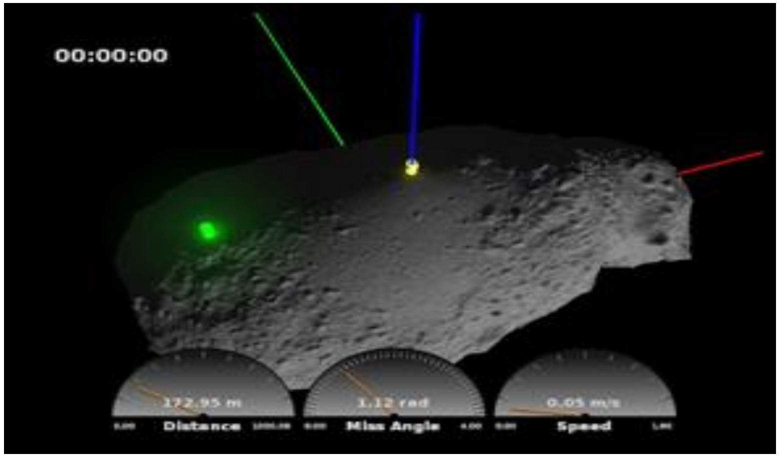
Fast and Accurate Simulation of Gravitational Field of Irregular-shaped Bodies using Sphere Packings

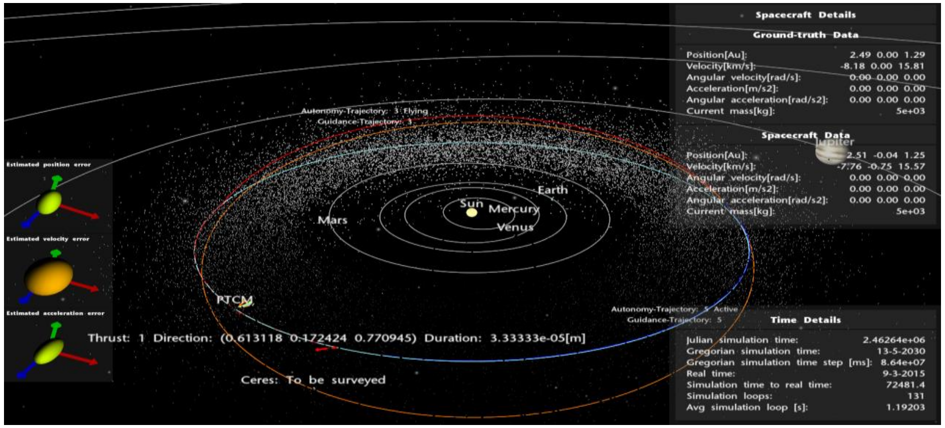
A. Srinivas, R. Weller, G. Zachmann
University of Bremen, Germany
cgvr.informatik.uni-bremen.de

ICAT-EGVE'17, Nov 2017, Adelaide, Australia

Motivation

- Surge in space missions to small bodies
- Cost and time efficient way to simulate missions in virtual test beds
- Planning optimal trajectories
  - Requires to compute gravitational acceleration several thousand times using onboard computers with limited capacity
- Physical simulation of gravitational field of small bodies
  - Complex irregularly shaped bodi





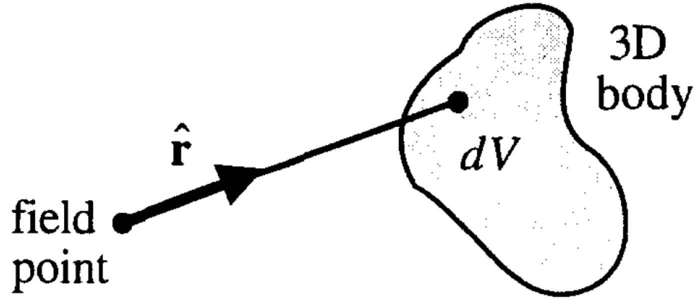
Universität Bremen

Universität der Bundeswehr München

DLR

Previous Work

- Modelling gravitational fields can be broadly classified into
  - Spherical harmonic methods
  - Polyhedral methods
  - Mascons (finite mass elements) methods (distribution of finite mass elements)
- Spherical harmonic methods are fast but suffer from accuracy issues within Brillouin sphere (LSOS simulator [NBC08])
- [WS96] provides a closed form solution to (Darts/Dshell [LIM09] & SEAS [BCJ11] from JPL)
  - Computationally expensive
  - Applicable only to bodies with constant density

