Fresnel Equations, Schlick Approximation, Metals, and Dielectrics

If you look at a renderer you will probably see some references to "Schlick" and "Fresnel" and see some 5th order polynomial equations. This is all about smooth metals and "dielectrics".

You will notice that the reflectivity of smooth surfaces vary with angle. This is especially easy to see on water:

Note how the reflectivity increases as the angle of the viewer increases as the incident angle of the viewing direction to the normal increases.   Work by Escher.

The fraction of how much light is transmitted and how much is reflected adds to one. This is true tracing the light in either direction:
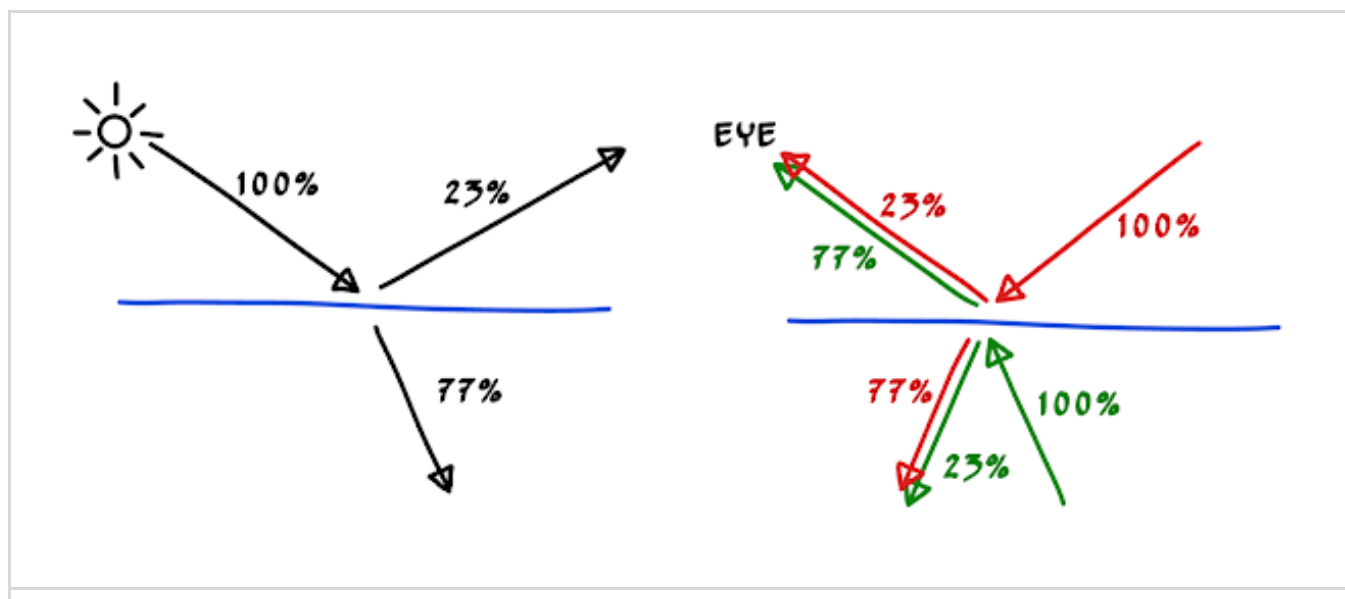
Left: the light hitting the water divides between reflected and refracted based on angle (for this particular one it is 23% reflected). **Right**: the eye sees 77% of whatever color comes from below the water and 23% of whatever color comes from above the water.

So in a ray tracer we would have the color of the ray be

color = 0.77*color(refracted_ray) + 0.23*color(reflected_ray)

But how do we compute those reflected and refracted constants?

