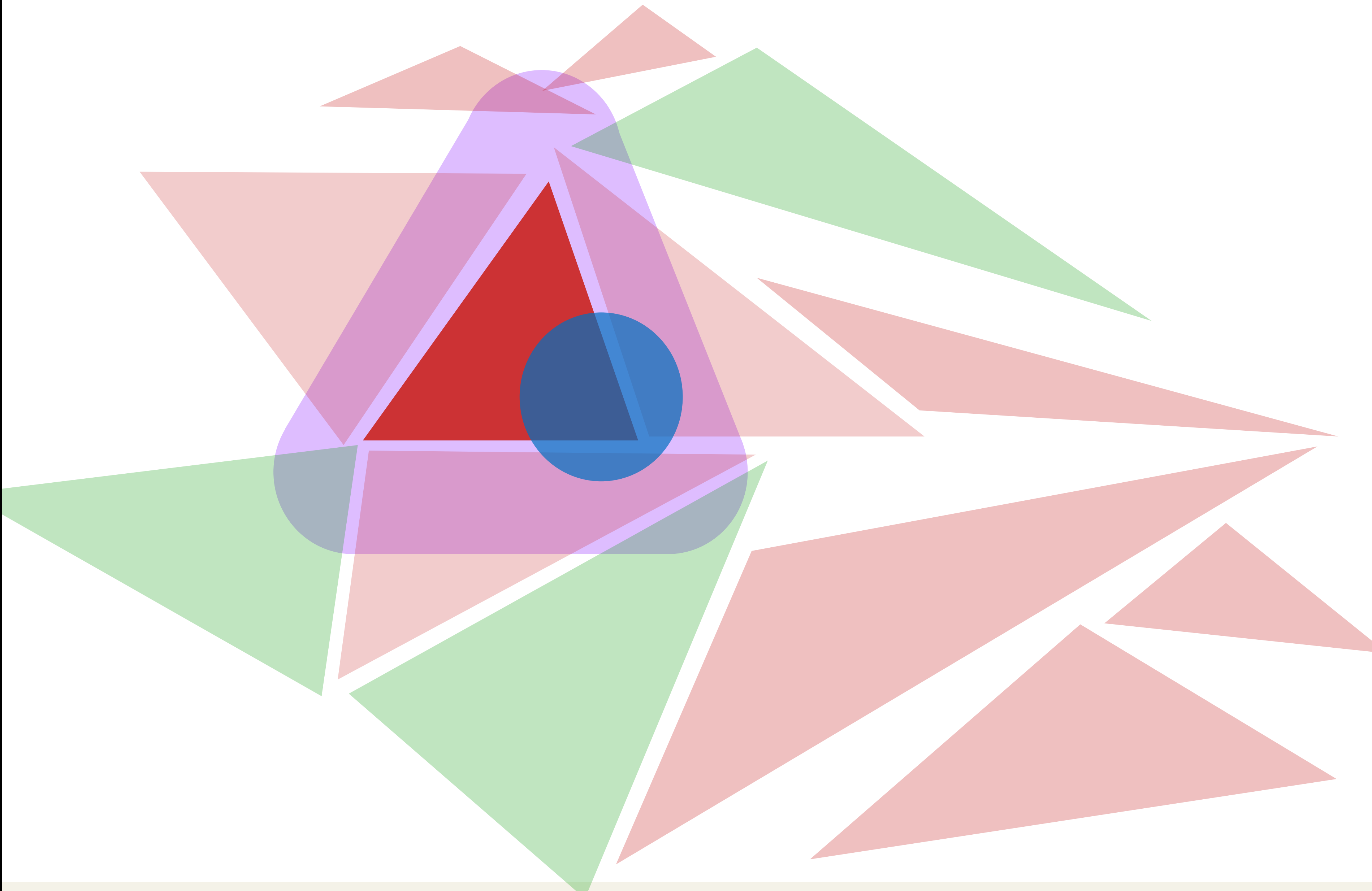
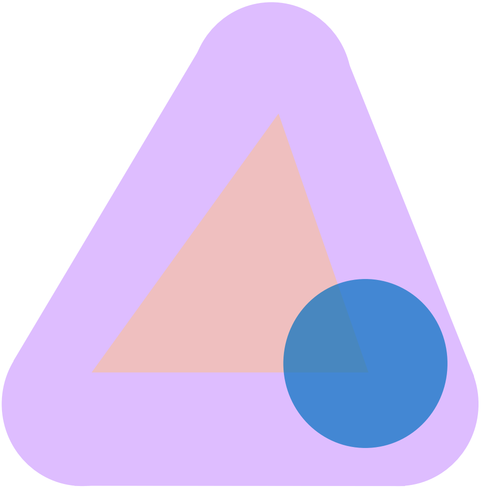


Lemma

- Let A be a k-free set of triangles and let t $\notin A$ be an arbitrary triangle that is not included in A. Then t intersects at most a constant number of larger triangles $t_j \in A$.

  More precisely, the number of intersections between t and larger triangles $t_j \in A$ is at most $3k$.



Theorem

- Let A and B be two k-free sets of triangles. Then the total number of colliding triangles of A and B is in $O(n)$, where n is the number of triangles in A and B. More precisely, the number of intersections is at most $3nk$.

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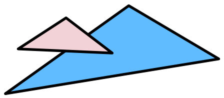

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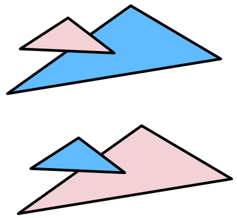

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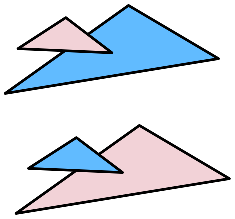
  $\Rightarrow$ Total number of intersection $\leq 3kn$

  $\Rightarrow O(n)$ intersections



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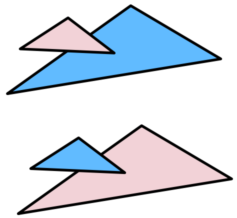
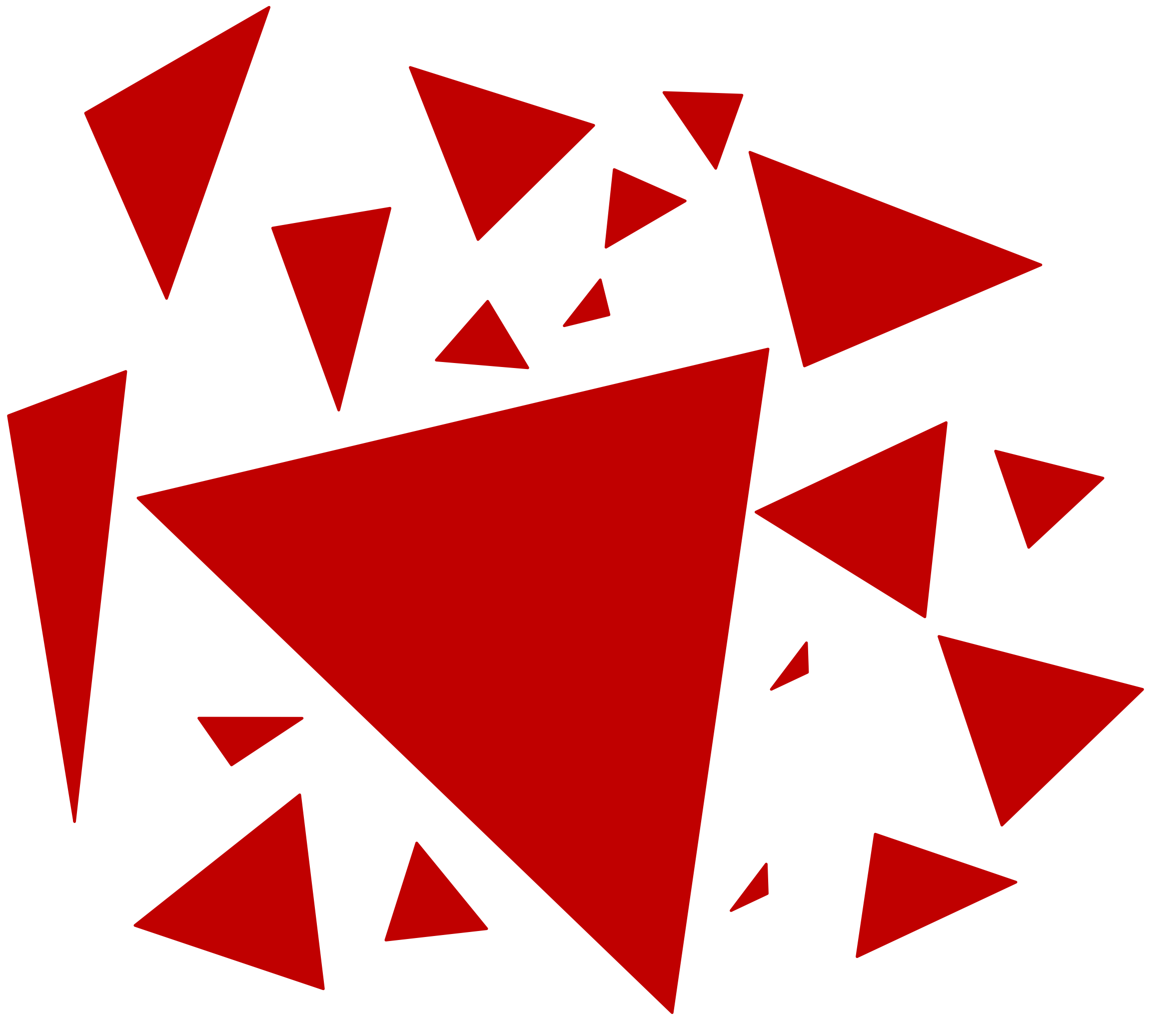
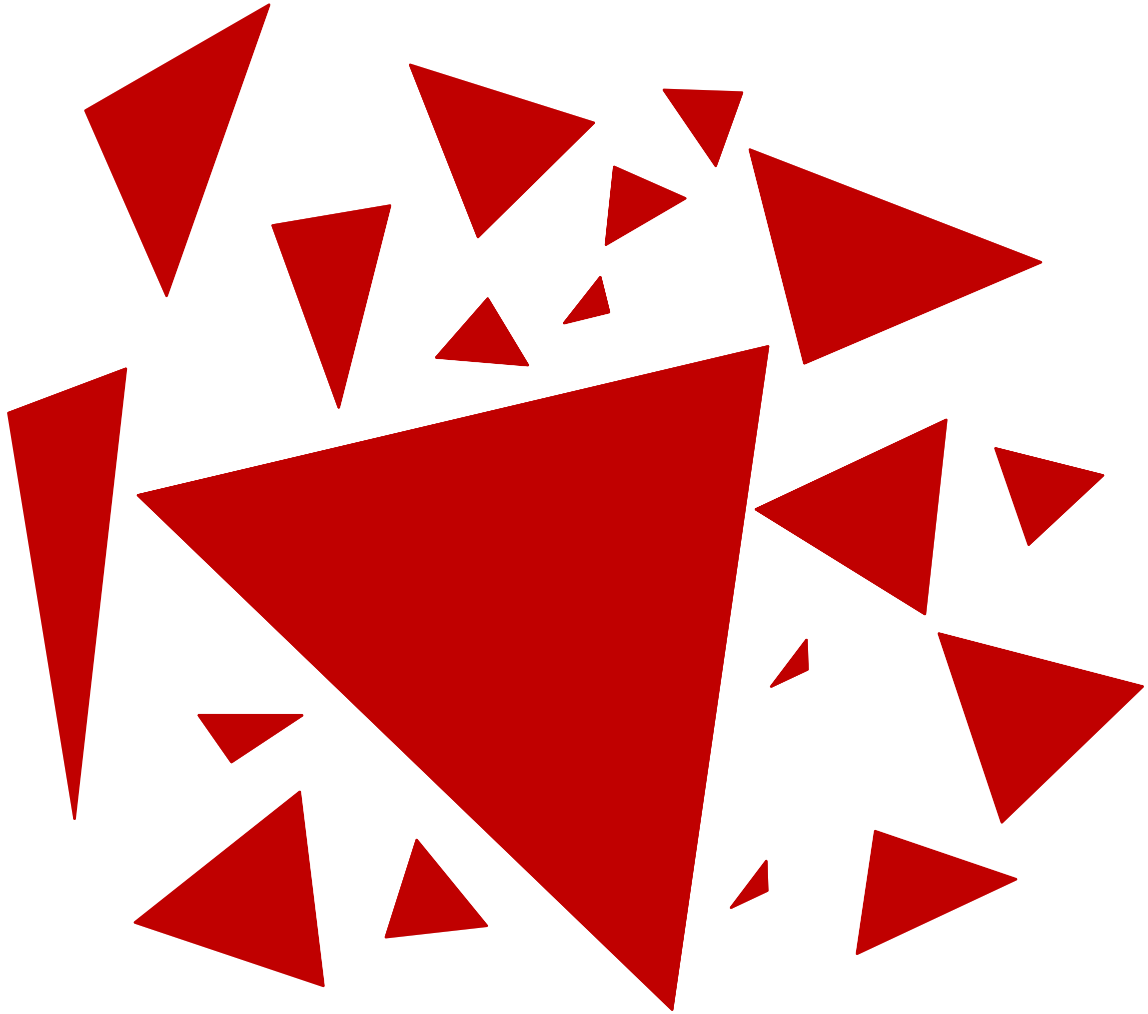
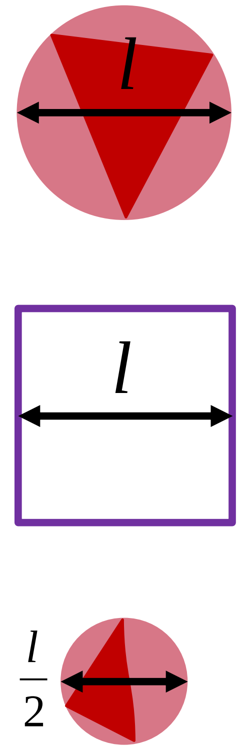


