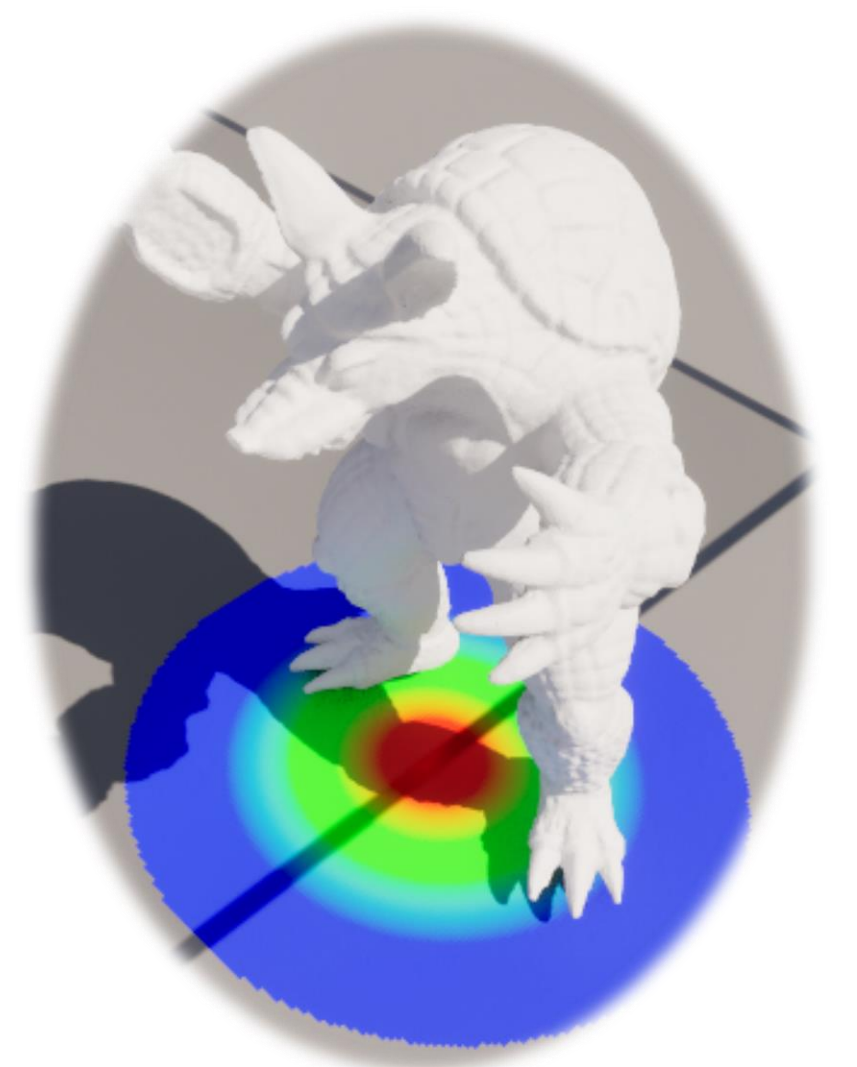


# Exploring Approaches for Rigid Body Dynamics with Uncertainty

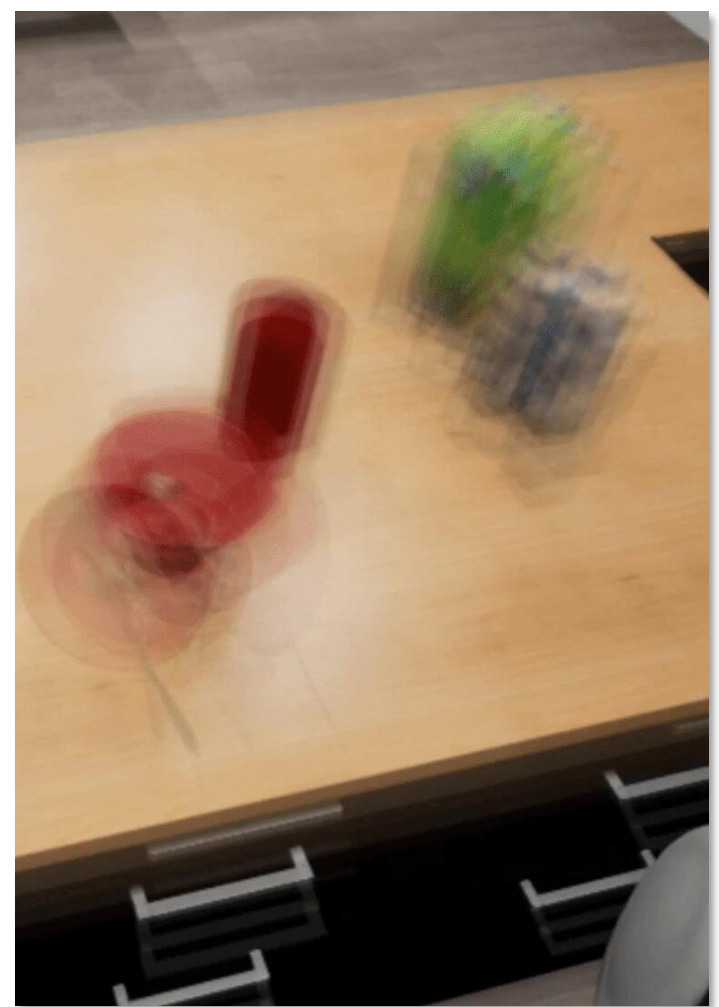
**Hermann Meißenhelter**, Franklin Kenghagho Kenfac, René Weller, Gabriel Zachmann

University of Bremen, Germany

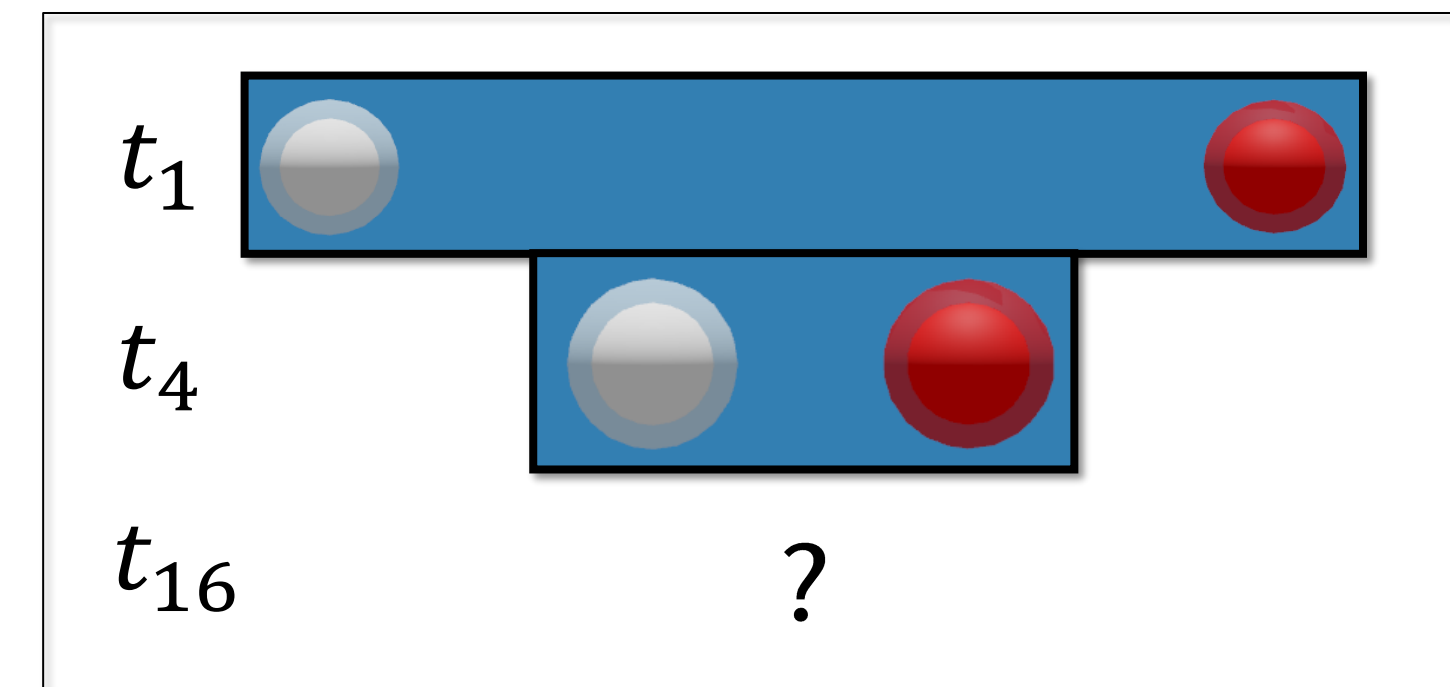
[meissenhelter@cs.uni-bremen.de](mailto:meissenhelter@cs.uni-bremen.de)