These are given by the [Fresnel Equations](). For a non-metal like water, all that you need for them is the [refractive index]() of the material. Back in the dark ages, I was very excited about these equations and used them in my ray tracer. Here they are from that wikipedia page:

$$R_s = \left| \frac{n_1 \cos\theta_i - n_2 \cos\theta_t}{n_1 \cos\theta_i + n_2 \cos\theta_t} \right|^2 = \left| \frac{n_1 \cos\theta_i - n_2 \sqrt{1 - \left(\frac{n_1}{n_2}\sin\theta_i\right)^2}}{n_1 \cos\theta_i + n_2 \sqrt{1 - \left(\frac{n_1}{n_2}\sin\theta_i\right)^2}} \right|^2,$$

$$R_p = \left| \frac{n_1 \cos\theta_t - n_2 \cos\theta_i}{n_1 \cos\theta_t + n_2 \cos\theta_i} \right|^2 = \left| \frac{n_1 \sqrt{1 - \left(\frac{n_1}{n_2}\sin\theta_i\right)^2} - n_2 \cos\theta_i}{n_1 \sqrt{1 - \left(\frac{n_1}{n_2}\sin\theta_i\right)^2} + n_2 \cos\theta_i} \right|^2.$$

Here R is the reflectance, theta is the angle to the normal of the incident direction, n1 is the refractive index of the material the incident light is in, and n2 is the refractive index of the material the light is going into. But there are two of them?! One is for one type of polarization, and one is for the other. The "types" are relative to the surface normal. This is a cool

topic but we would need would delve into some pretty serious optics we wont use (though many applications do where optical accuracy is needed and Alexander Wilkie has been doing great work in that area). So what if I don't have polarization in my ray tracer. I used to just average them:

R = (R_s + R_p) /2

So how do those look? Here is from that same wikipedia page as above:
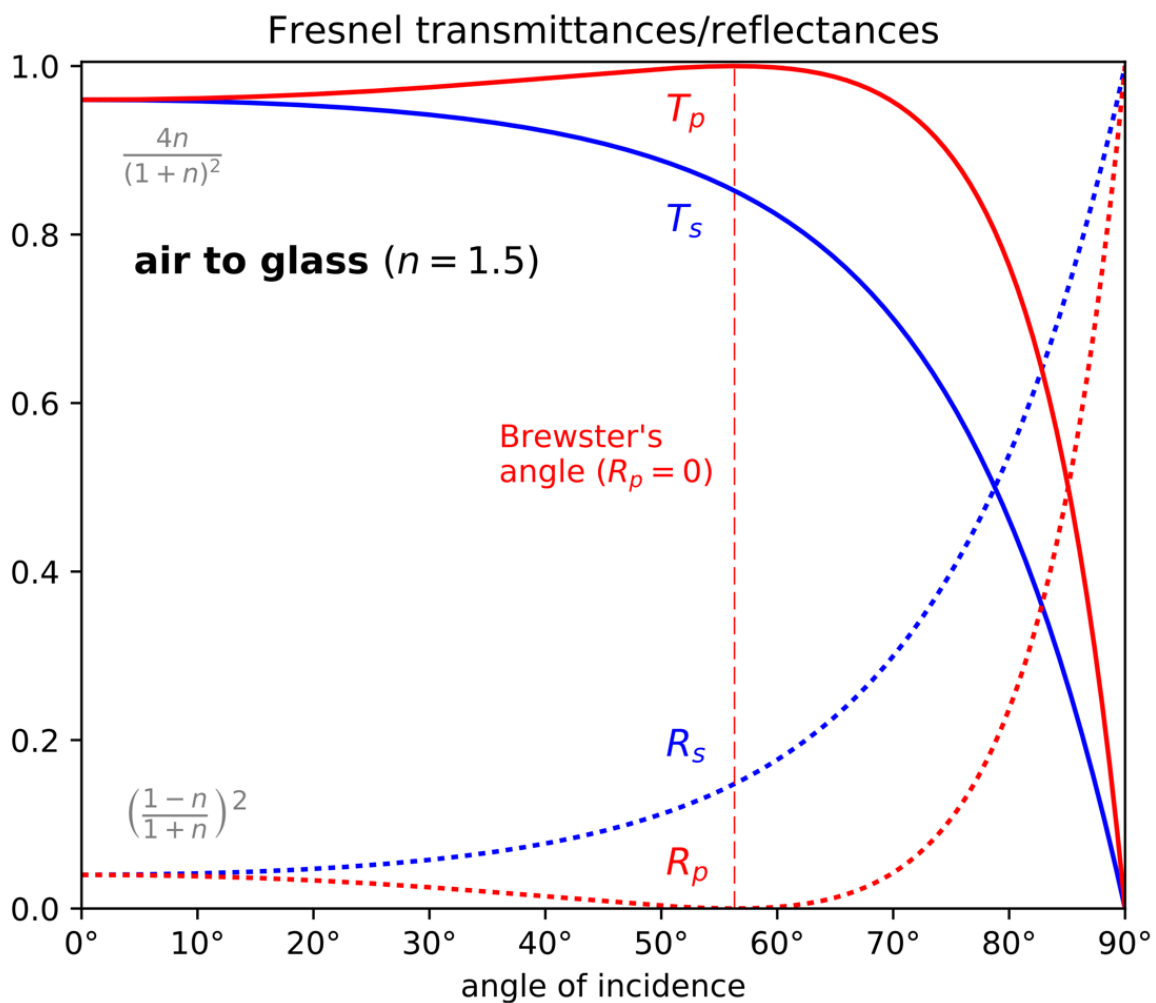
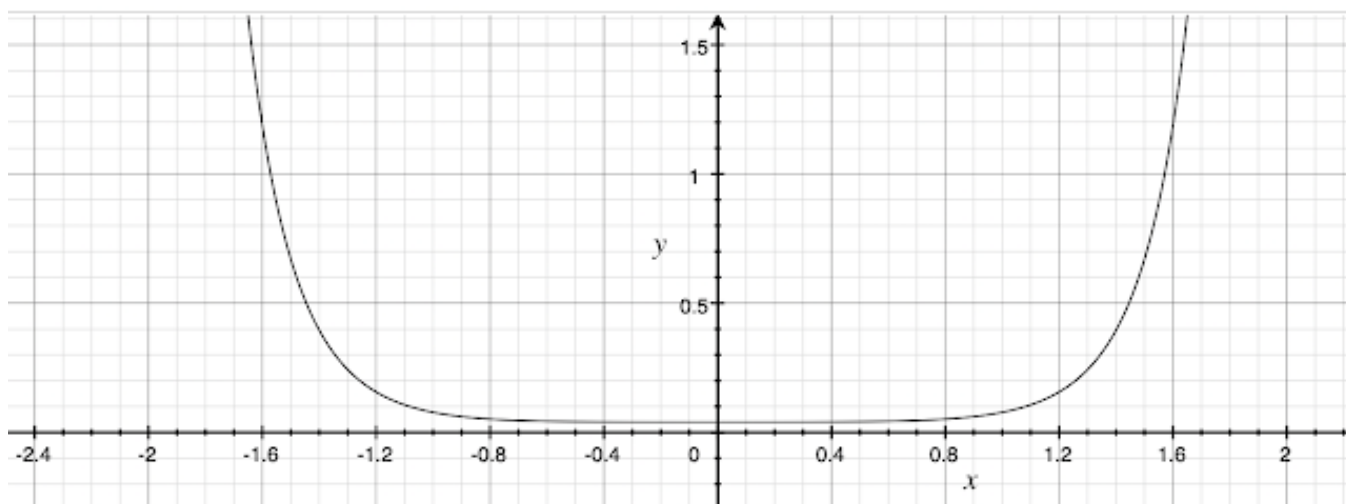
Well there are some cool things there. Especially that R_p goes to zero for one angle. This is Brewster's Angle and why I have polarized sun glasses-- so I can get x-ray vision into water! But I wont simulate that in a