Our Algorithm

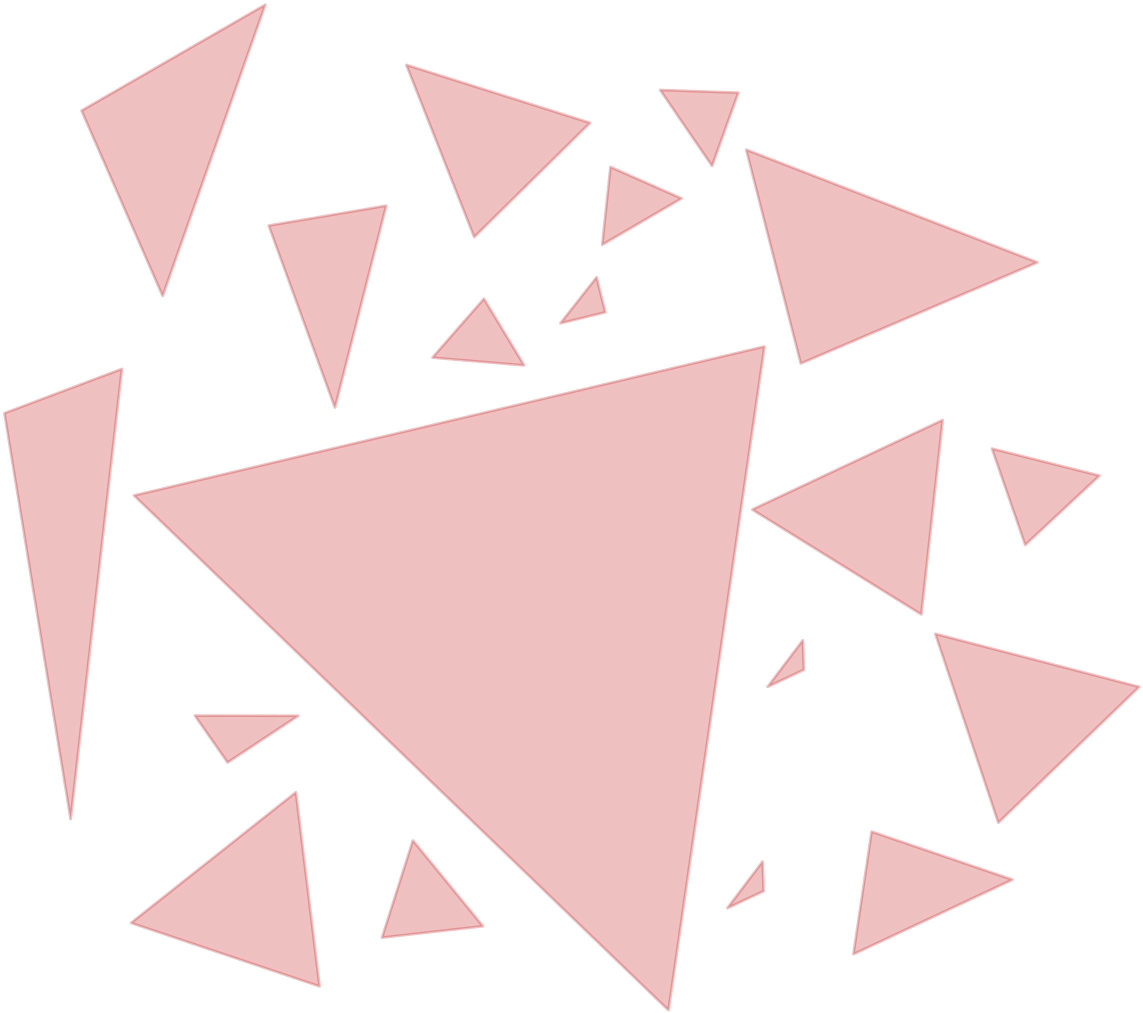


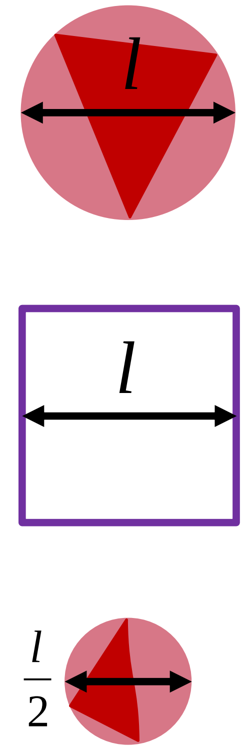
Our Algorithm





Our Algorithm





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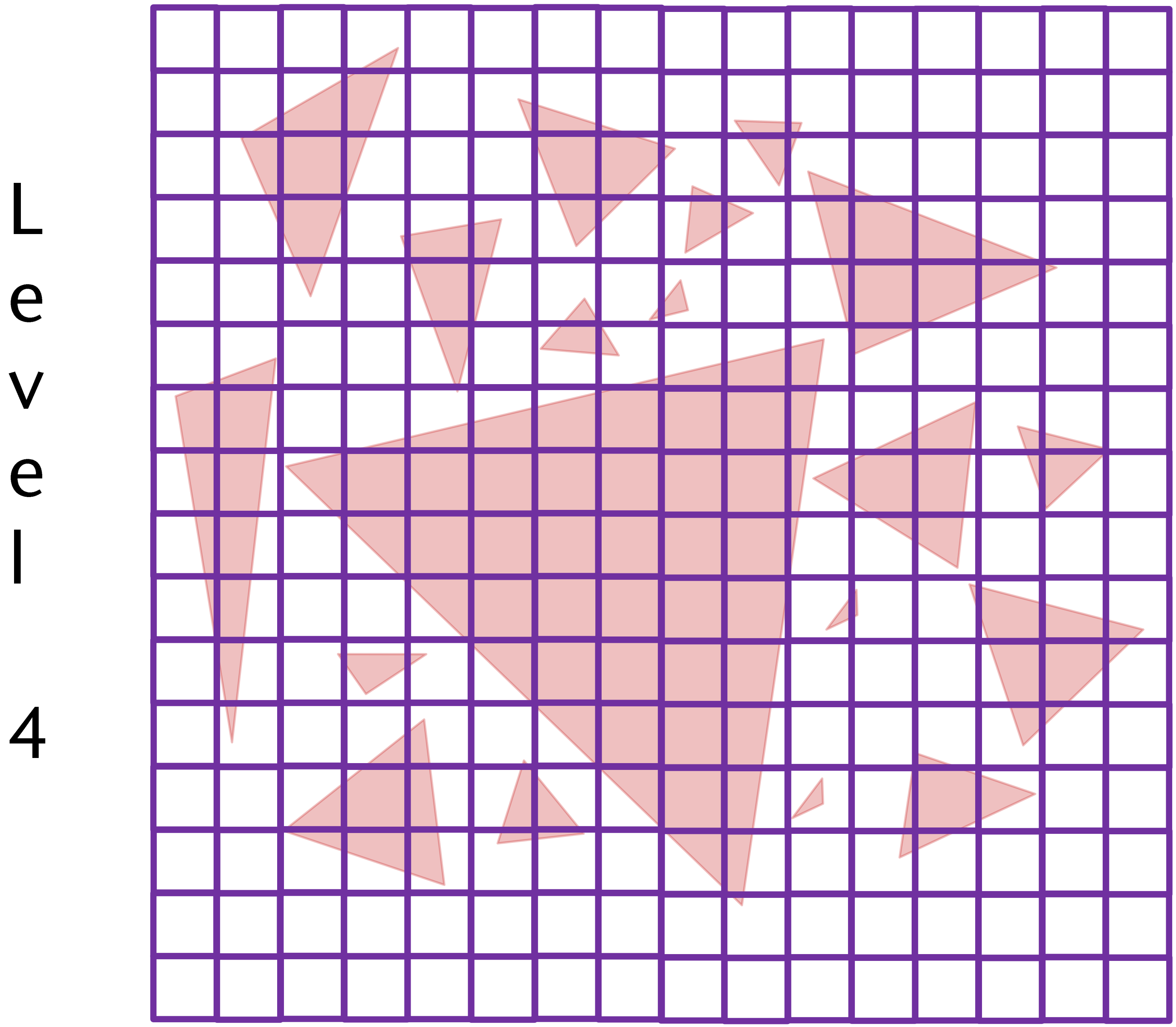
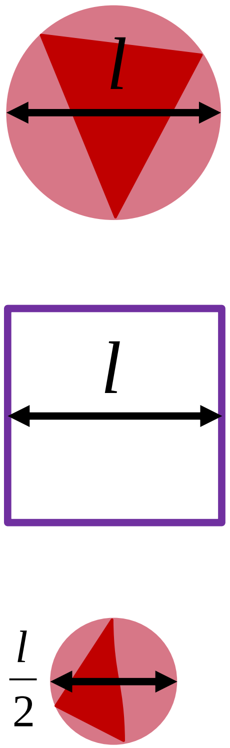
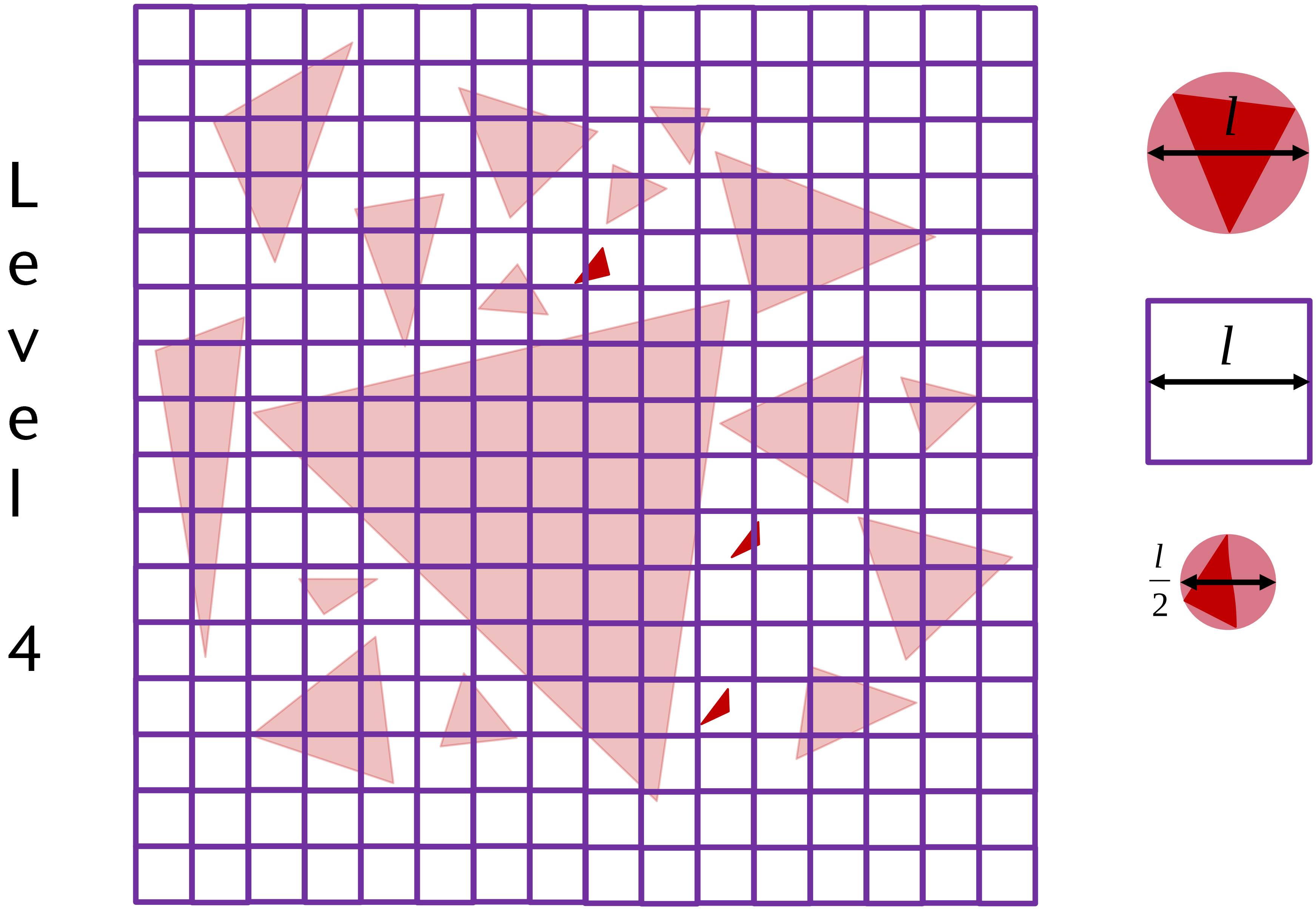