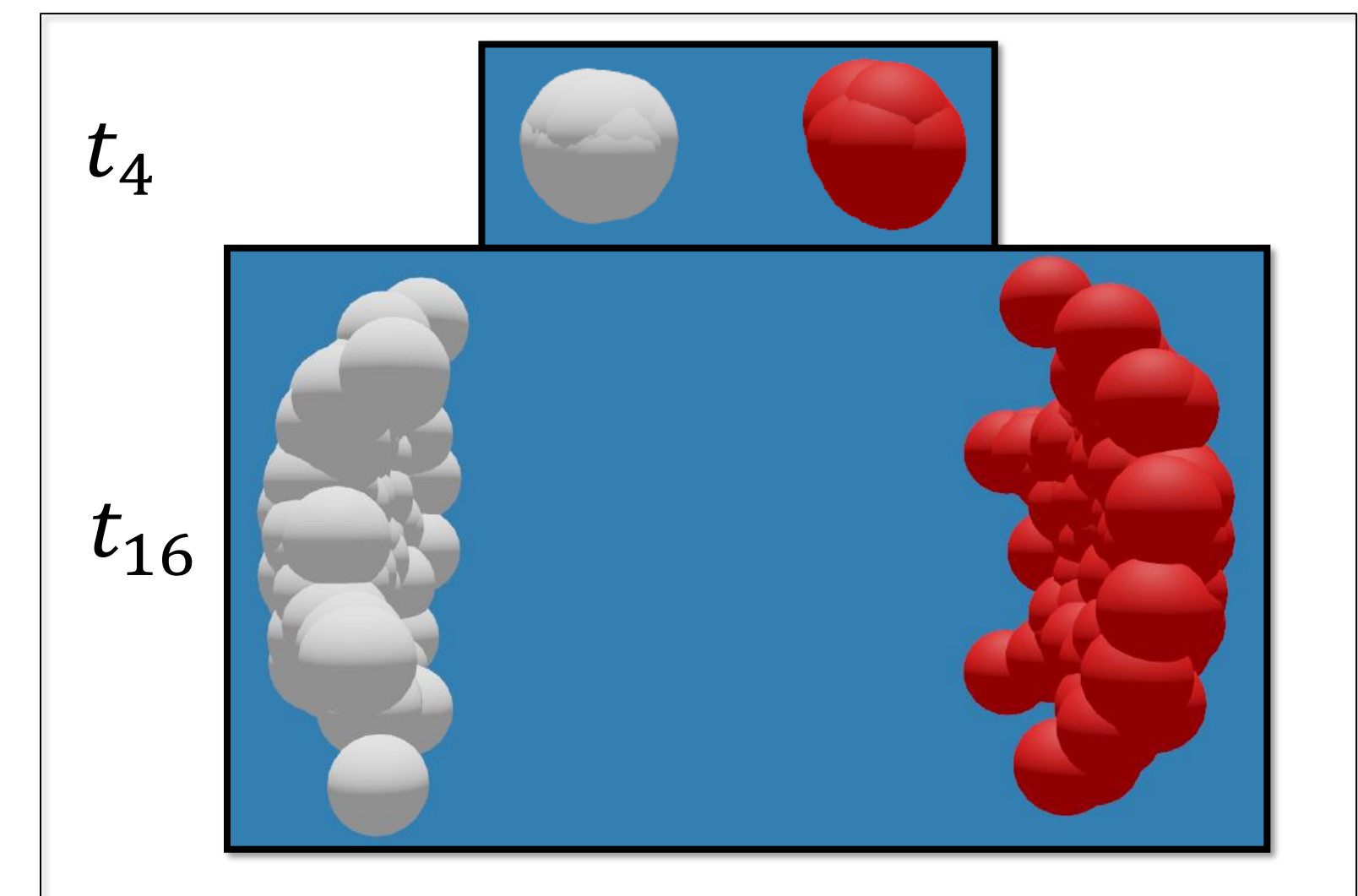
- Real-world scenarios are often uncertain:
  - Sensor noise and incomplete models
- Why Simulation + Uncertainty?
  - Real-world inputs are uncertain: position, orientation, contact
  - Safety & reliability in real world for robotics
  - Current physics engines ignore variability, limiting robustness
  - Collisions can amplify uncertainty



- Standard approach: sample many initial states and simulate each case
- More samples improve fidelity
- But sampling is computationally expensive
- Challenge: efficient simulation while accounting for uncertainty

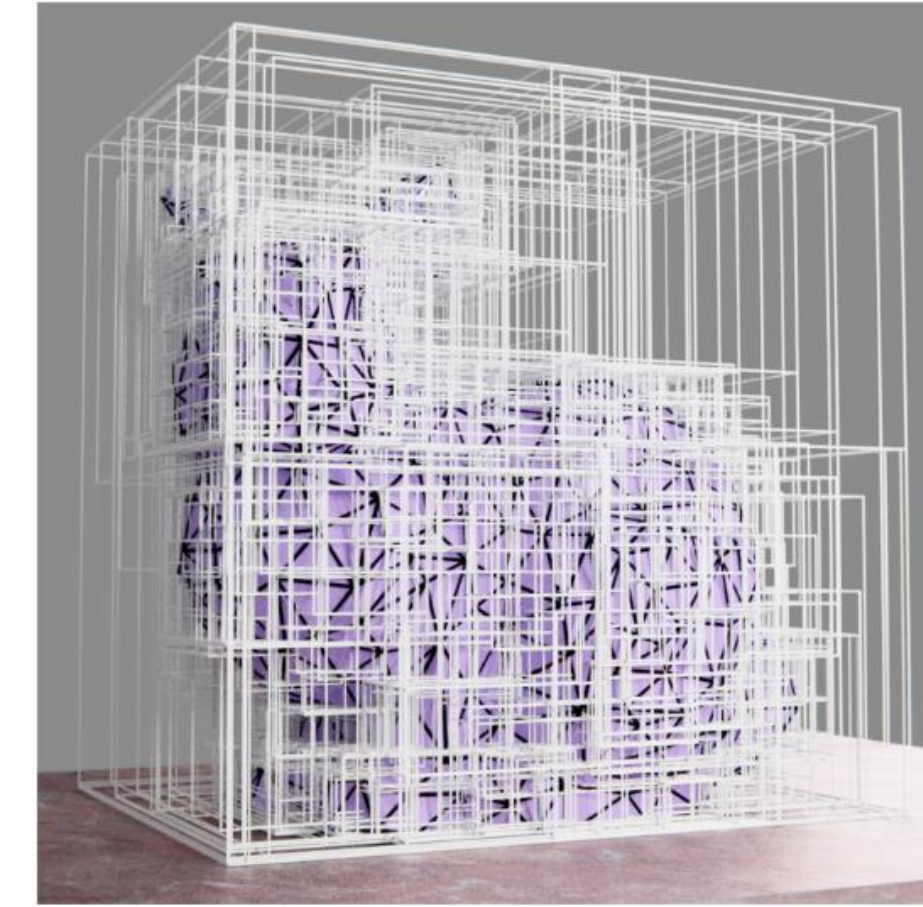


**Problem example:** Particles approaching with initial positional uncertainty

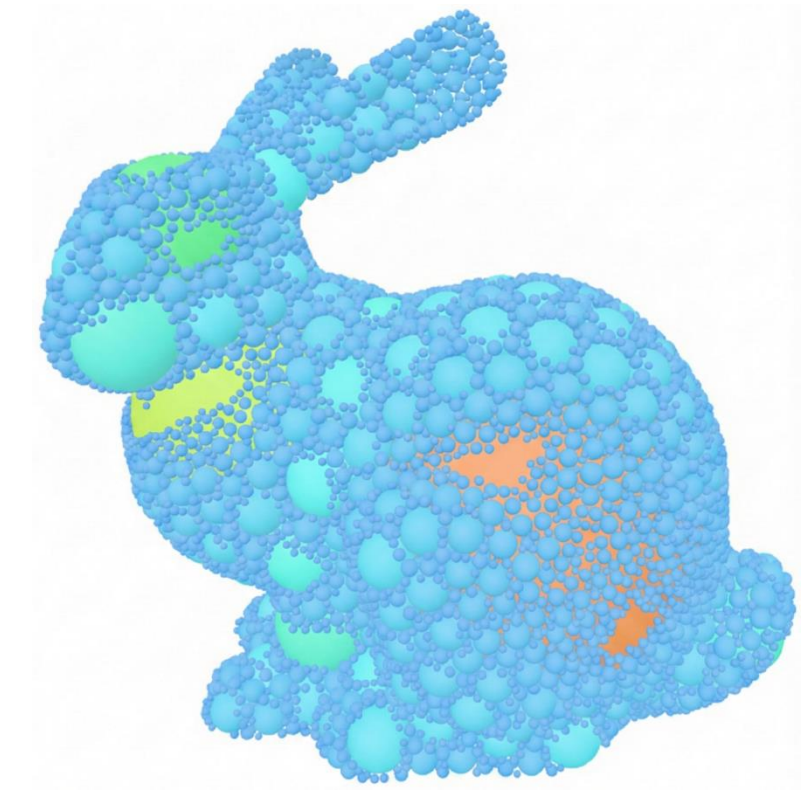


**Baseline:** Sample initial uncertainty and run  $n$  simulations

- Collision detection (CD)
  - Bounding Volume Hierarchies (BVH) [Zachmann98]
  - Grid-based methods [Teschner03]
  - Sphere packings [Weller10]
- Methods for physics-based animation:
  - Penalty [Terzopoulos87][Tang12]
  - Impulse [Mirtich96]
  - Constraint [Macklin16]
- Numerical integration:
  - Symplectic Euler, Runge-Kutta



BVH[Chitalu20]



Sphere Packing



Penalty[Jansson01]

# Related Work - Uncertainty

- Error propagation rules
  - Analytic (Gaussian)
  - If non-linear function  $\rightarrow$  approximating methods, non-Gaussian
- Bayesian Filter
  - Kalman Filter [Kalman60]
  - Extended Kalman filter [Sorenson85]
  - Unscented transform (sigma points) [Julier97]
  - Particle filters [Gordon93] / Monte Carlo
- We got no measurement/correction