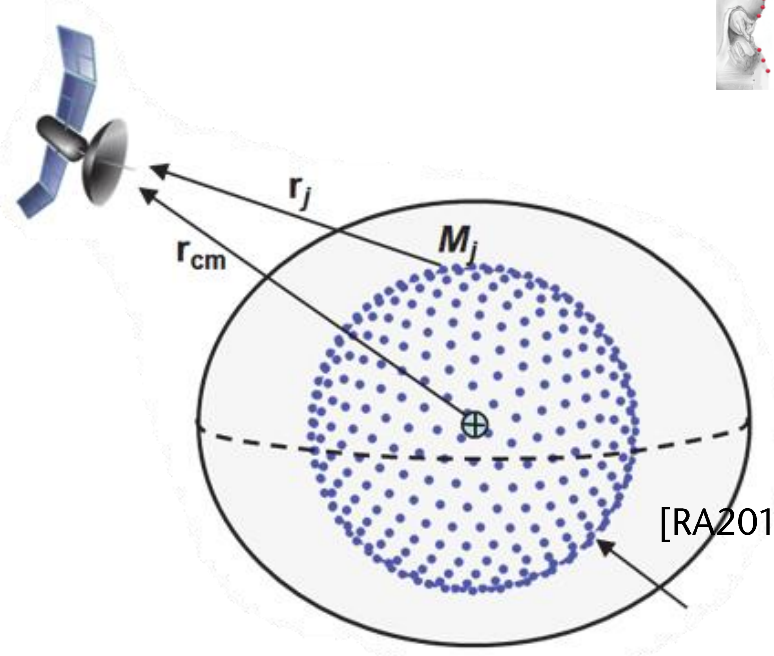


$$\hat{\mathbf{r}} = \hat{\mathbf{i}}\frac{\Delta x}{r} + \hat{\mathbf{j}}\frac{\Delta y}{r} + \hat{\mathbf{k}}\frac{\Delta z}{r}$$

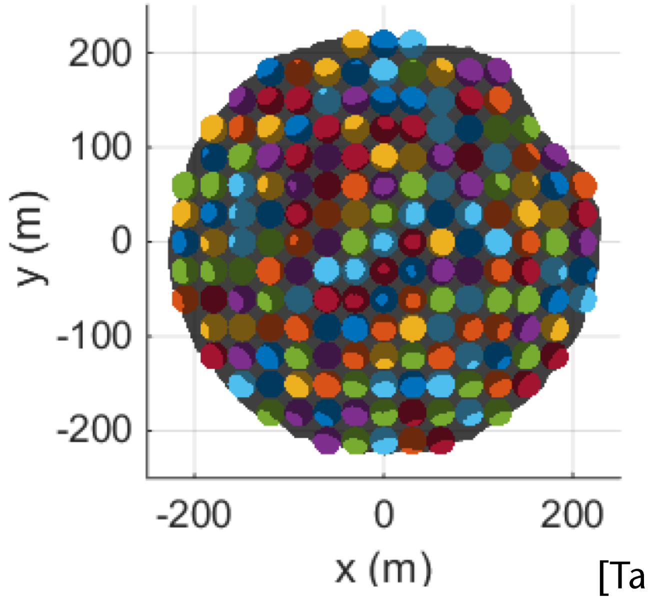
[WS96]

Previous Work

- Mascon based methods use point mass concentrations
- Simple computation and easy to parallelize
- [RA2012] usually need large number of mascons and only suitable for almost spherical shaped bodies
- [PWB08] uses both, cubes and spheres as mascons with simple space filling arrangements
  - Errors due to shape or voids
- [Tar16] uses trivial spheres arrangement with voids still contributing for the errors



[RA2012]



[Tar2016]

Contribution

- Utilizes better spheres arrangement algorithm (Protosphere) with higher object packing ratio and novel methods for distributing total mass among these spheres
- Provides level of detail representation for gravitational field
- Out performs the traditional polygon based method by 2 orders of magnitude
- Able to generate gravitational field of polyhedral model with relative difference of < 0.3%
  - Three times lower relative difference than other mascon based methods
- Extendable to bodies with variable densities

Mascon Method Basics

- Subdivide the body into smaller parts called mascons
  - Spheres, voxels, points etc.
- Compute the acceleration due to each mascon and accumulate the accelerations

```
forall mascons i in parallel do   // n threads
```