Our Algorithm



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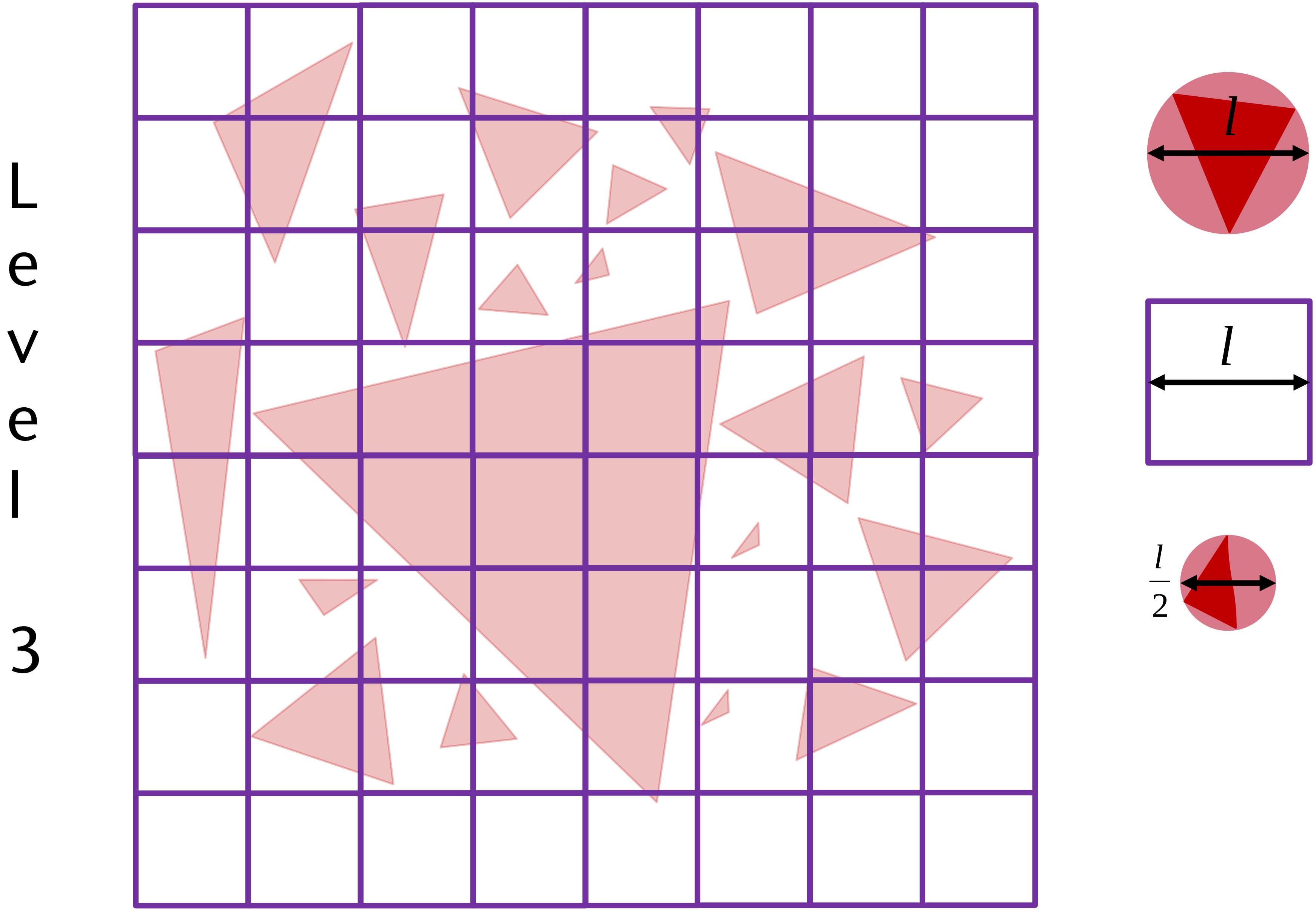
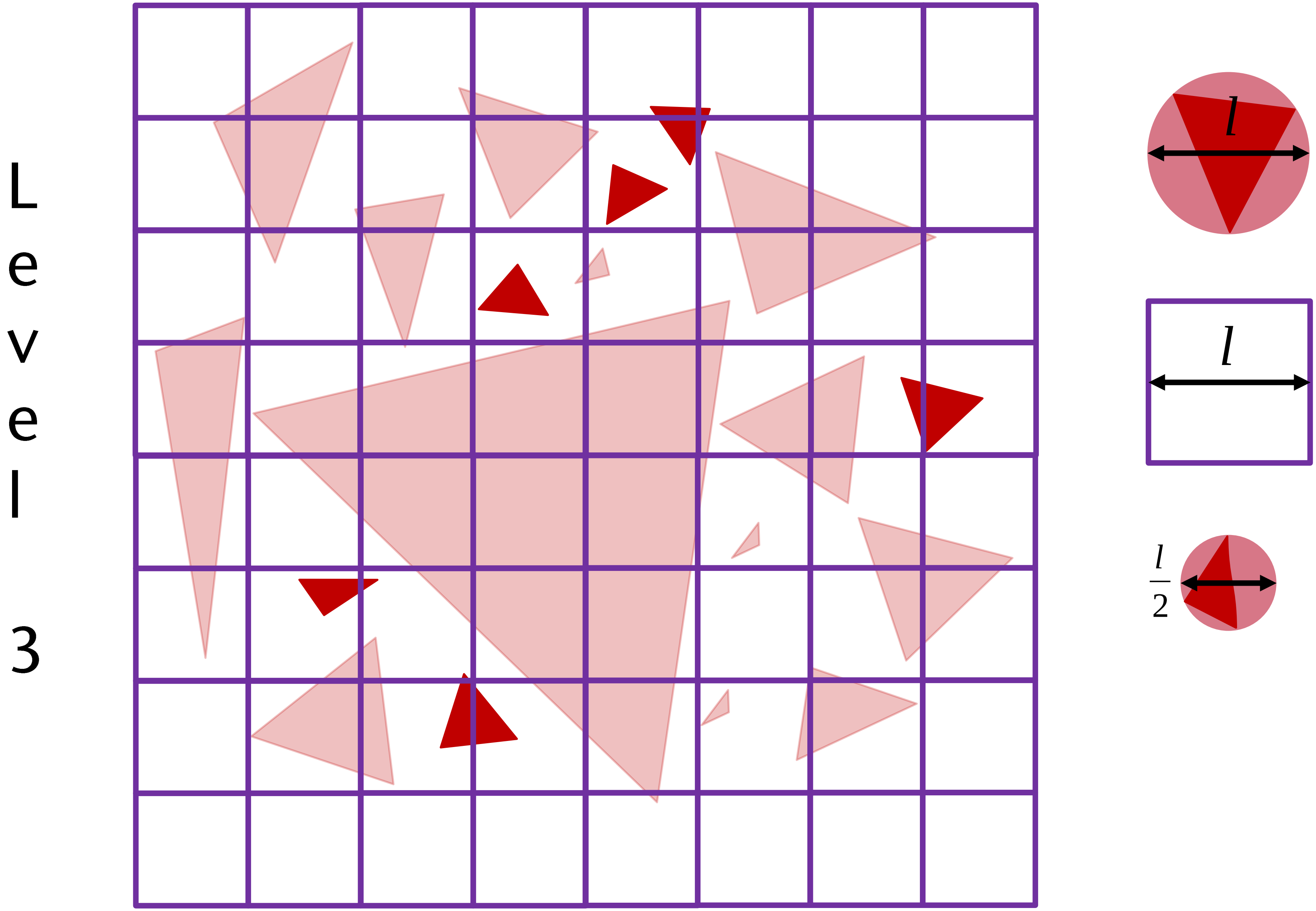
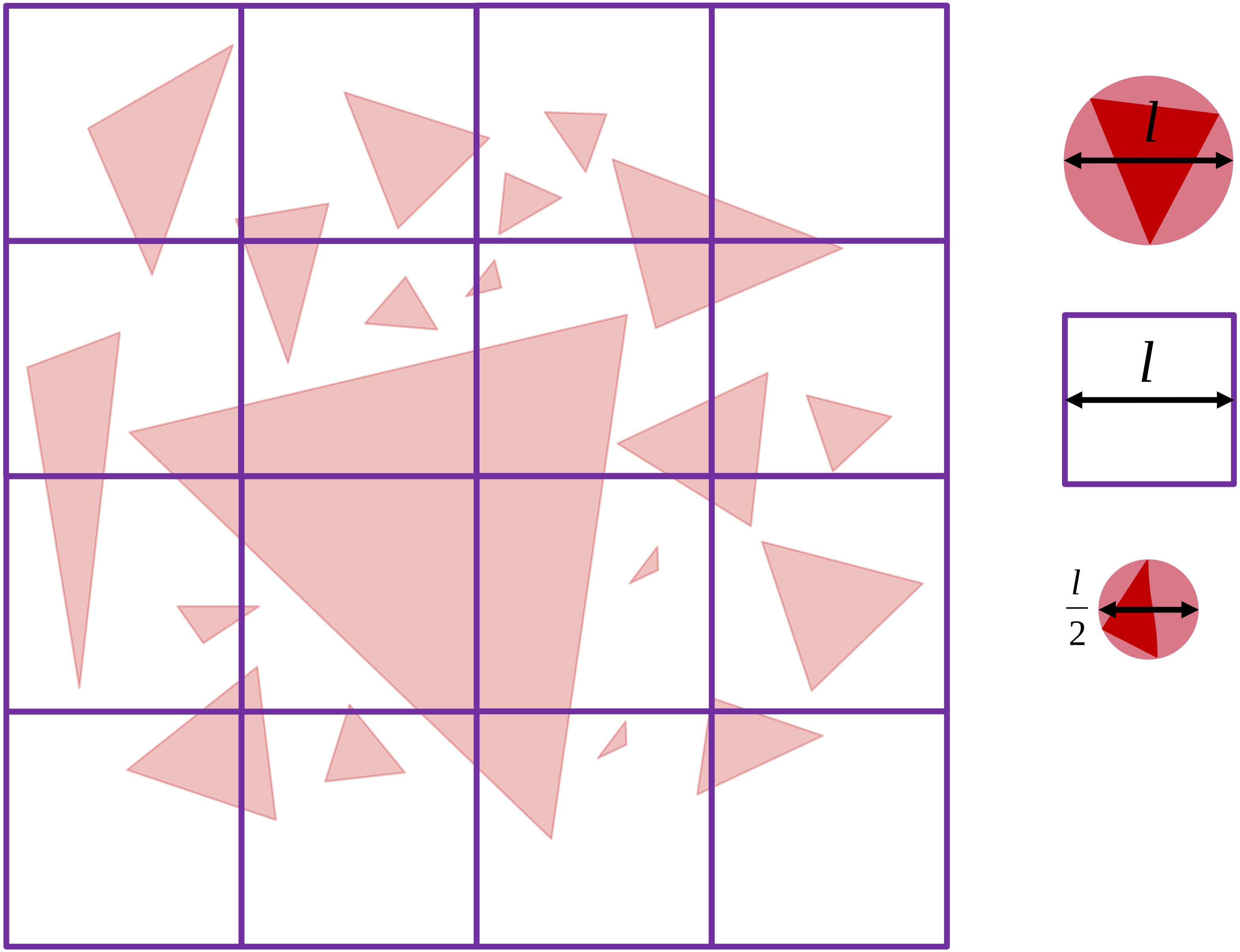