Adding two 1D Gaussians

$$f = A + B \quad \text{Mean values}$$

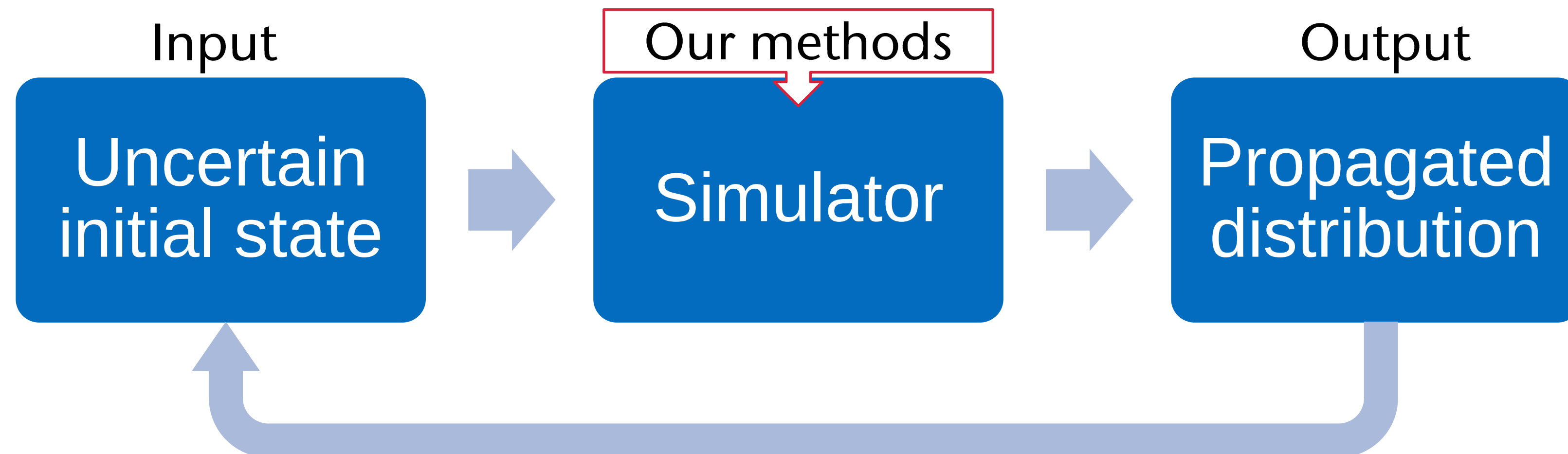
$$\sigma_f^2 = \sigma_A^2 + \sigma_B^2 + 2\sigma_{AB}$$

|               | Smooth Dynamics | Collision Dynamics |
|---------------|-----------------|--------------------|
| Deterministic | ✓               | ✓                  |
| Uncertain     | ✓               | Limited work       |

Our work is a first step

# Overview

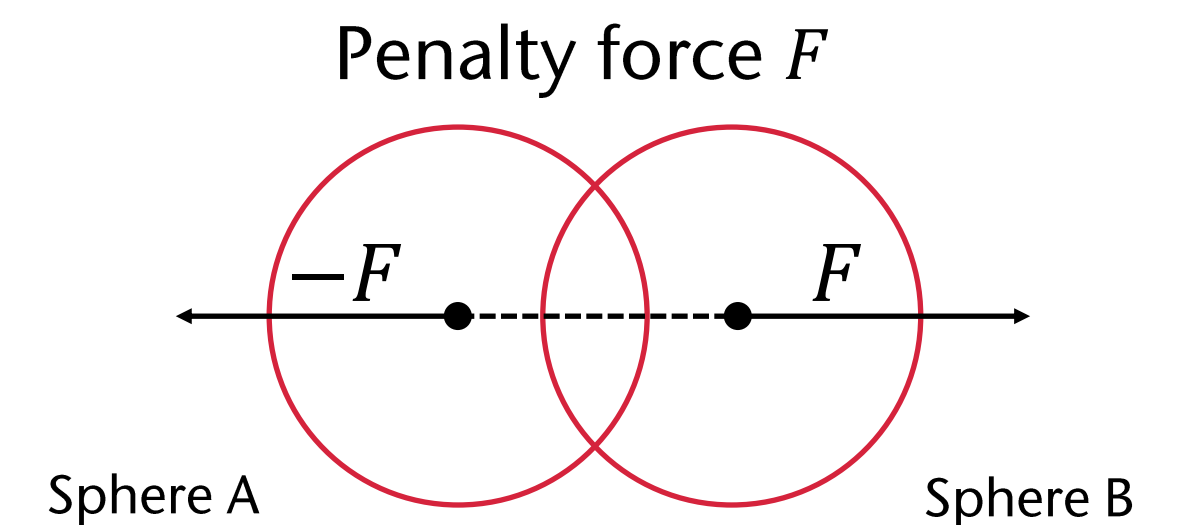
- Input: rigid bodies with Gaussian uncertainty in the state
- Simulator: penalty-based rigid-body dynamics
- Output: predicted distribution after simulation
- Formulate and compare four methods against Monte Carlo ground truth



- State includes position, velocity, orientation, and angular velocity
- Position uncertainty is Gaussian in Euclidean space
- Rotation uncertainty is represented in the tangent space of  $SO(3)$
- Collision detection uses non-uniform sphere packings
- Contact response uses penalty forces based on sphere overlaps



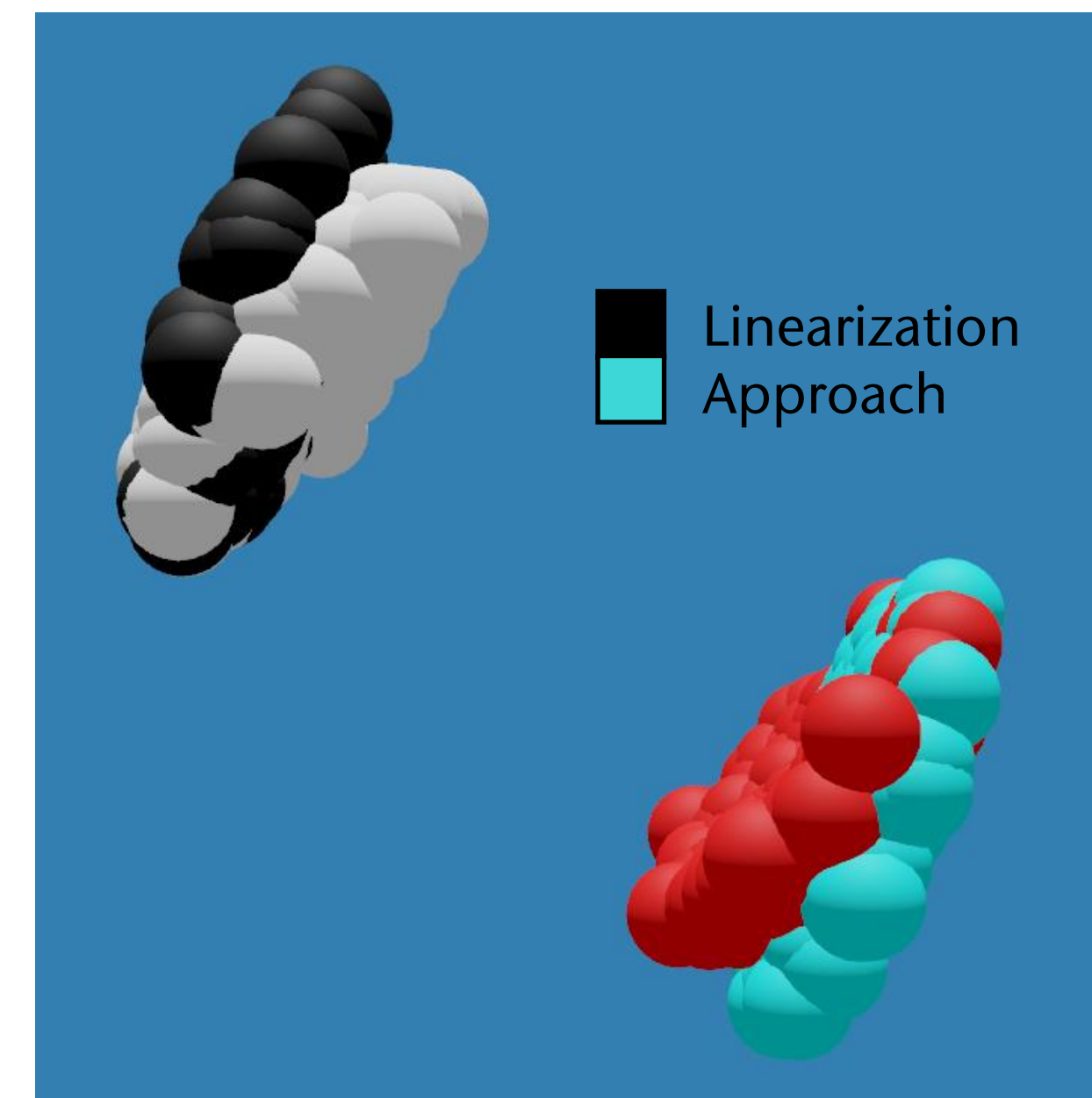
Sphere Packings & Test models



# Handling Uncertainty: Linearization Approach

- Idea: Linearize the full simulator around the nominal state
- Propagates the full covariance through a Jacobian
- Captures cross-body correlations
- Challenge: collisions make linearization unstable
- Uses a dominant-contact restriction to reduce covariance inflation