ray tracer.

So what does R = (R_s + R_p) /2 look like for the example above. I'll type it into grapher on my mac, where x is in theta_i in radians:

$$y=0.5 \cdot \frac{\left(\cos(x)-1.5\,\mathrm{sqrt}\left(1-\left(\frac{4}{9}\right)\sin^2(x)\right)\right)^2}{\left(\cos(x)+1.5\,\mathrm{sqrt}\left(1-\left(\frac{4}{9}\right)\sin^2(x)\right)\right)^2}+0.5 \cdot \frac{\left(-1.5\cos(x)+\mathrm{sqrt}\left(1-\left(\frac{4}{9}\right)\sin^2(x)\right)\right)^2}{\left(1.5\cos(x)+\mathrm{sqrt}\left(1-\left(\frac{4}{9}\right)\sin^2(x)\right)\right)^2}$$

Wow that is certainly a complicated expensive function for such a smooth curve. The x axis in in radians so 90 degrees is x = PI/2 = 1.57. So 0 to 1.57 is the interesting part.

Now I ran that code for years. But Christophe Schlick has some sweet papers in the 1990s about using simple approximations to smooth functions in rendering. I mean why are we solving the approximate equation (we threw away polarization) with exact, expensive equations??! His equation for approximate fresnel reflection was almost immediately adopted by all of us. Here it is from that wikipedia page:

According to Schlick's model, the specular reflection coefficient R can be approximated by:

$$R(\theta) = R_0 + (1 - R_0)(1 - \cos\theta)^5$$

$$R_0 = \left(\frac{n_1 - n_2}{n_1 + n_2}\right)^2$$

That R_0 is the reflectance for theta=0, so light going straight at the surface. That is right where transparent surfaces are most clear. It is just the big fancy equations above which both collapse to that for theta=0.   If I graph that too for the n=1.5 I get:
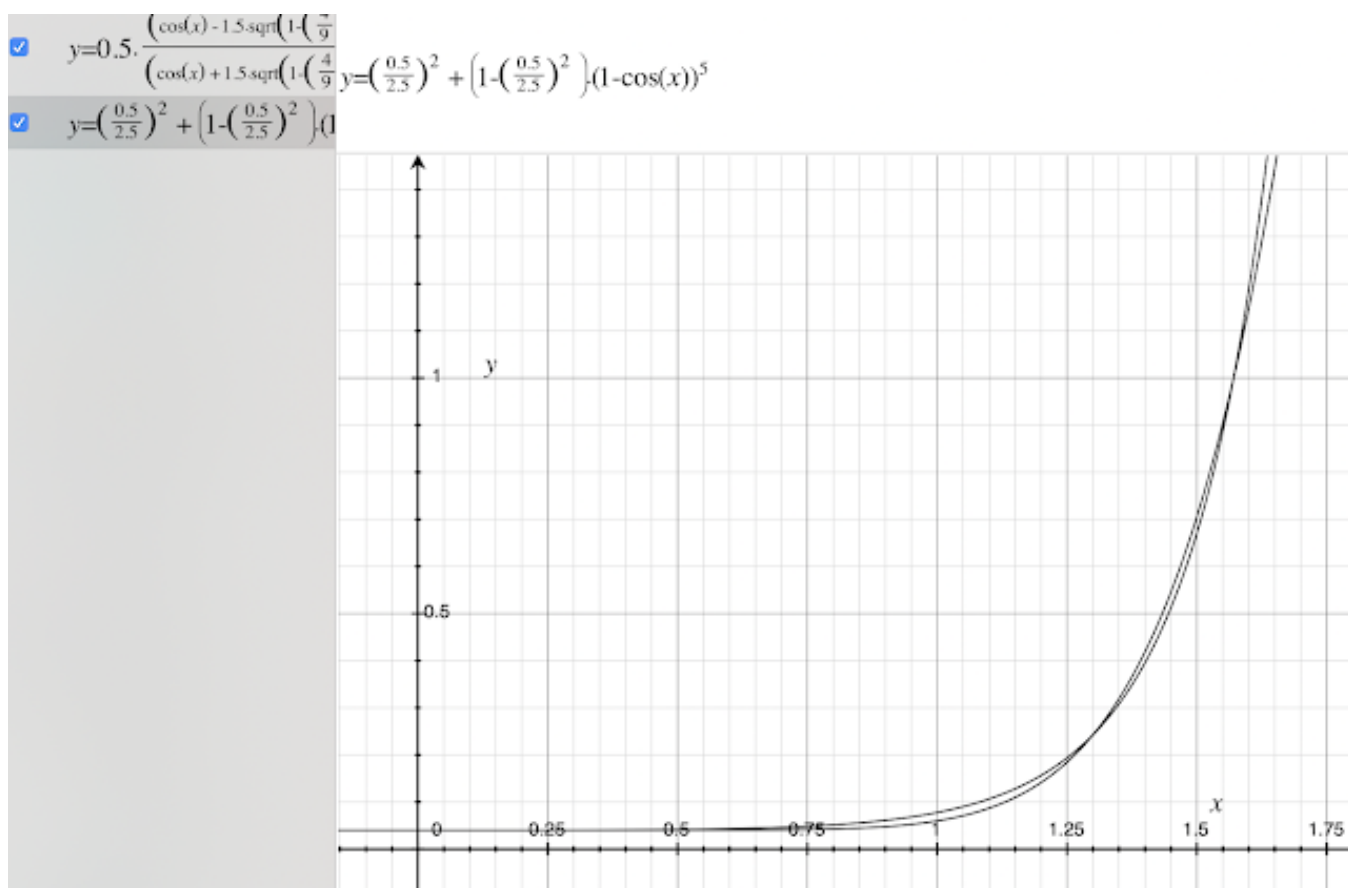
OK that is close! And a lot easier and cheaper.