Our Algorithm



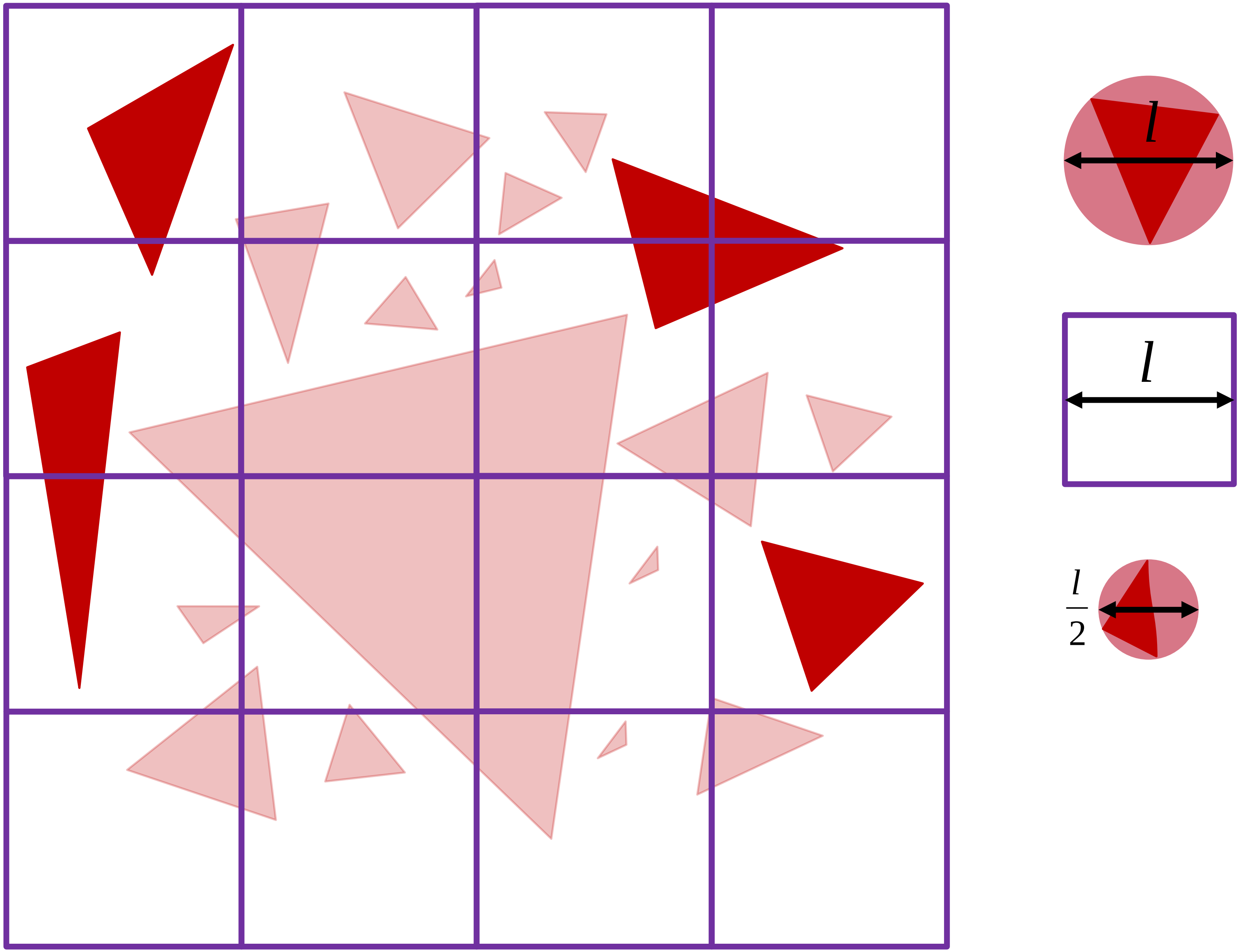
Our Algorithm

Level 2



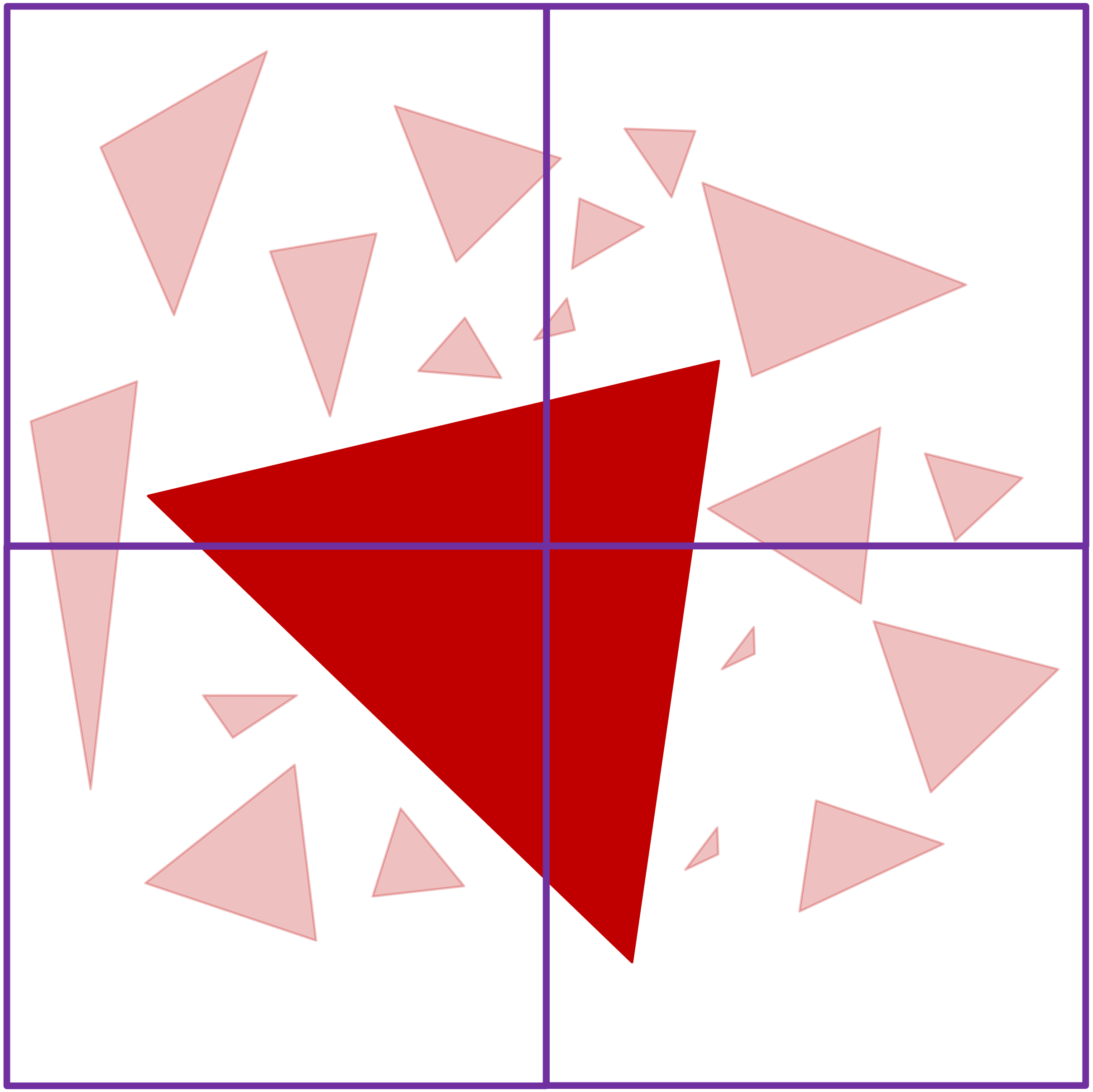
Our Algorithm

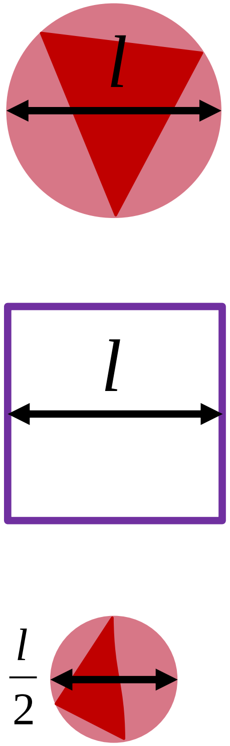
Level 2



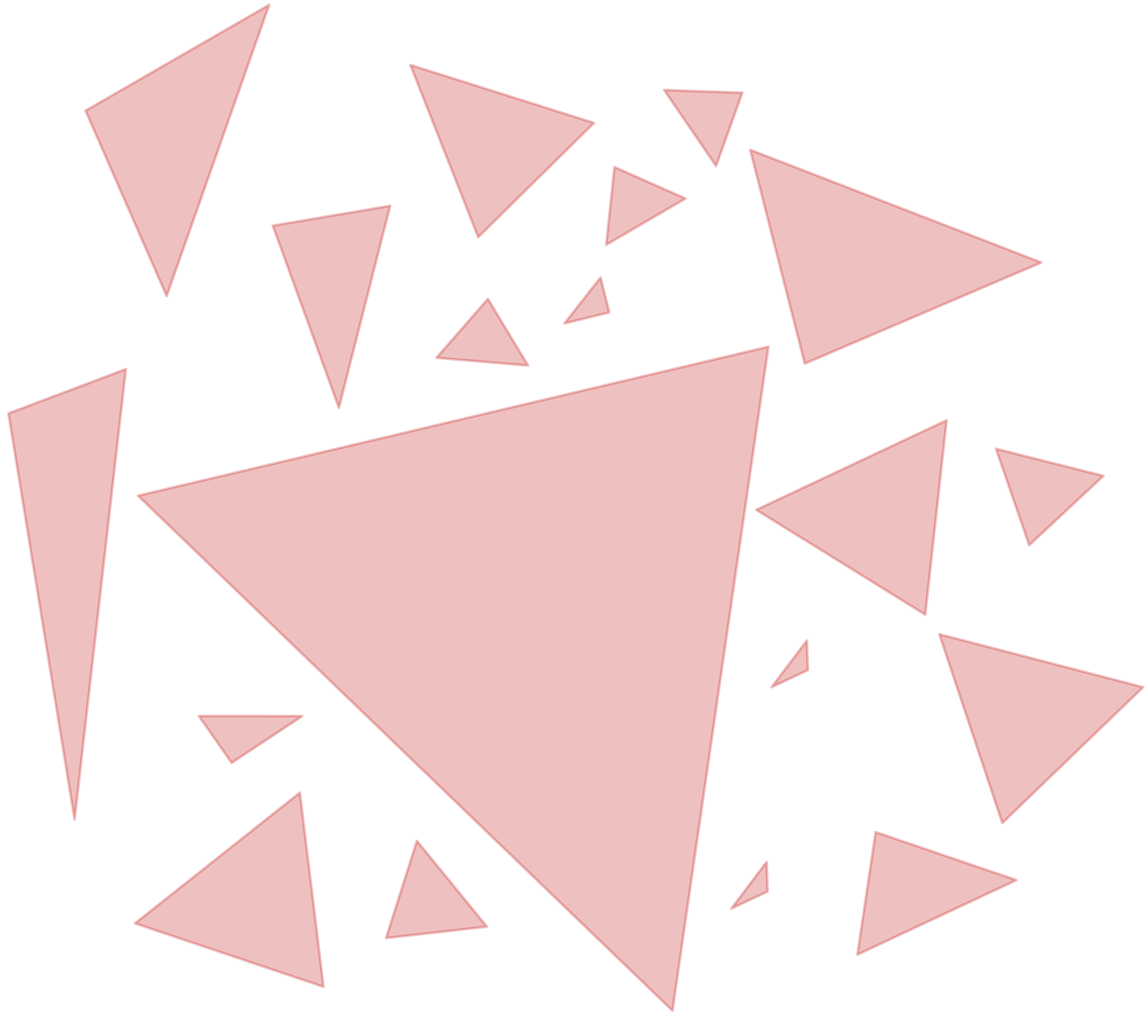
Our Algorithm

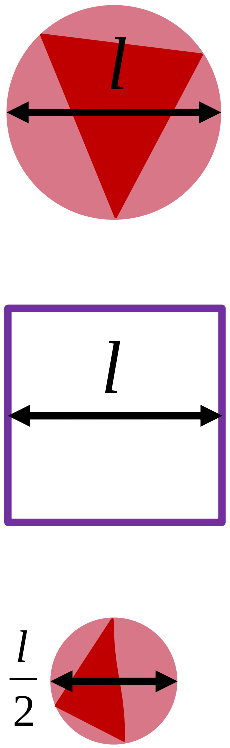