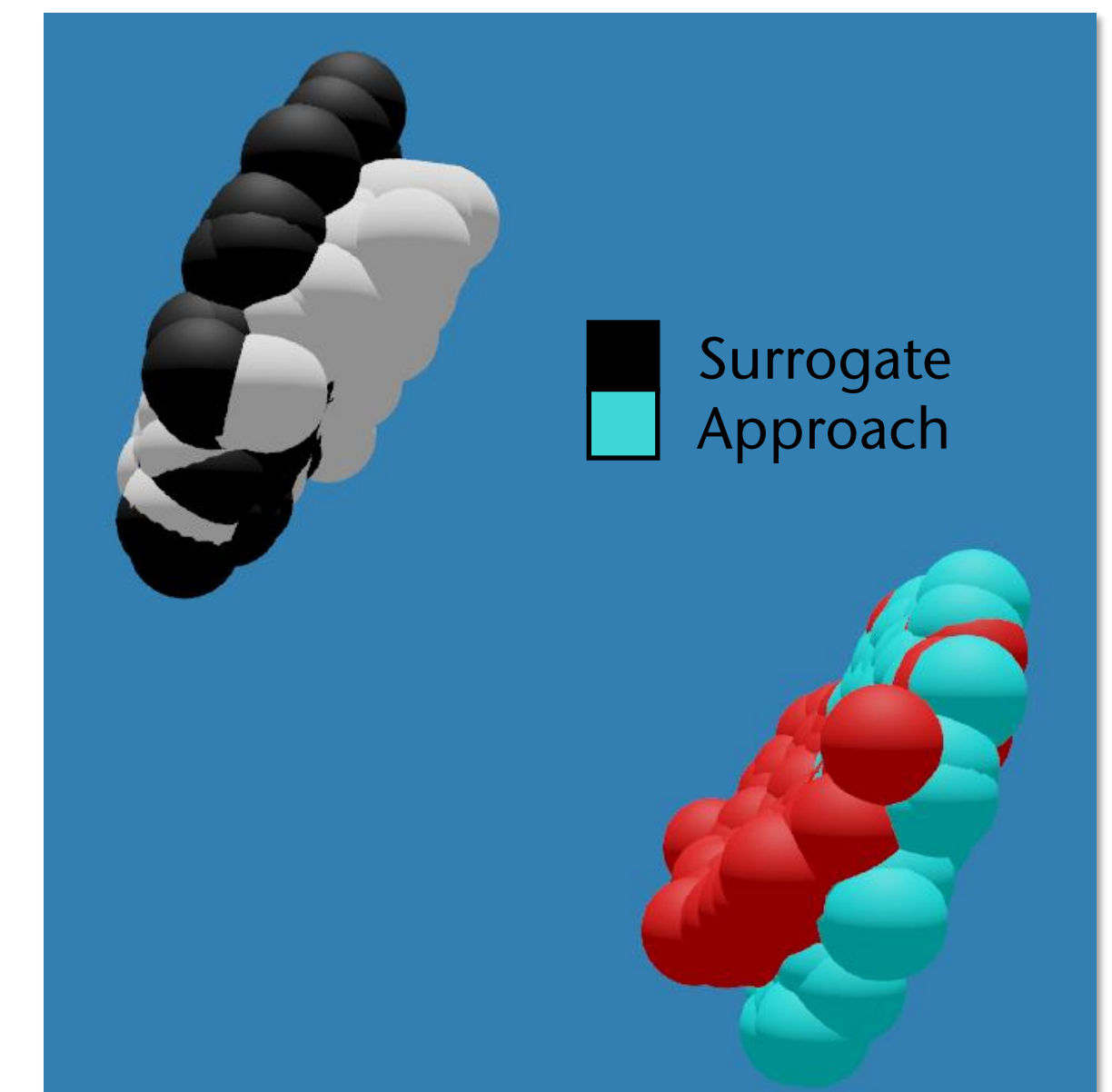
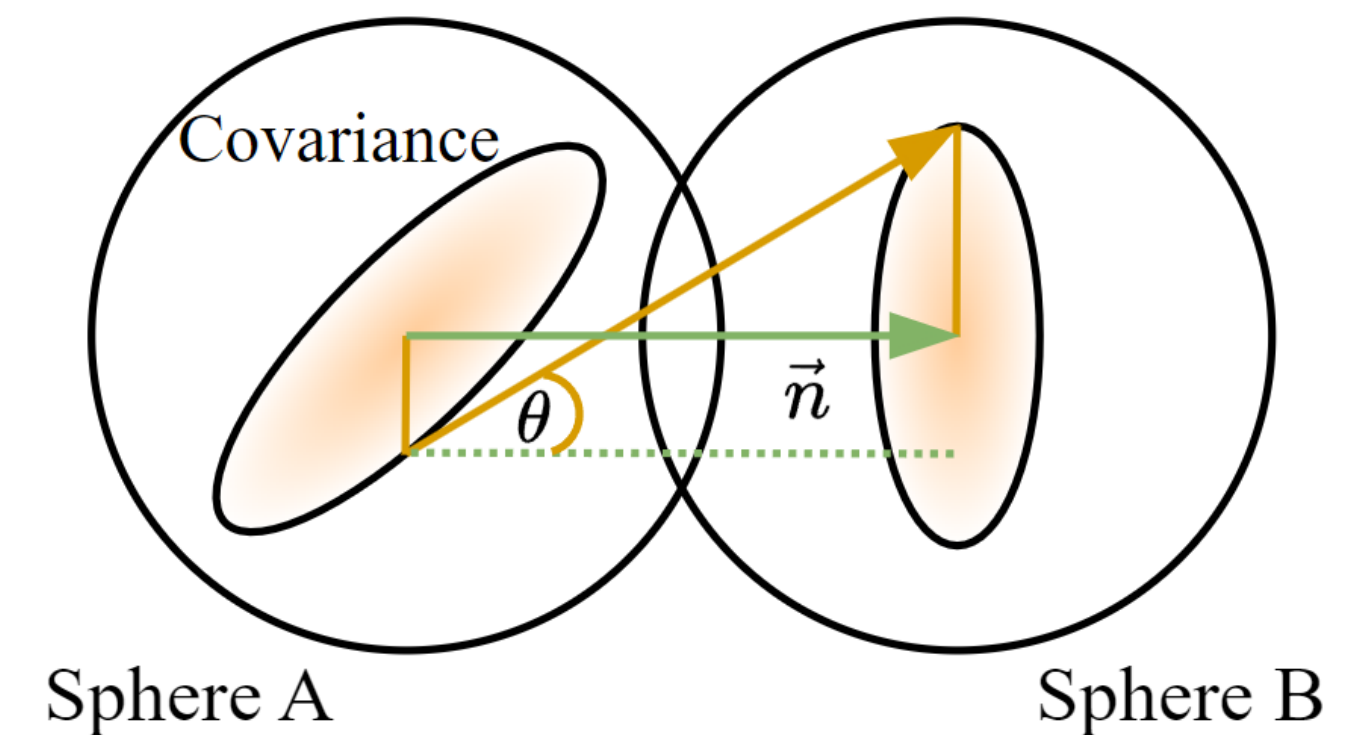
$$\Sigma_{t+1} = J \Sigma_t J^\top$$