So where do you get n1 and n2?   If one of the surfaces is air use 1.0 for that. From that wikipedia article on refractive index above we have:

Selected refractive indices at λ=589 nm. For
references, see the extended List of refractive
indices.

Material	n
Vacuum	1
Gases at 0 °C and 1 atm	
Air	1.000 293
Helium	1.000 036
Hydrogen	1.000 132
Carbon dioxide	1.000 45
Liquids at 20 °C	
Water	1.333
Ethanol	1.36
Olive oil	1.47
Solids	
Ice	1.31
Fused silica (quartz)	1.46[11]
PMMA (acrylic, plexiglas, lucite, perspex)	1.49
Window glass	1.52[12]
Polycarbonate (Lexan™)	1.58[13]
Flint glass (typical)	1.69
Sapphire	1.77[14]
Cubic zirconia	2.15
Diamond	2.42
Moissanite	2.65

When in doubt, use 1.5.

Now what happens if you want the rainbows you can see along edges in glass? That is dispersion and it is a pain in RGB renderers... you probably need more samples than 3 in color and you need to convert to a spectral renderer. This has been done a few times (here is a fun one with lovely images) but I wont here!

Note that this same equation is used not only for transparent surfaces but also for the reflective coat of "diffuse-specular" surfaces. Here is one way to think of these surfaces-- a chunk of glass/plastic/whatever with junk in it:
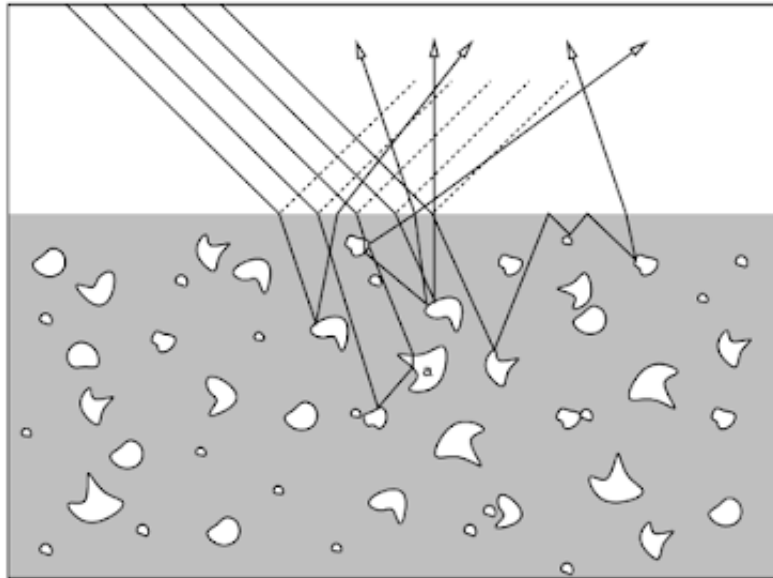
Figure 2.11: Detailed view of a glossy paint.

Note that the *surface* (specular) component is just like glass-- use the Schlick for that! What refractive index? When in doubt, 1.5 :)

Now what about metals? Does the above apply? No, they are nothing like the Fig 2.11 above. All the action is at the surface. But the Fresnel equations are more complex-- literally-- the refractive index of metals is **complex**!   This is sometimes written as *real refractice index* and *conductivity* (the real and imaginary part).