Level 1





Our Algorithm





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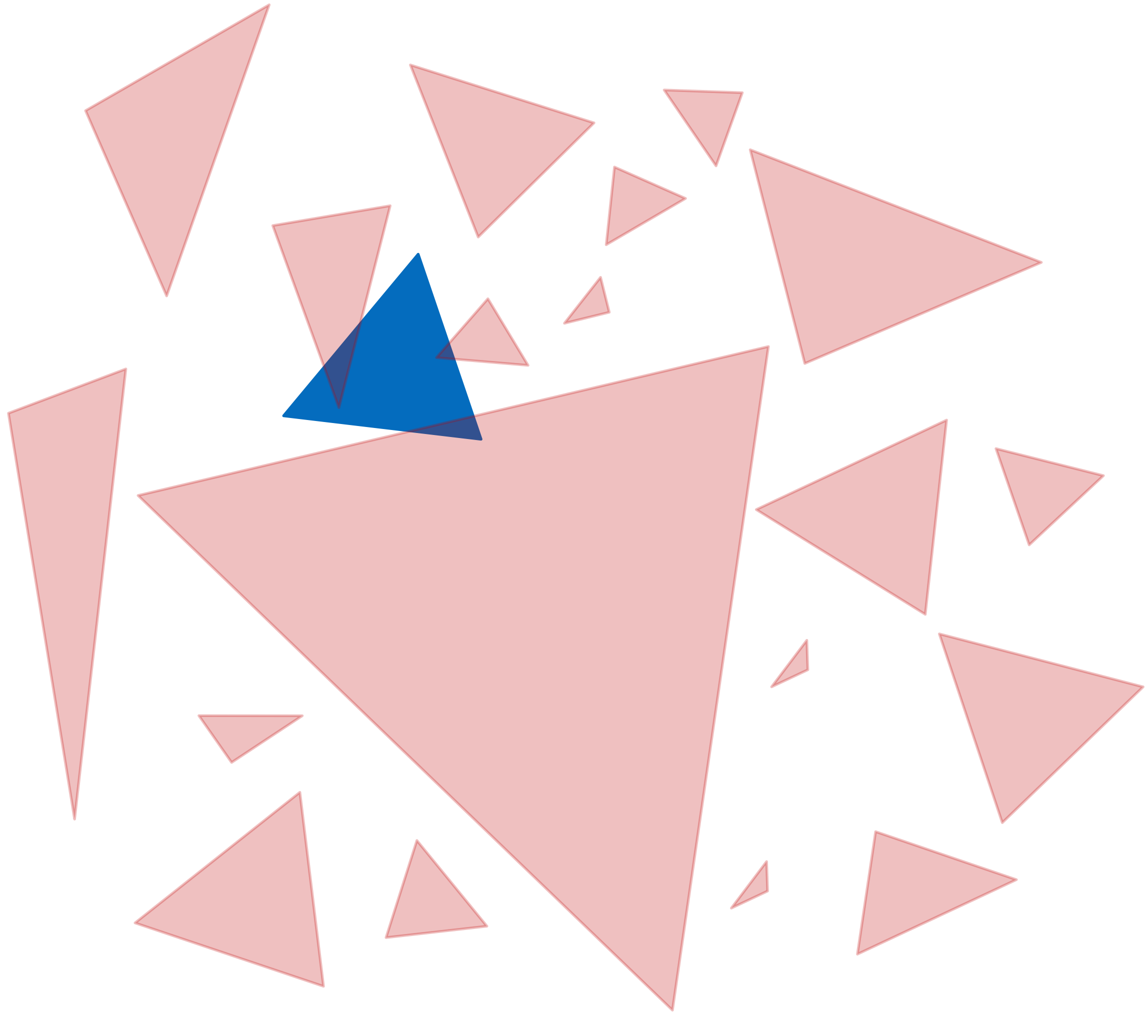

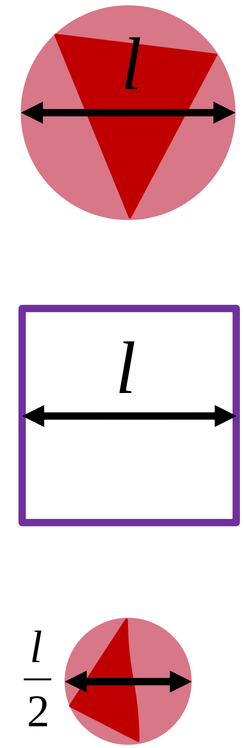

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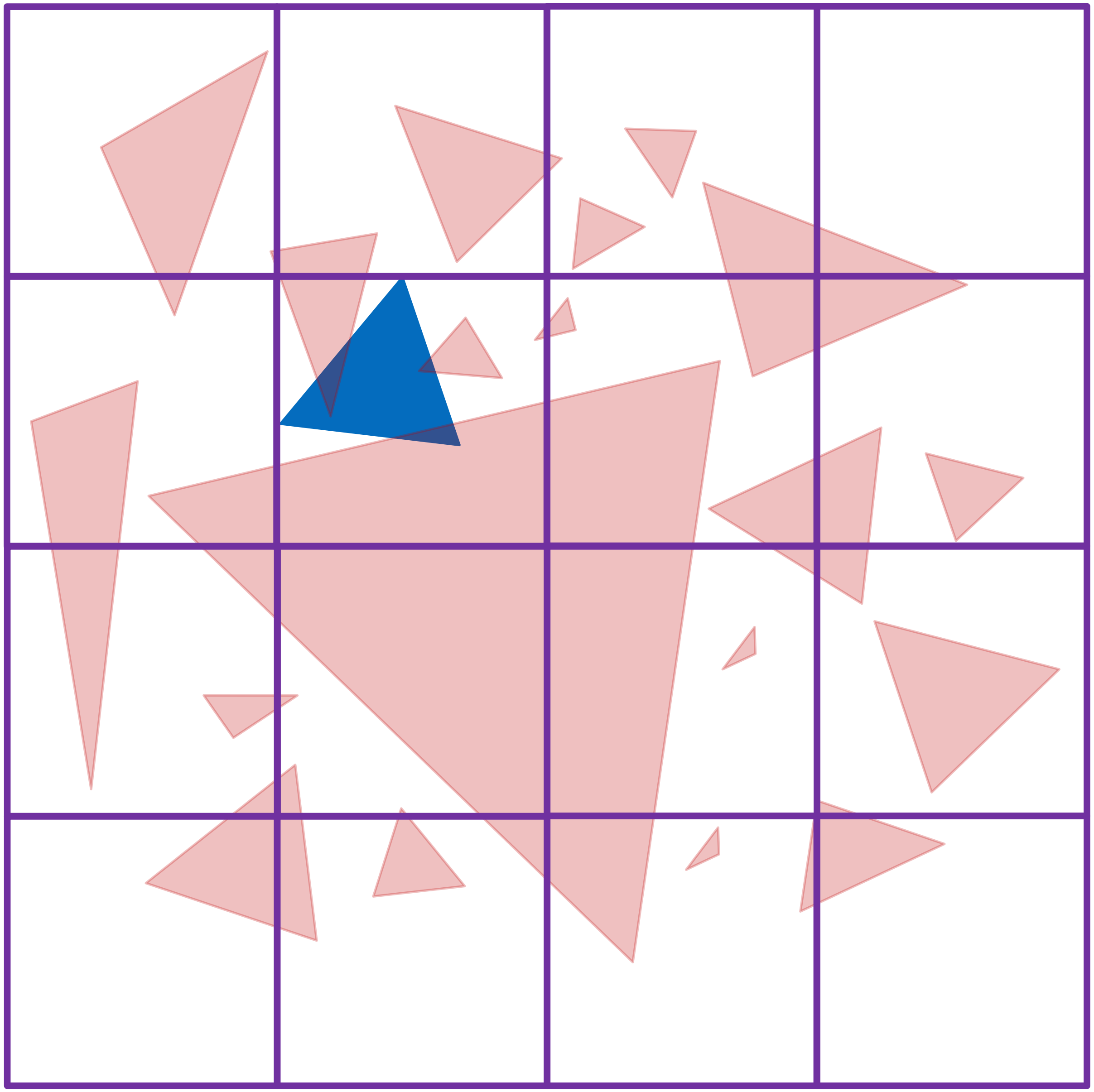