# Handling Uncertainty: Surrogate Model

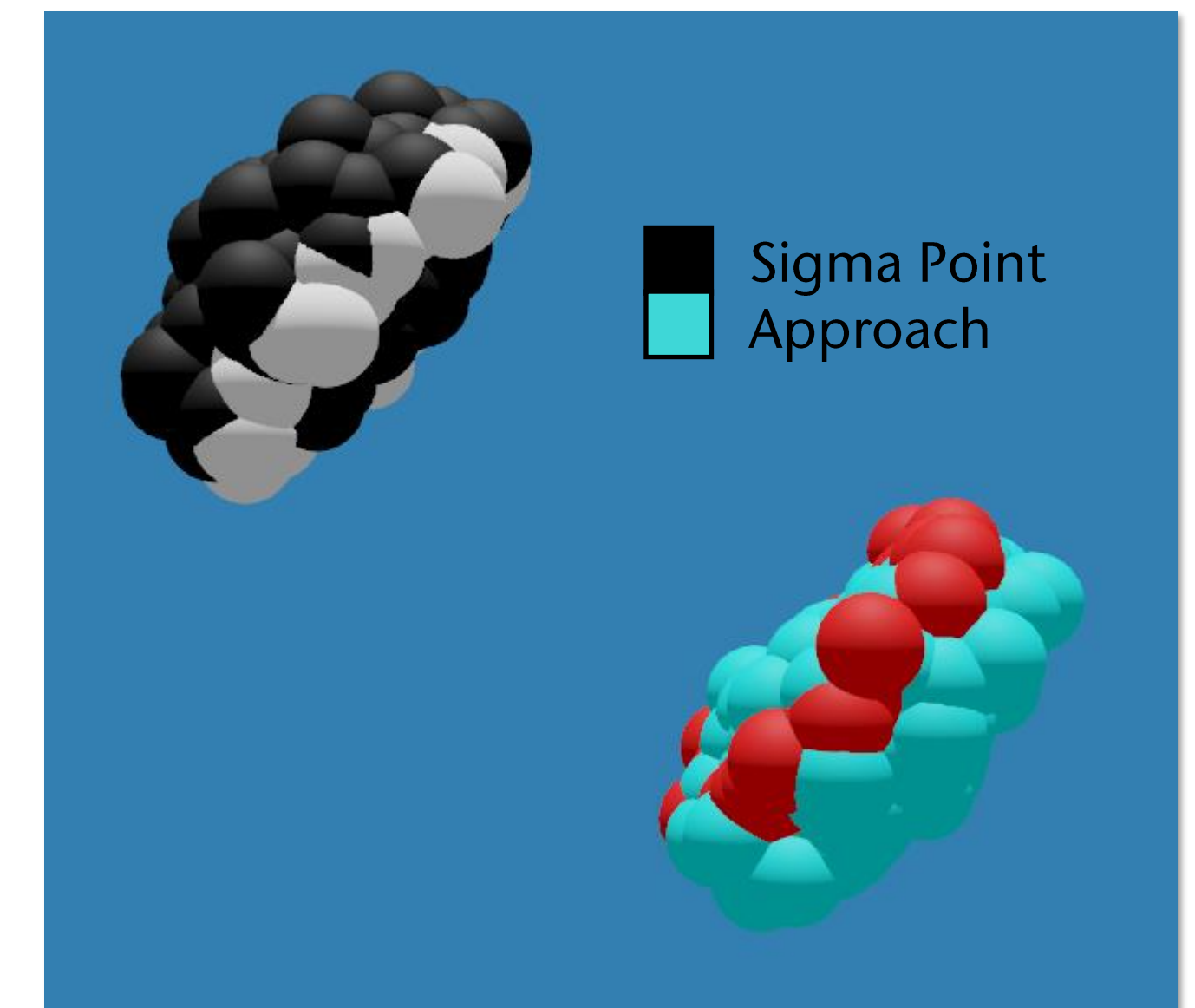
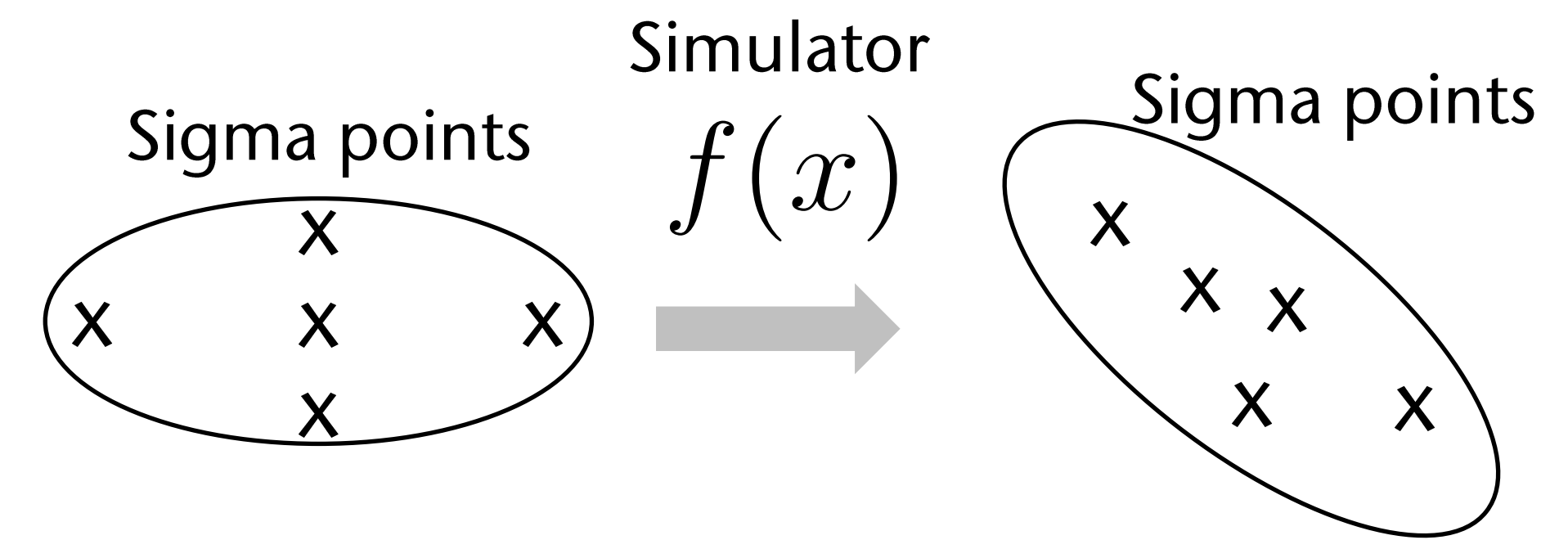
Uncertain contact -> many possible normal directions

- Idea is to estimate contact normal uncertainty
  - Maximum cone angle
  - Converts this into tangential velocity and angular-velocity uncertainty
- Linearization for integration
- Designed as a fast, stable approximation
  - Uncertainty growth is bounded



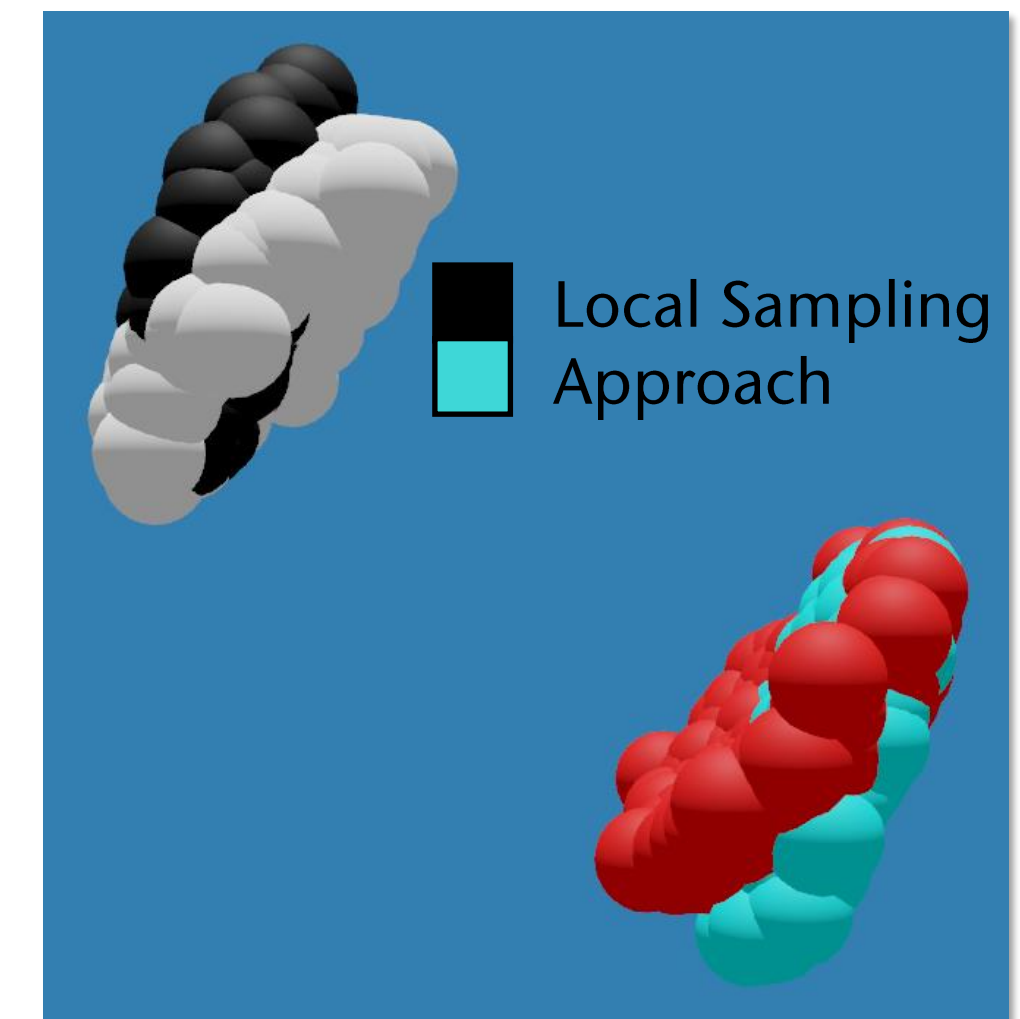
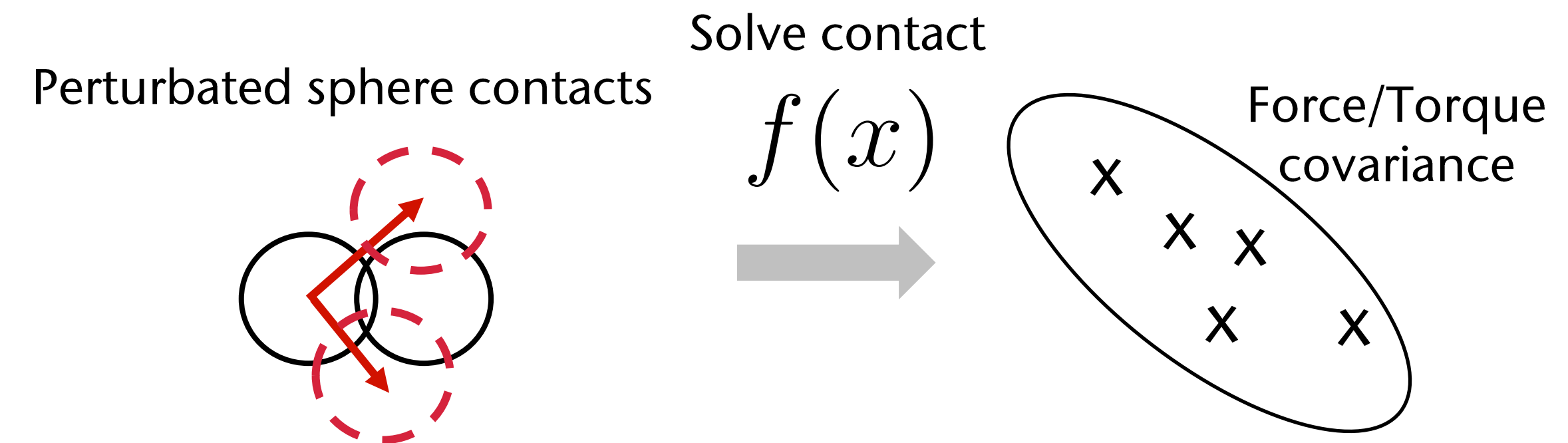
# Handling Uncertainty: Persistent Sigma Points

- Idea is to simulate representative states through time
- For two bodies: 49 sigma points
- Propagate them through the full simulator
- Challenge: unstable with collisions
  - Do not regenerate them every timestep