Here are the equations:

$$R_s = \frac{a^2 + b^2 - 2a\cos\theta + \cos^2\theta}{a^2 + b^2 + 2a\cos\theta + \cos^2\theta} \tag{2.4}$$

$$R_p = R_s \frac{a^2 + b^2 - 2a\sin\theta\tan\theta + \sin^2\theta\tan^2\theta}{a^2 + b^2 + 2a\sin\theta\tan\theta + \sin^2\theta\tan^2\theta} \tag{2.5}$$

Where R_s and R_p are the reflectivities for the two planes of polarization, θ is the angle of incidence, and a and b are given by

$$a^2 = \frac{1}{2n_i^2}\left\{ \sqrt{(n^2 - k^2 - n_i^2\sin^2\theta)^2 + 4n^2k^2} + n^2 - k^2 - n_i^2\sin^2\theta \right\} \tag{2.6}$$

$$b^2 = \frac{1}{2n_i^2}\left\{ \sqrt{(n^2 - k^2 - n_i^2\sin^2\theta)^2 + 4n^2k^2} - (n^2 - k^2 - n_i^2\sin^2\theta) \right\} \tag{2.7}$$

Wow. Those are even worse (with k, the conductivity set to zero, you get the non-conductor ones). So for some applications, this is needed. But not for most graphics programs. I rarely use them. Note that they are real valued. So can we use Fresnel? The answer is **yes**!   So where do I get *n* and *k*? I implemented them in the dark ages and tried it for a few metals. Here is one:
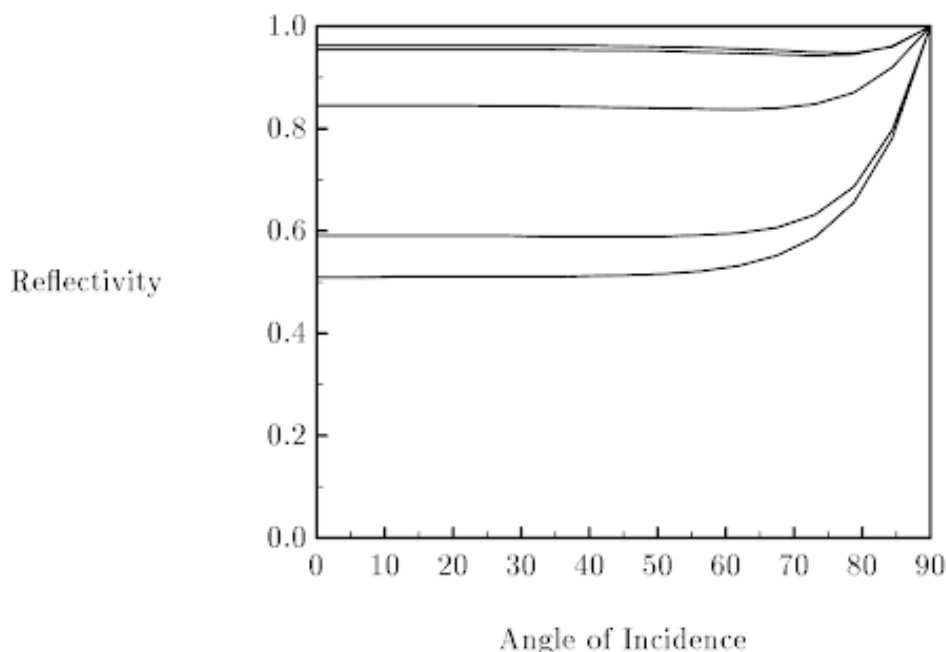
Figure 2.8: Reflectivity of copper for wavelengths 4000, 5000, 6000 7000, and 8000 angstroms (bottom to top). Curves were generated directly from Fresnel Equations using data from Palik [82].

That looks like we can probably use the Schlick equations but with different base cases for R, G, B. So where do we get the normal incident reflectance? We could by evaluating normal incidence.   But the data isn't widely available and a lot of metals are alloys of unknown composition. So we make R0 up (like grab them from an image-- it's an RGB color, but be careful about gamma!)

vec3 R0 = (guess by looking)
vec3 R = R0 + (1-R0)*pow(1-cosine,5)

But with RGB not just floats. This is because the refractive index for metals can vary a lot with wavelength (that is why some metals like gold are colored).

Now most graphics programs have two Schlick Equations running around-- one for dielectrics that are float and set by a refractive index, and one

that is RGB and set by an RGB R0. This seems odd at first, because it is, but it makes sense if you go searching for the refractive index of glass, then gold (try it!).

Naty H writes:

I think this is good advice by Naty.

For the curious-- does light refract into metal? Yes it does, but it gets absorbed quickly. This is why a gold film is transparent if it is thin enough.   Here is [information on refraction for metals](#) (I have never implemented this) using Snell's law:

Thanks to Alain Galvan for pointing out some errors!