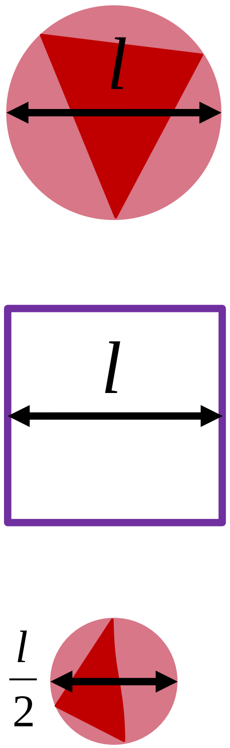

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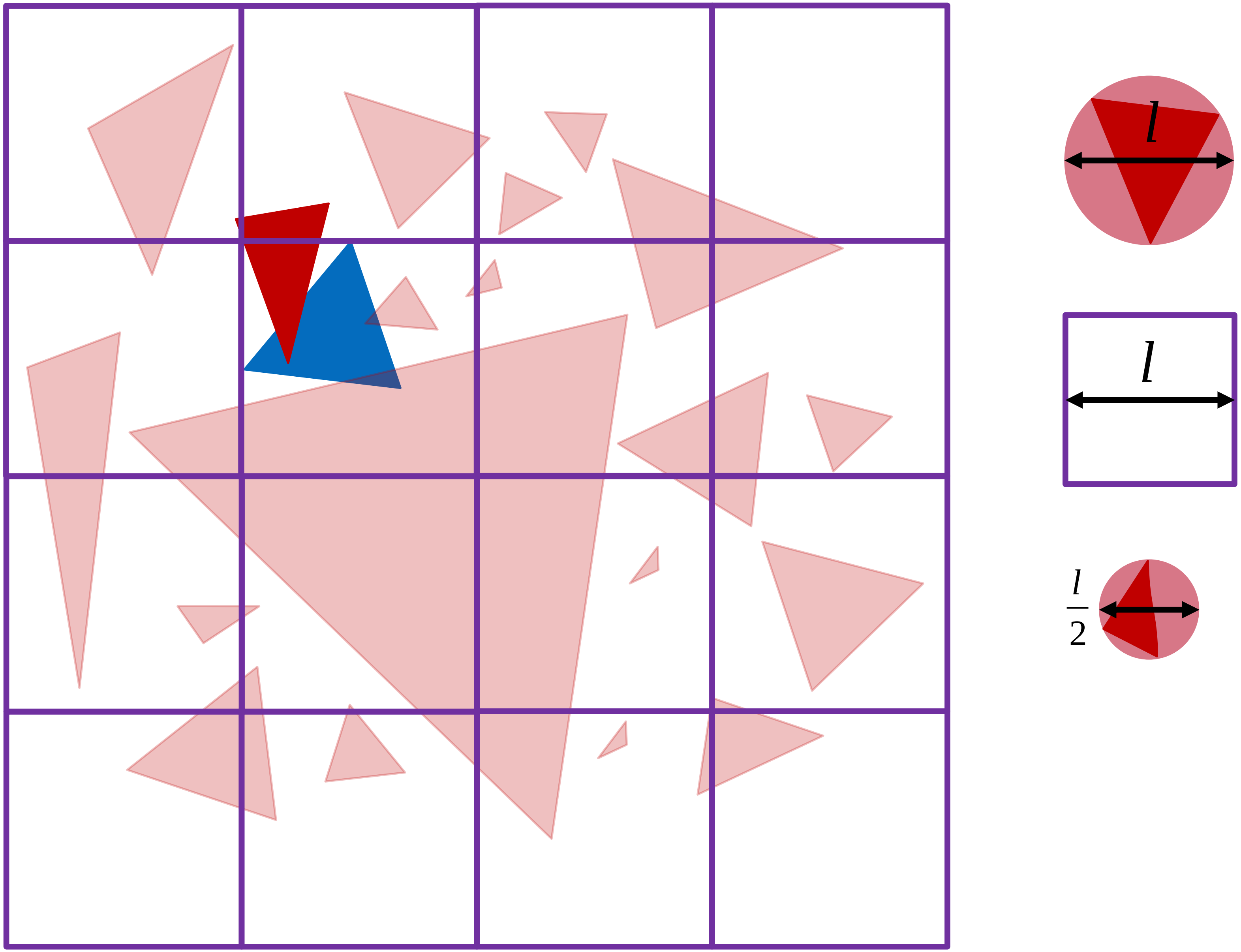
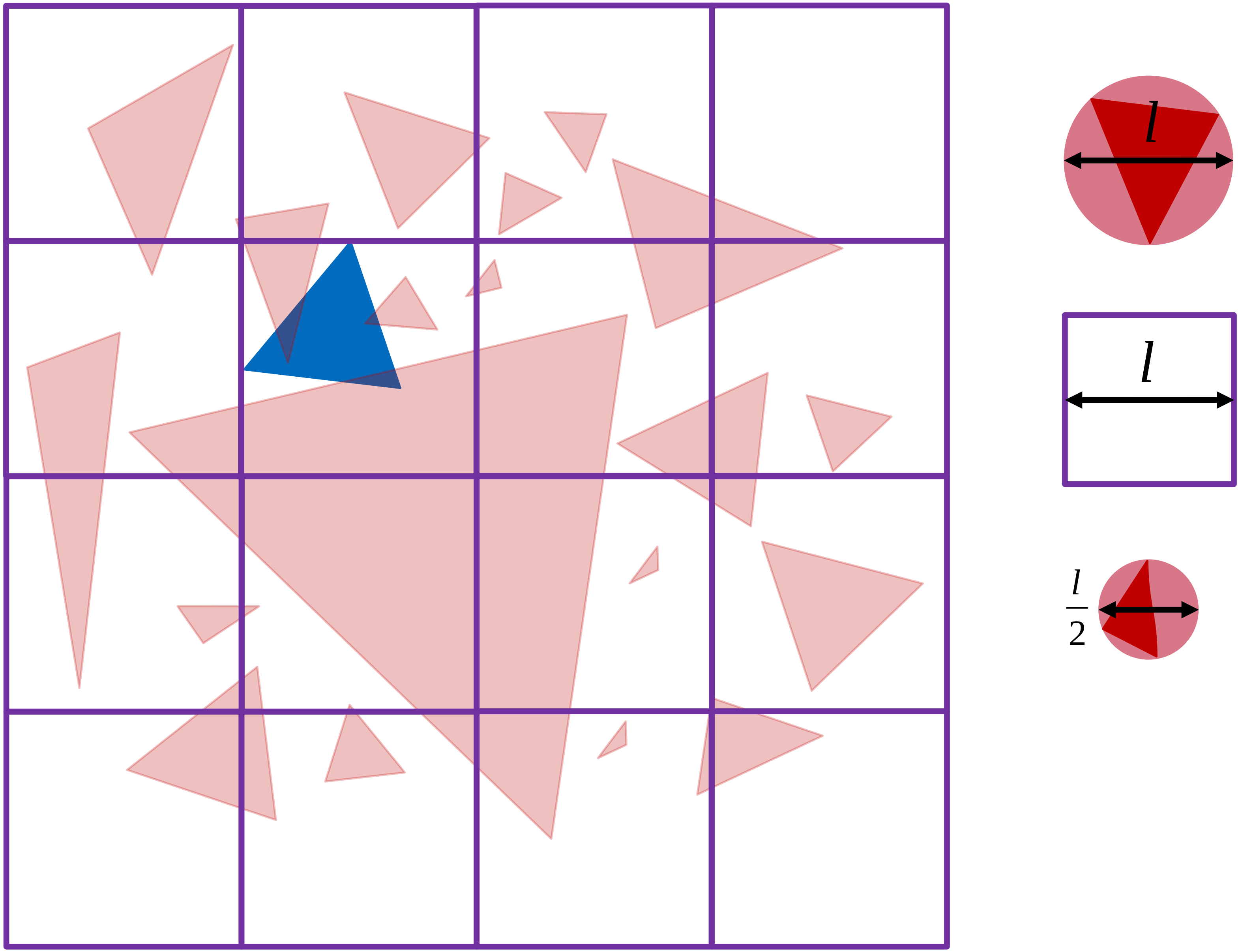
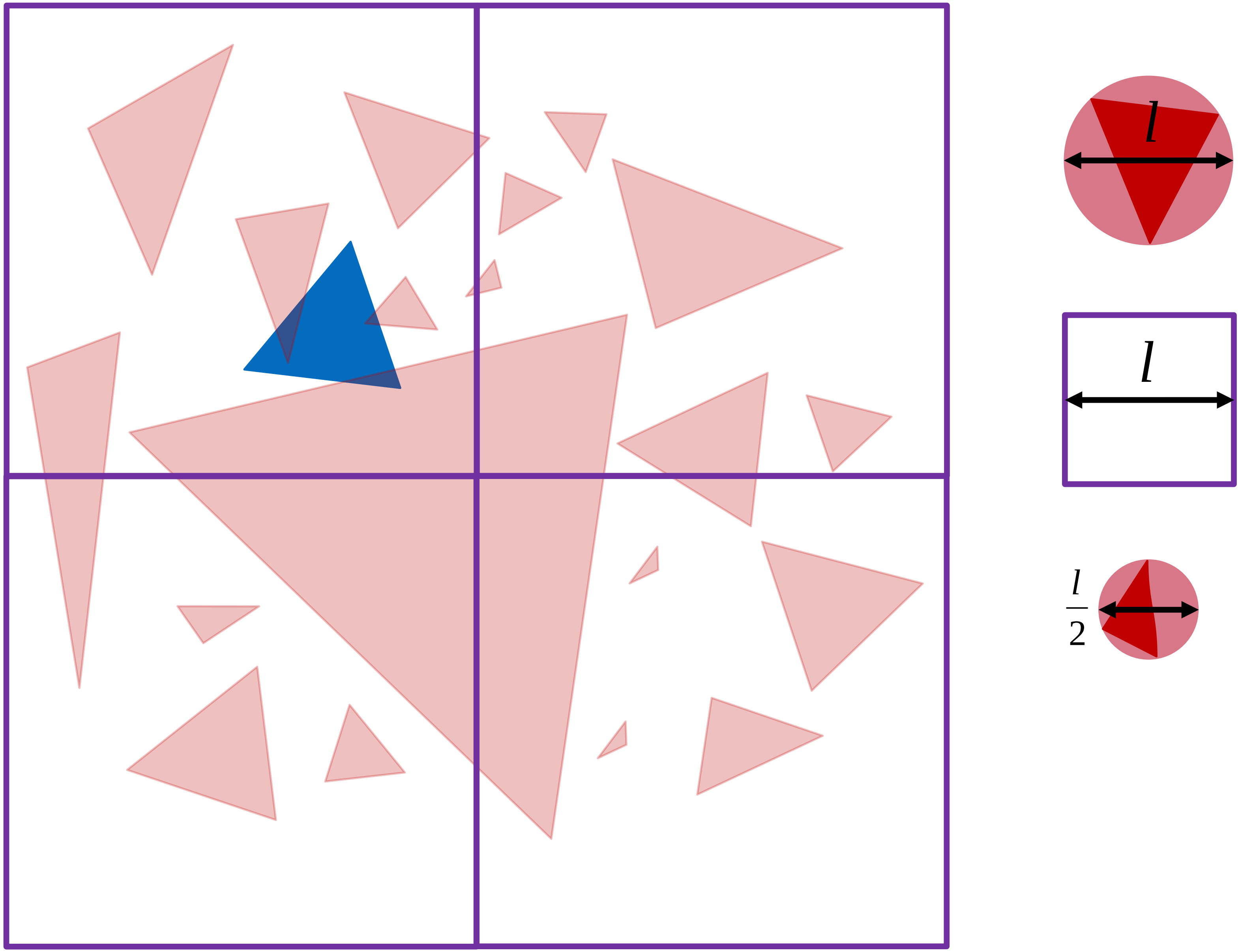


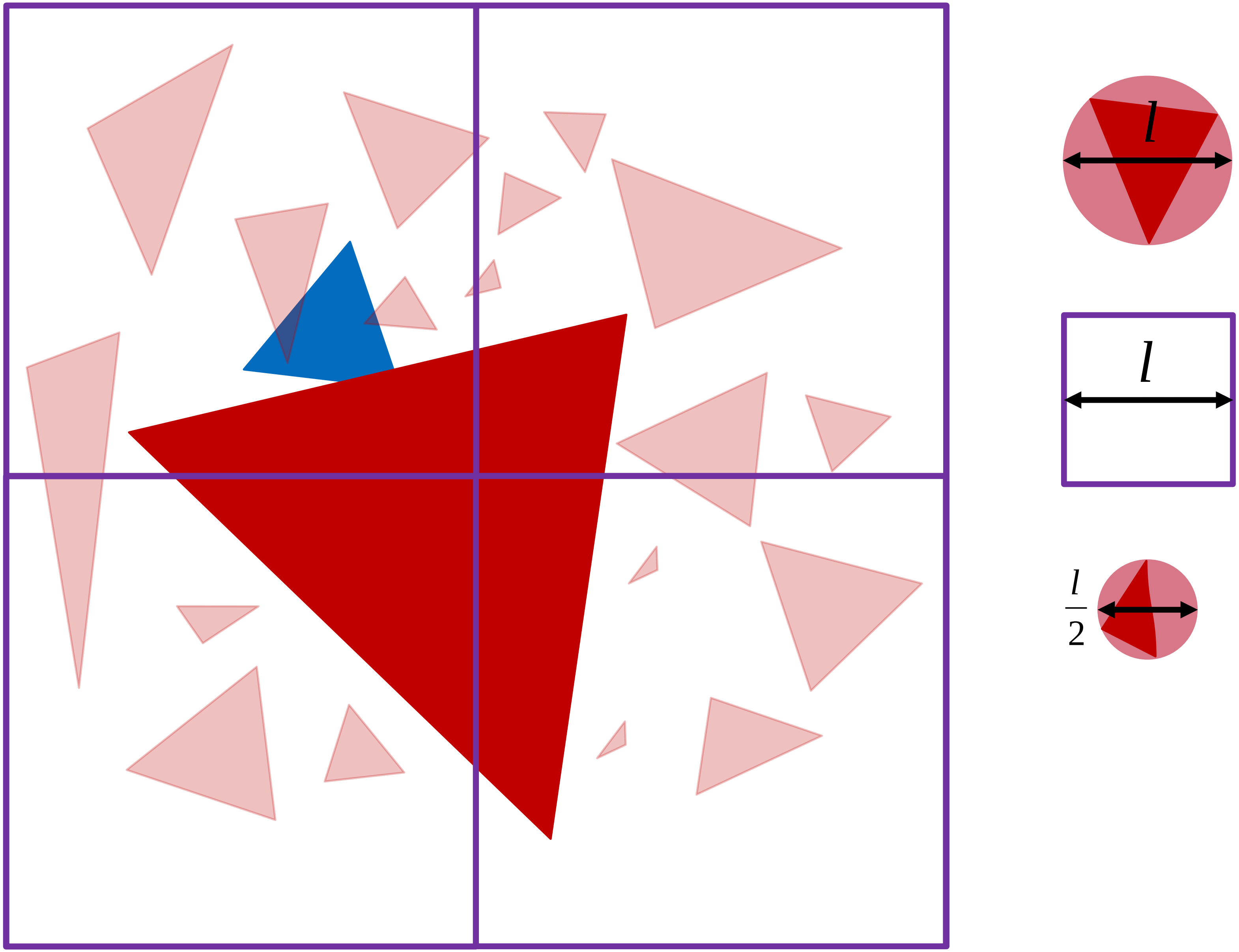
Our Algorithm



Our Algorithm



Our Algorithm



Analysis

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For each triangle t ∈ B:
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For each triangle t ∈ B:
    Compute hierarchy level l
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For each triangle t ∈ B:
    Compute hierarchy level l
    For all levels l_i, l ≤ l_i ≤ l_max:
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For each triangle t ∈ B:
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$\Rightarrow O(1)$