# Handling Uncertainty: Local Sampling Approach

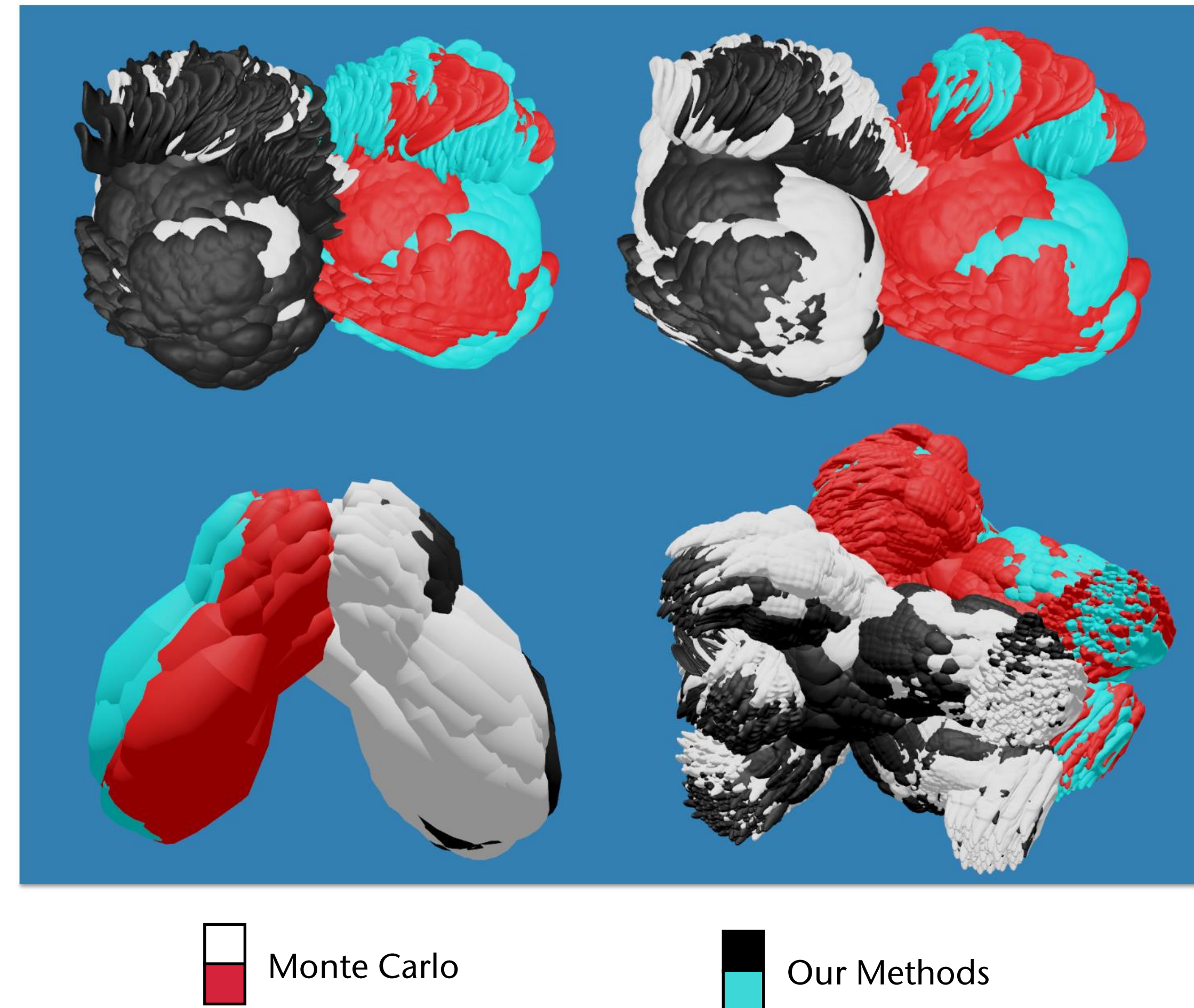
- Idea is to sample the uncertainty for contact solving function
- Generate  $x$  samples of state
  - Mean + perturbation
- Estimate covariance out of  $x$ -forces and torques
- Inject uncertainty into velocity and angular velocity



# Evaluation Setup

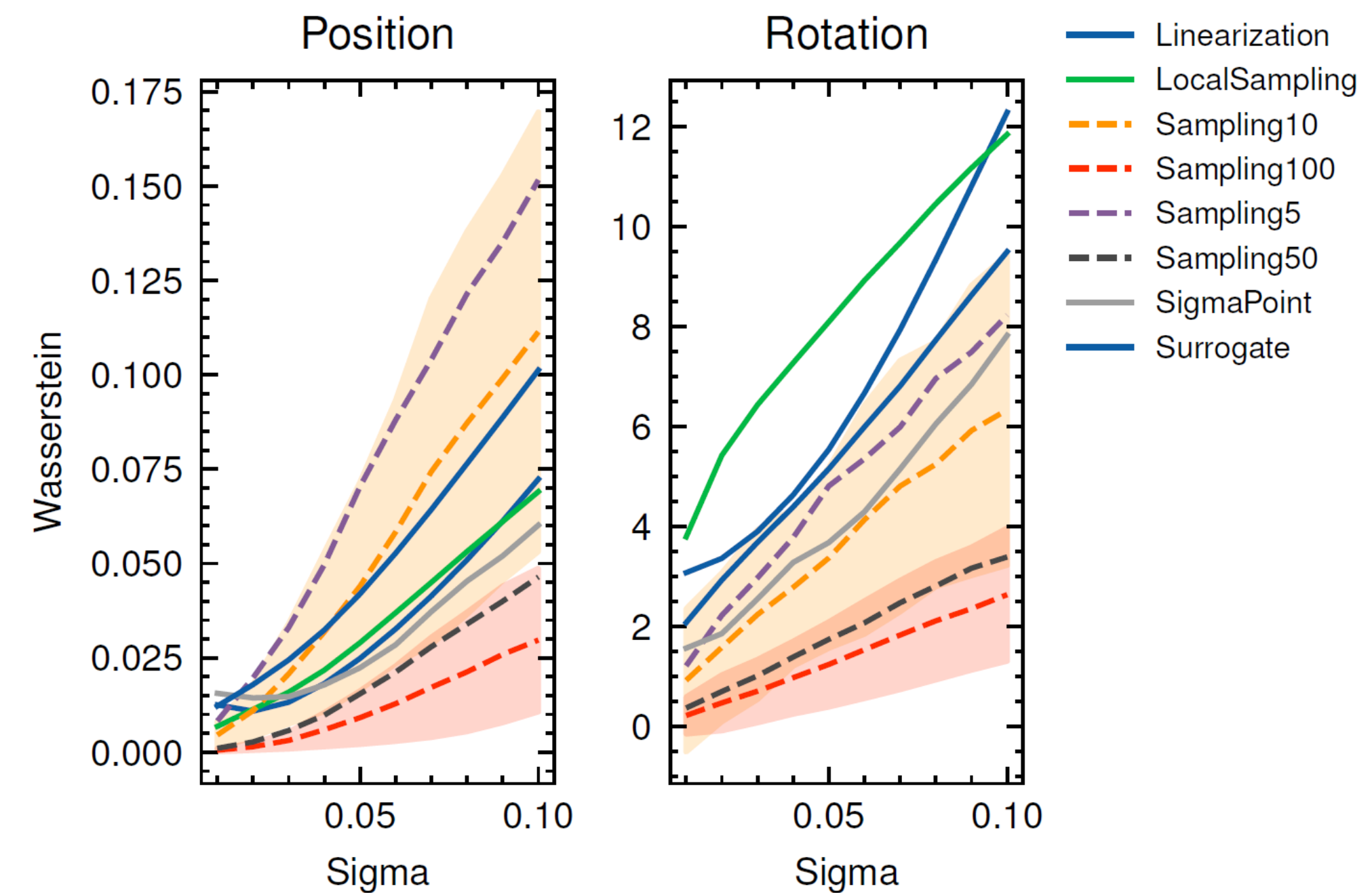
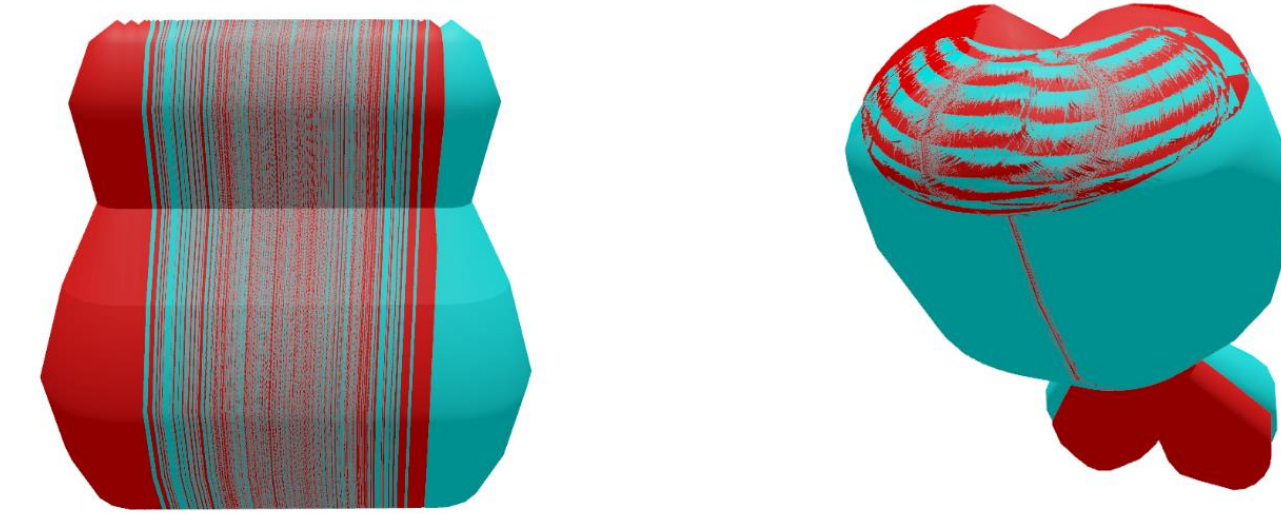
- Monte Carlo with 1000 samples as ground truth
- Metric: Wasserstein distance
- Collision-free and collision settings
- Test objects: Armadillo, Bunny, Pin
- With different initial conditions:
  - Sigma
  - Number of spheres
  - Method

Visual Comparison after Collision



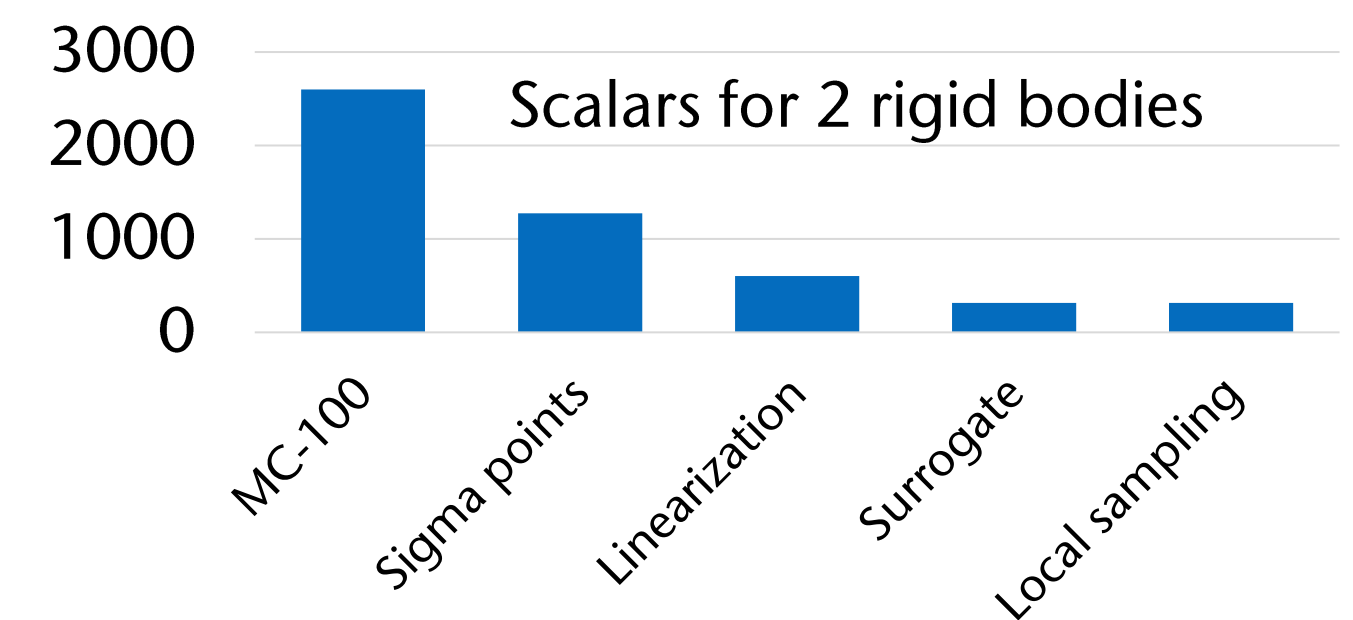
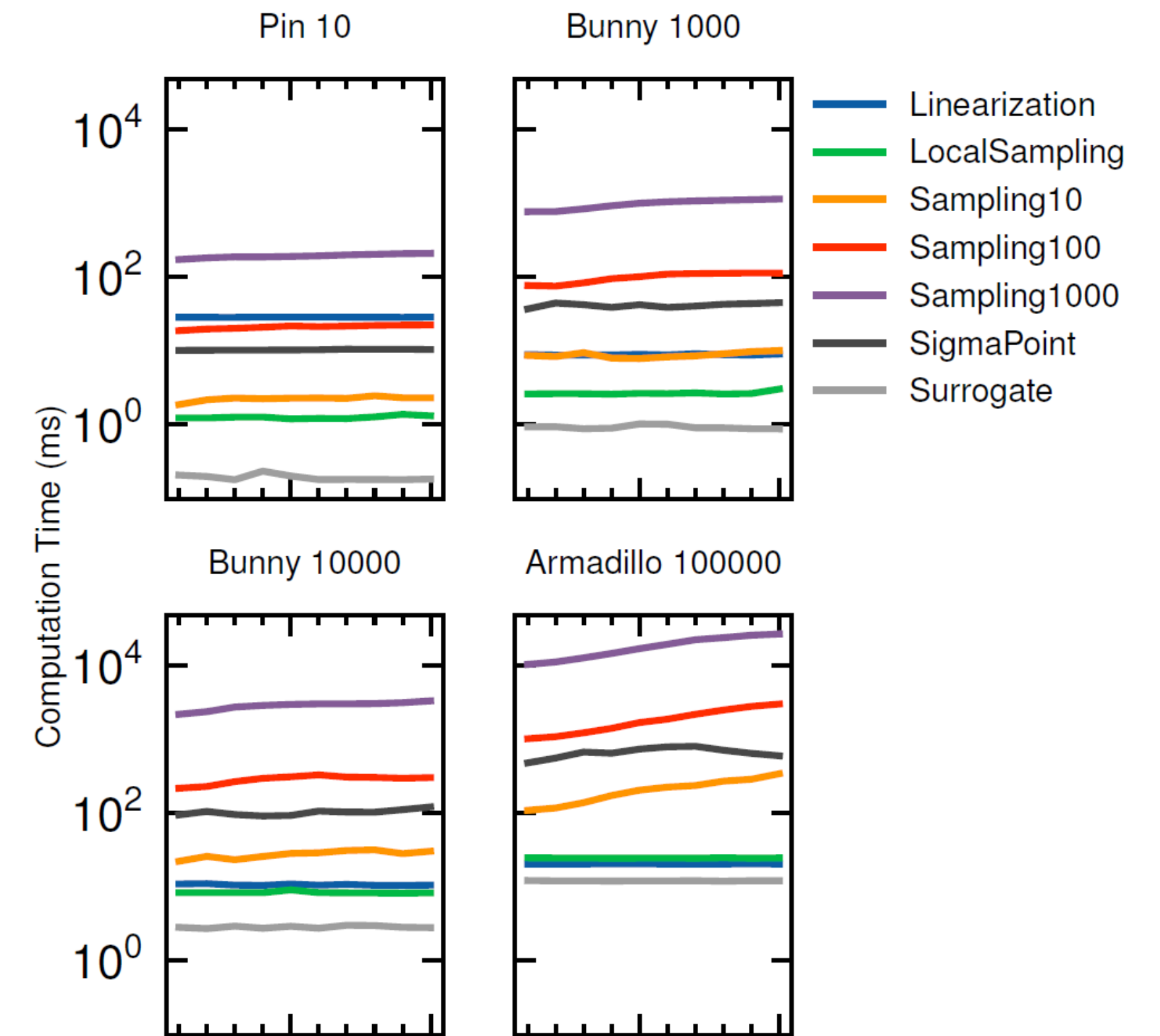
# Results: Quality/Distance wrt. Uncertainty

- Collision-free experiments validate the implementation
- In collision cases, error increases with larger uncertainty
- Persistent sigma points achieve the best overall position and rotation accuracy
- Linearization and local sampling perform well for position
- Rotation is more difficult than position



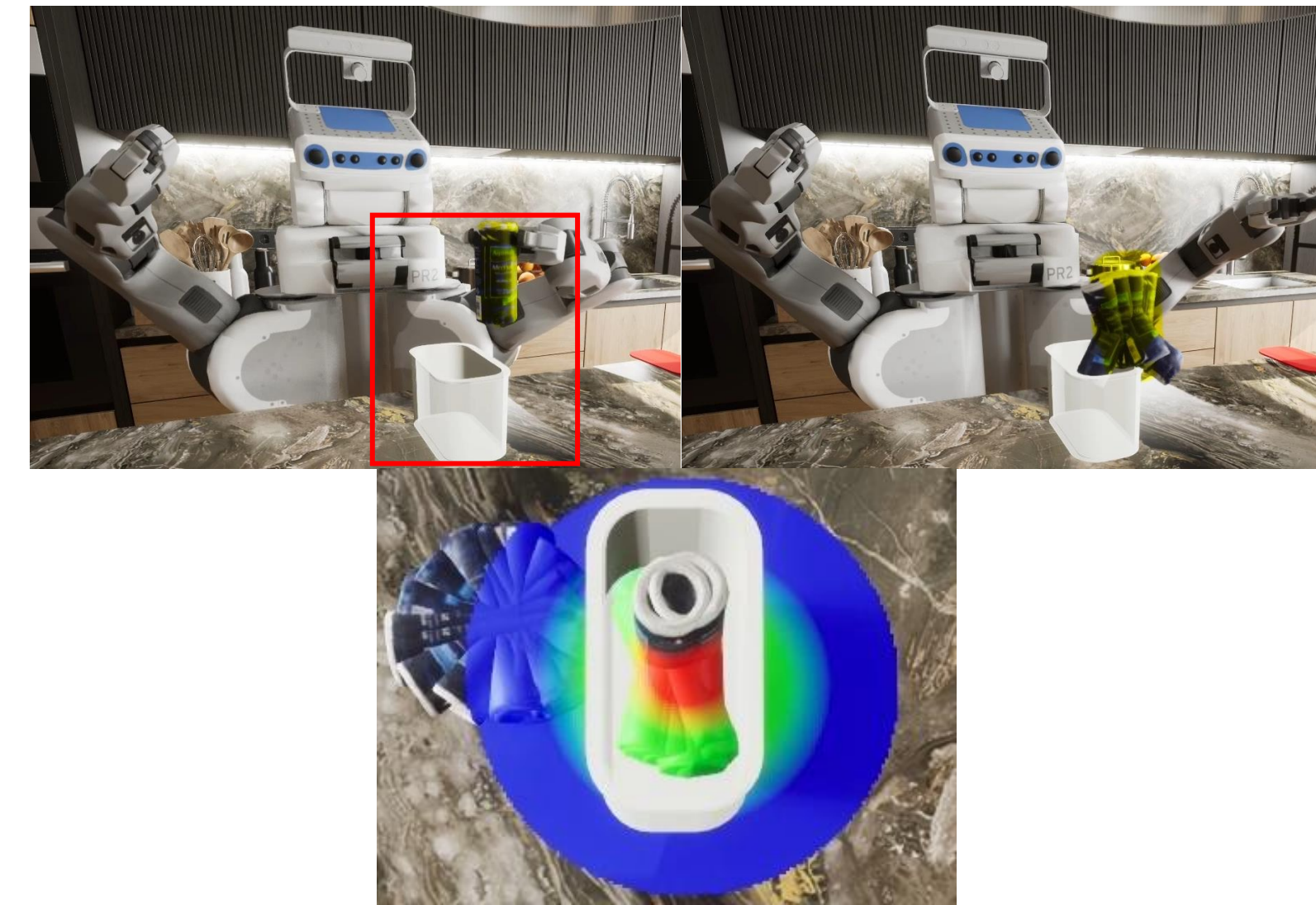
# Results: Runtime Performance and Storage