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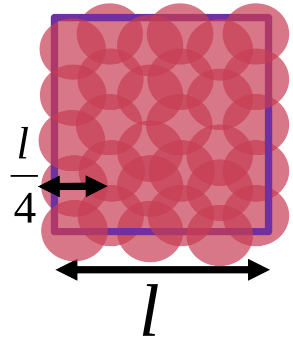

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    For all levels l_i, $l \le l_i \le l_{max}$:
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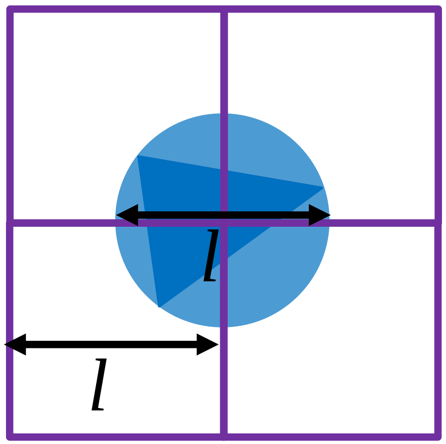
            For all triangles $t_k \in c_j$ $\Rightarrow O(1)$
                Compute intersection for t and t_k $\Rightarrow O(1)$



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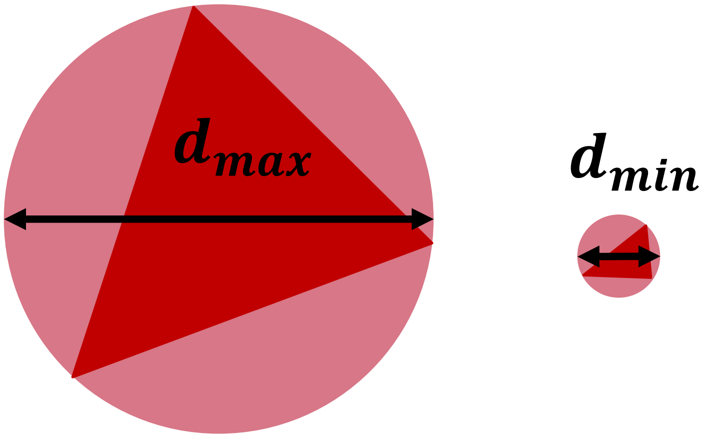

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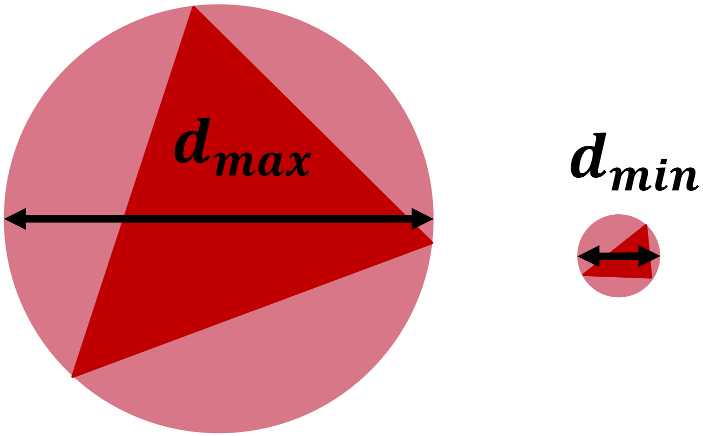

$$\Rightarrow O\left(\log \frac{d_{\max}}{d_{\min}}\right)$$

Analysis

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    Compute hierarchy level l
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        For all cells c_j in level l_i overlapped by t          ⇒ O(1)
            For all triangles t_k ∈ c_j                          ⇒ O(1)
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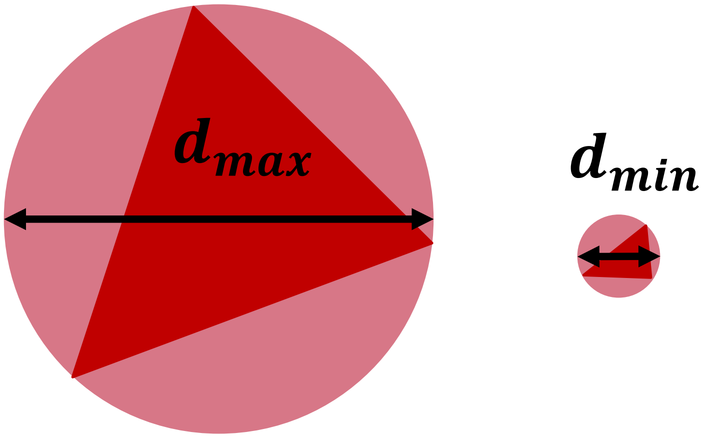
```



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```

- For all levels: $\Rightarrow O(1)$
- For all cells: $\Rightarrow O(1)$
- For all triangles: $\Rightarrow O(1)$
- Compute intersection: $\Rightarrow O(1)$

Analysis

```
For each triangle t ∈ B:                                              ⇒ O(n)
    Compute hierarchy level l
    For all levels l_i, l ≤ l_i ≤ l_max:                               ⇒ O(1)
        For all cells c_j in level l_i overlapped by t                ⇒ O(1)
            For all triangles t_k ∈ c_j                               ⇒ O(1)
                Compute intersection for t and t_k                    ⇒ O(1)
```

Analysis

```
For each triangle t ∈ B:                                       ⇒ O(n)
    Compute hierarchy level l
    For all levels l_i, l ≤ l_i ≤ l_max:                       ⇒ O(1)
        For all cells c_j in level l_i overlapped by t         ⇒ O(1)
            For all triangles t_k ∈ c_j                        ⇒ O(1)
                Compute intersection for t and t_k             ⇒ O(1)
```

$$\text{Total Time:} \quad O(n)$$

Analysis

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For each triangle t ∈ B:                                   ⇒ O(n)
    Compute hierarchy level l
    For all levels l_i, l ≤ l_i ≤ l_max:                   ⇒ O(1)
        For all cells c_j in level l_i overlapped by t     ⇒ O(1)
            For all triangles t_k ∈ c_j                    ⇒ O(1)
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Total Time: $O(n)$

Analysis

```
In Parallel for all triangles t ∈ B:                     ⇒ O(n)
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    For all levels l_i, l ≤ l_i ≤ l_max:                   ⇒ O(1)
        For all cells c_j in level l_i overlapped by t     ⇒ O(1)
            For all triangles t_k ∈ c_j                    ⇒ O(1)
                Compute intersection for t and t_k         ⇒ O(1)
```

Total Time: $O(n)$

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In Parallel for all triangles t ∈ B:                         ⇒ O(n)
    Compute hierarchy level l
    For all levels l_i, l ≤ l_i ≤ l_max:                     ⇒ O(1)
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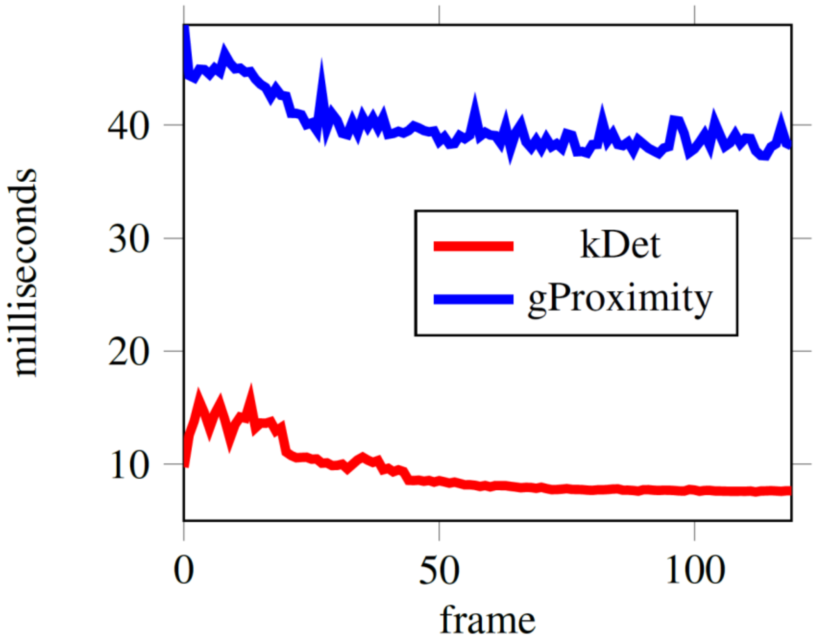
            For all triangles t_k ∈ c_j                      ⇒ O(1)
                Compute intersection for t and t_k           ⇒ O(1)
```

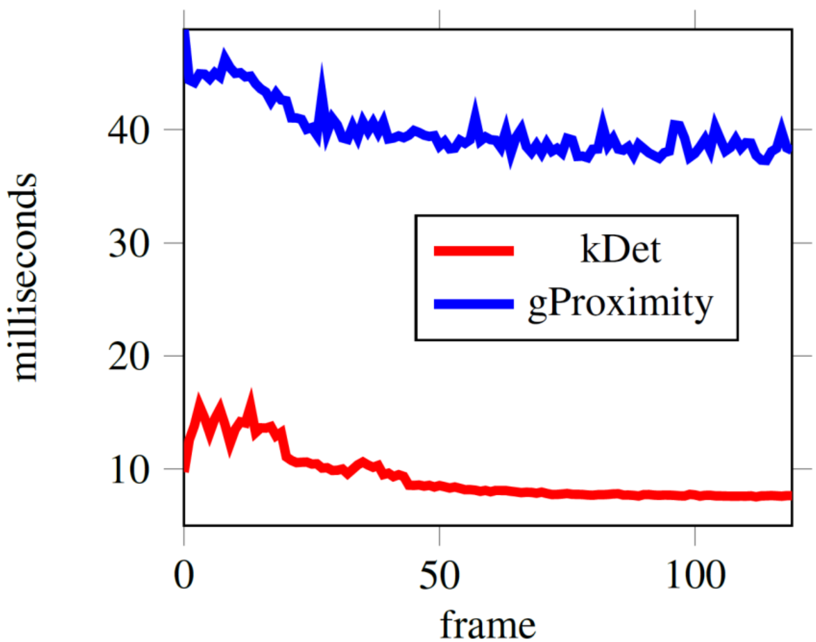
Total Time: $O(n)$

Total Parallel Time: $O(1)$

Results



Results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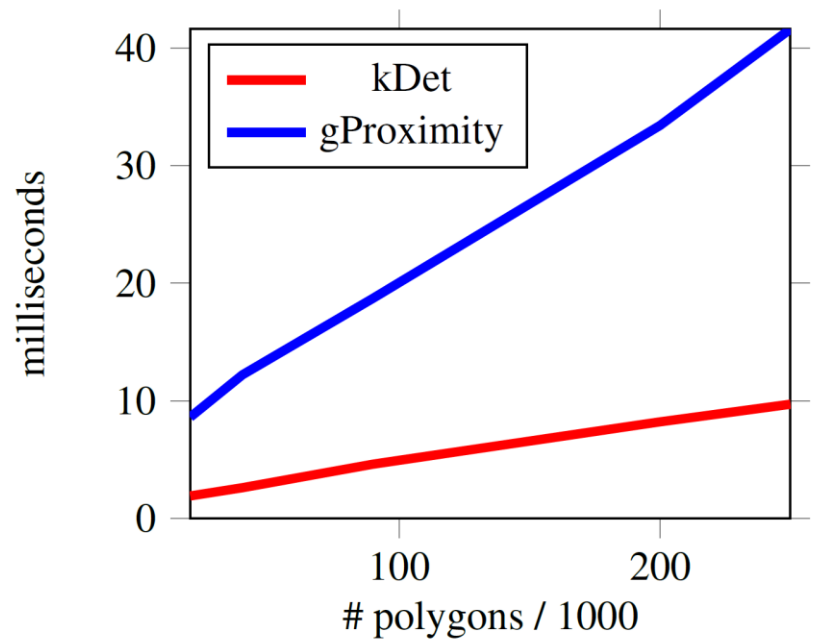
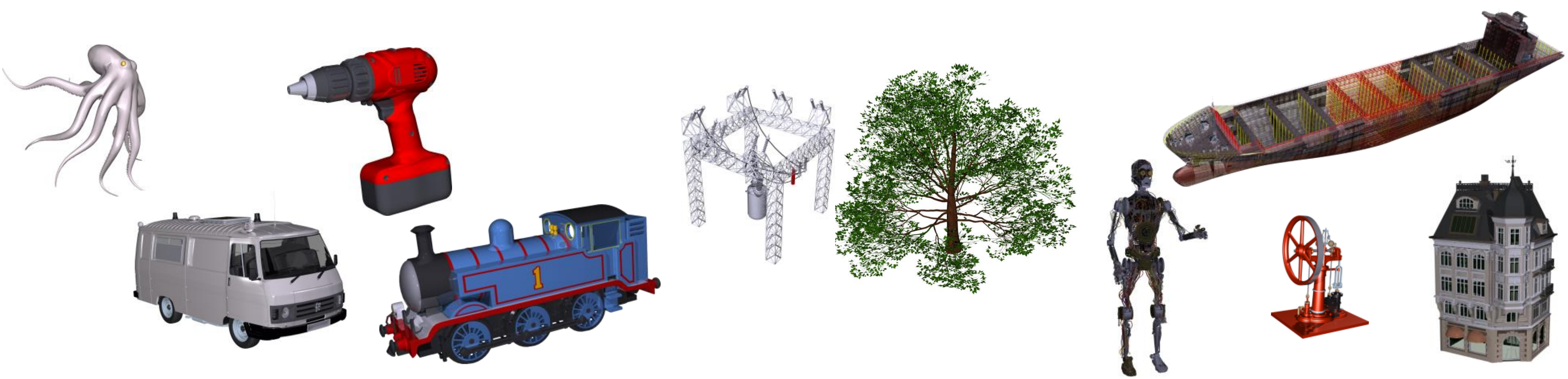
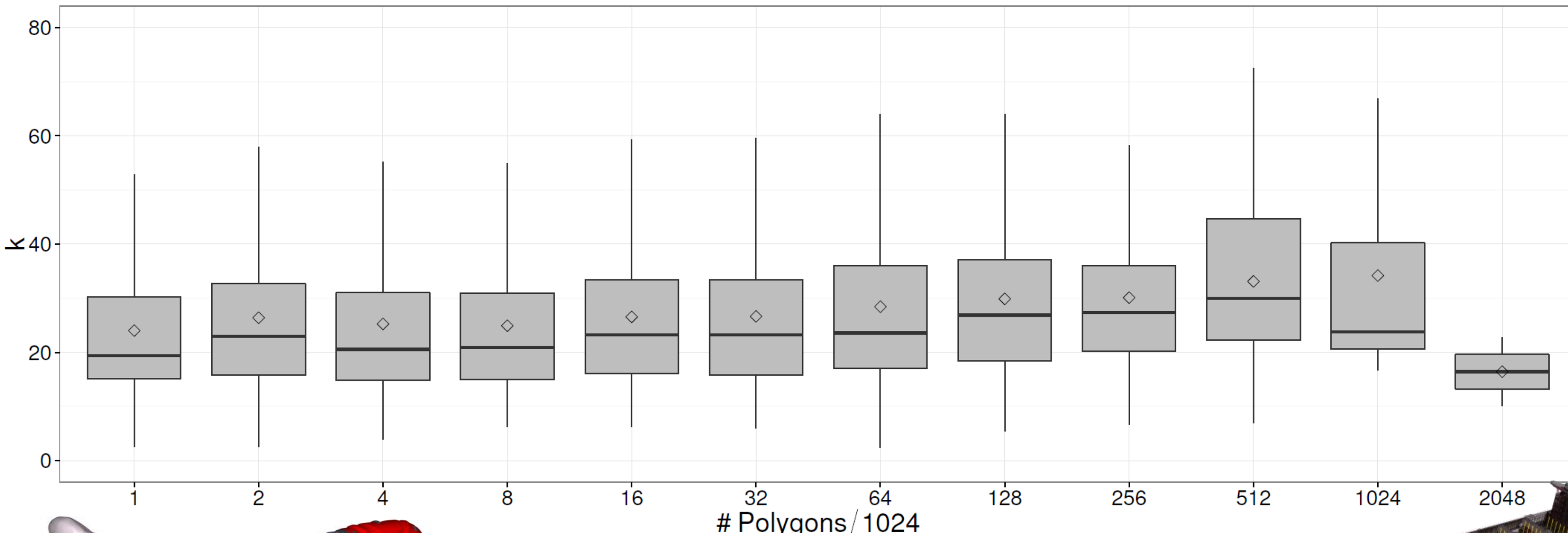
40
30
20
10
0
50
100
frame
kDet
gProximity

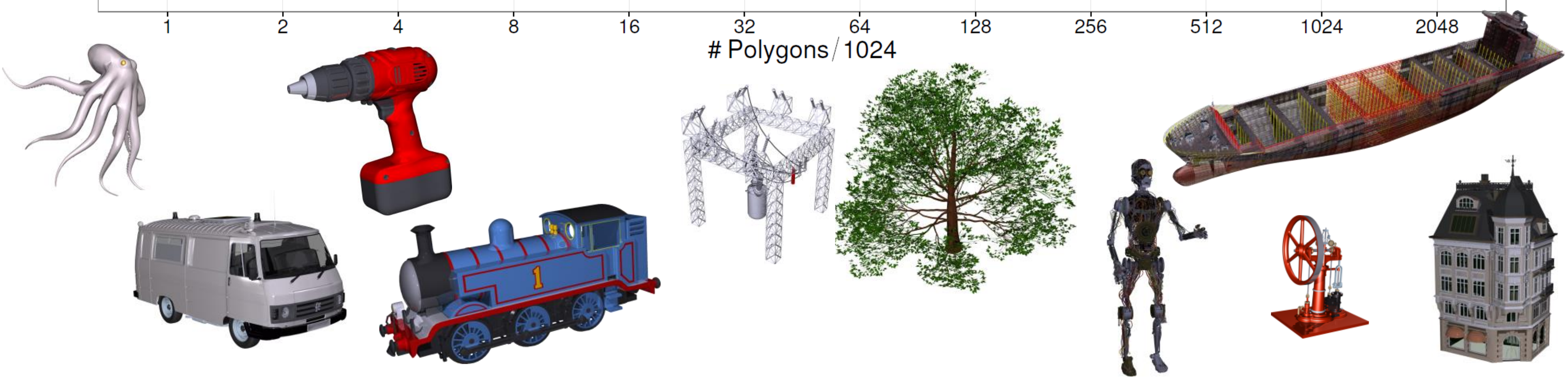

milliseconds
40
30
20
10
0
100
200
polygons / 1000
kDet
gProximity

Results



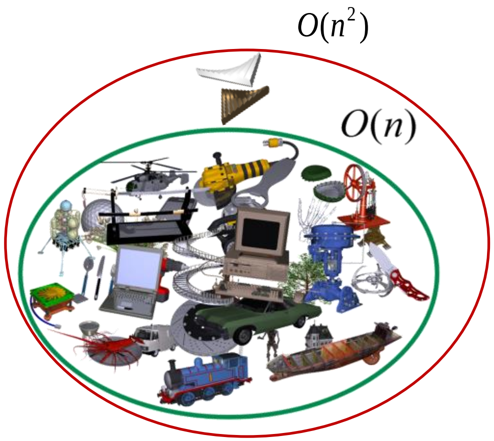
Results



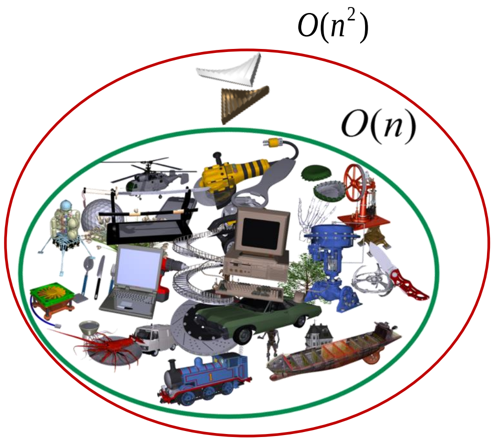


Conclusions

- Novel geometric predicate for polygonal objects defining a class with provable $O(n)$ worst-case intersecting polygon pairs
- Algorithm that realizes linear running-time
  - Parallel running-time: $O(1)$
- <10 msec to check two objects with 250k triangles for collision



Conclusions

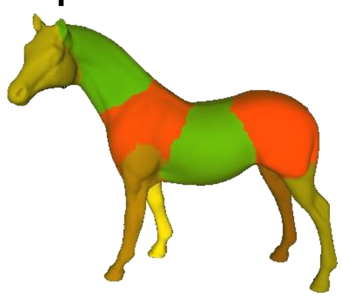


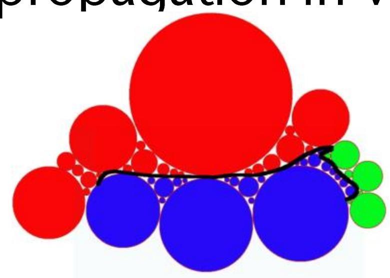
- Novel geometric predicate for polygonal objects defining a class with provable $O(n)$ worst-case intersecting polygon pairs
- Algorithm that realizes linear running-time
  - Parallel running-time: $O(1)$
- <10 msec to check two objects with 250k triangles for collision
- Future challenges:
  - Triangulations that optimize k and $\frac{d_{max}}{d_{min}}$
  - Deformation methods that maintain k and $\frac{d_{max}}{d_{min}}$
  - Application to other problems

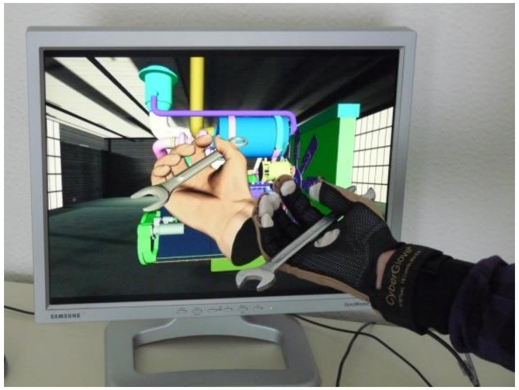
More related MSc Thesis Topics

- Massively-parallel Voronoi diagram computation with

- Massively-parallel machine learning algorithms for CG challenges (e.g. self-organizing-maps, k-means)
- Application of sphere packings to other CG challenges
  - Object classification
  - Object segmentation
- Sphere-based sound propagation in VR
-







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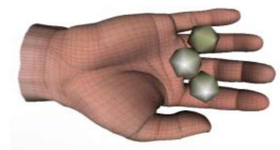
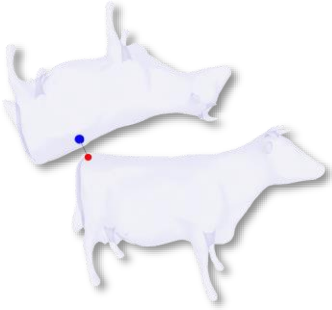
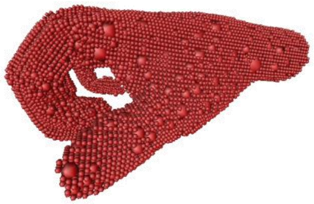


Protosphere

Kinetic CollDet





VR-Coralreef

KINAPTIK