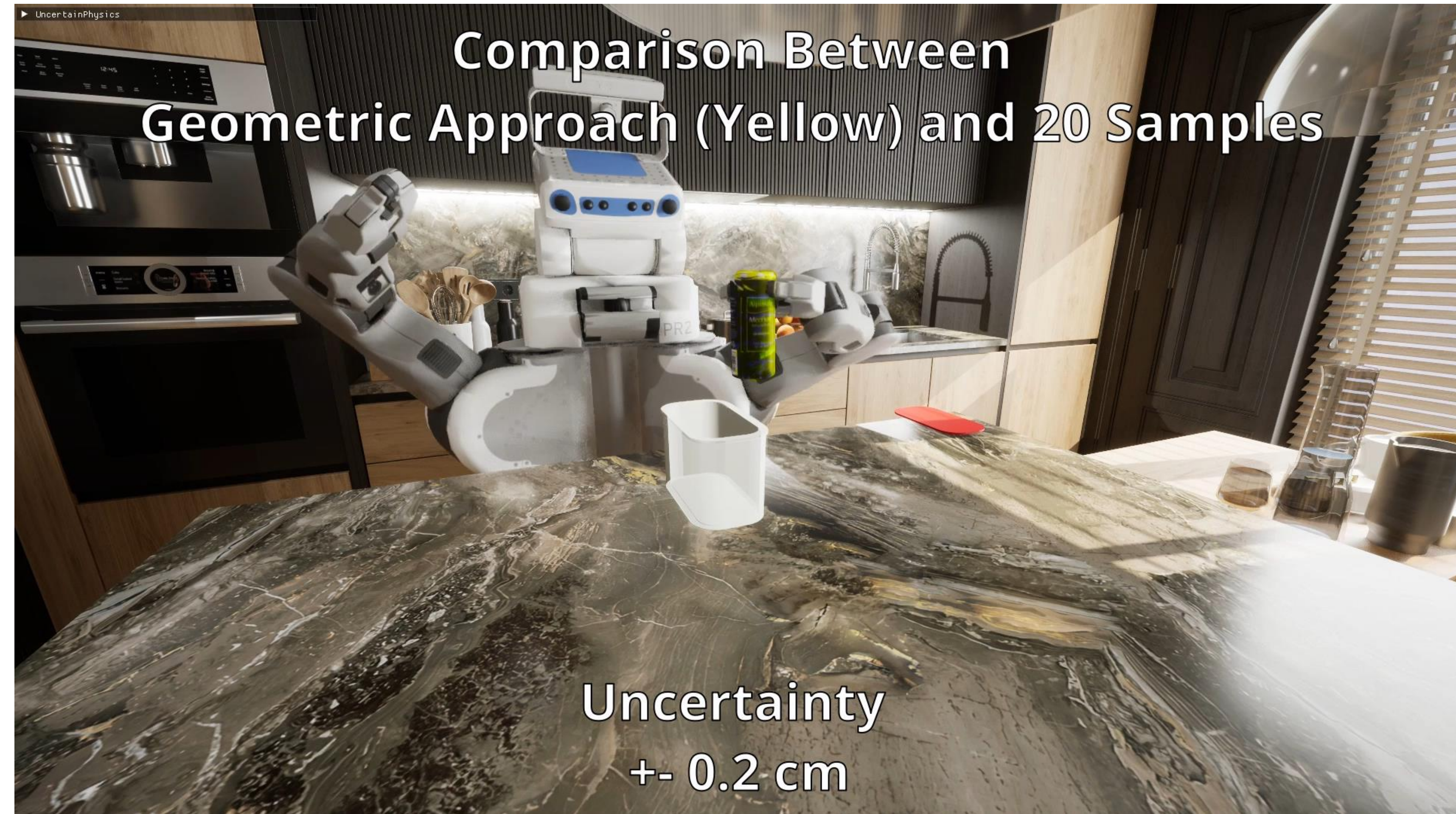
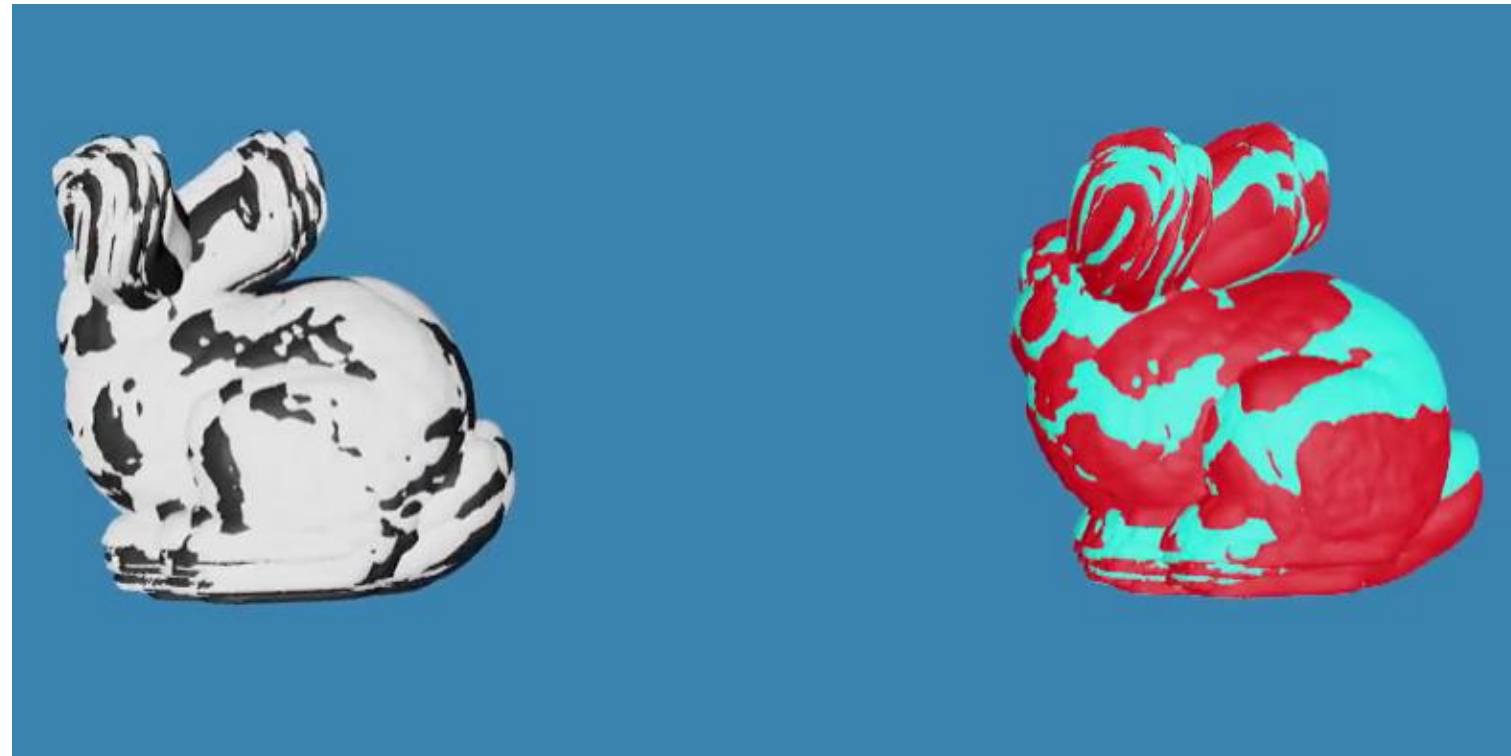
- Proposed methods are significantly faster than sampling baselines
- Surrogate model is fastest
- Sigma points are costliest among proposed approaches, but still much faster than high-sample MC
- Storage is lower than high-sample MC
- Per-body methods scale better in storage



# Conclusion and Future Work

- Up to 1-1.5 orders of magnitude faster
- Competitive accuracy with Monte Carlo
- Use cases: prediction, physical reasoning
- Future Work
  - Test more complex scenes beyond controlled two-body collisions
  - We plan to experiment with multi-modal distributions
  - Neural network to approximate uncertainty





{meissenhelter, fkenghag, weller, zach}@cs.uni-bremen.de