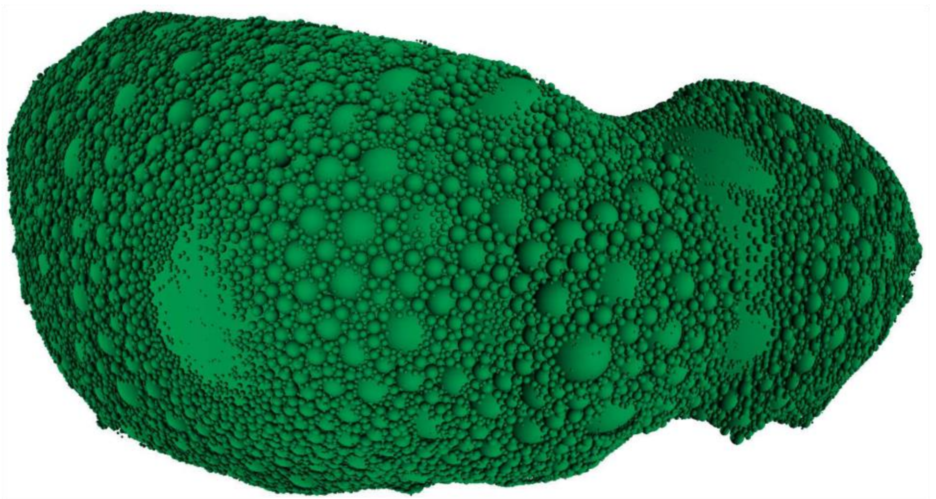
$$\boldsymbol{g}_i = \frac{Gm_i}{||\boldsymbol{r}_i||^3}\boldsymbol{r}_i$$

```
parallel scan over all
```
$\boldsymbol{g}_i$

- Arrangement of mascons and mass assignment methods are crucial for accuracy

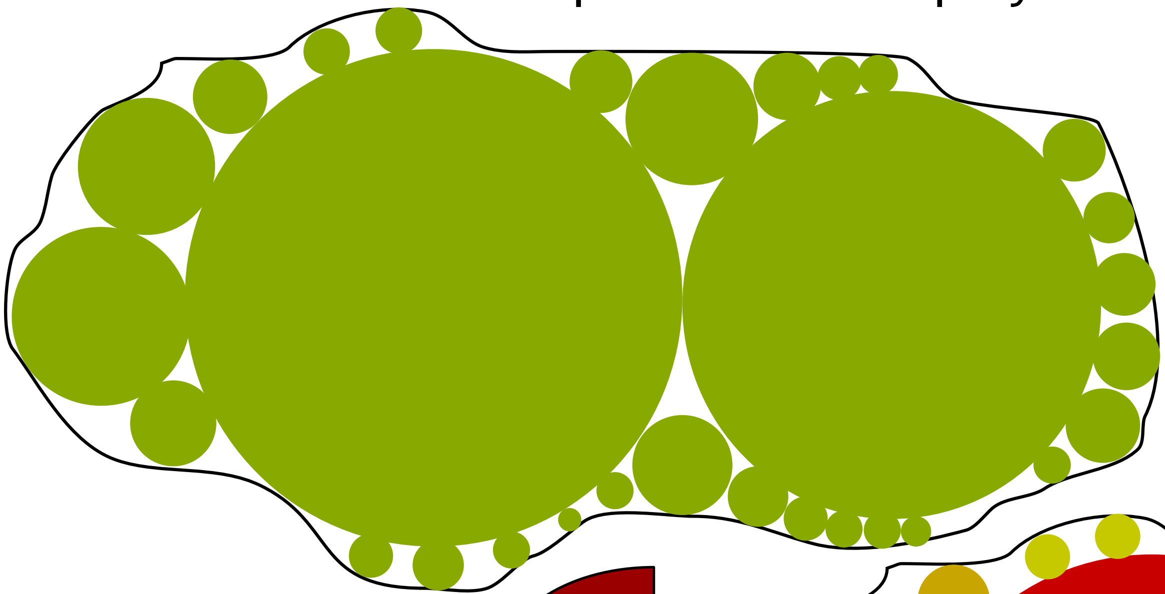
Mascons Arrangement & Representation

- Sphere mascons
  - Point mass computation (simple)
  - Computational efficiency
- Kepler conjecture: Uniform spheres can cover < 75% of space
- Protosphere algorithm [Weller2010]
  - Extends the idea of Apollonian sphere packings to arbitrary 3D objects
  - Greedily fills objects with the largest possible spheres (achieves ~90% packing fraction)
  - Level of detail representation



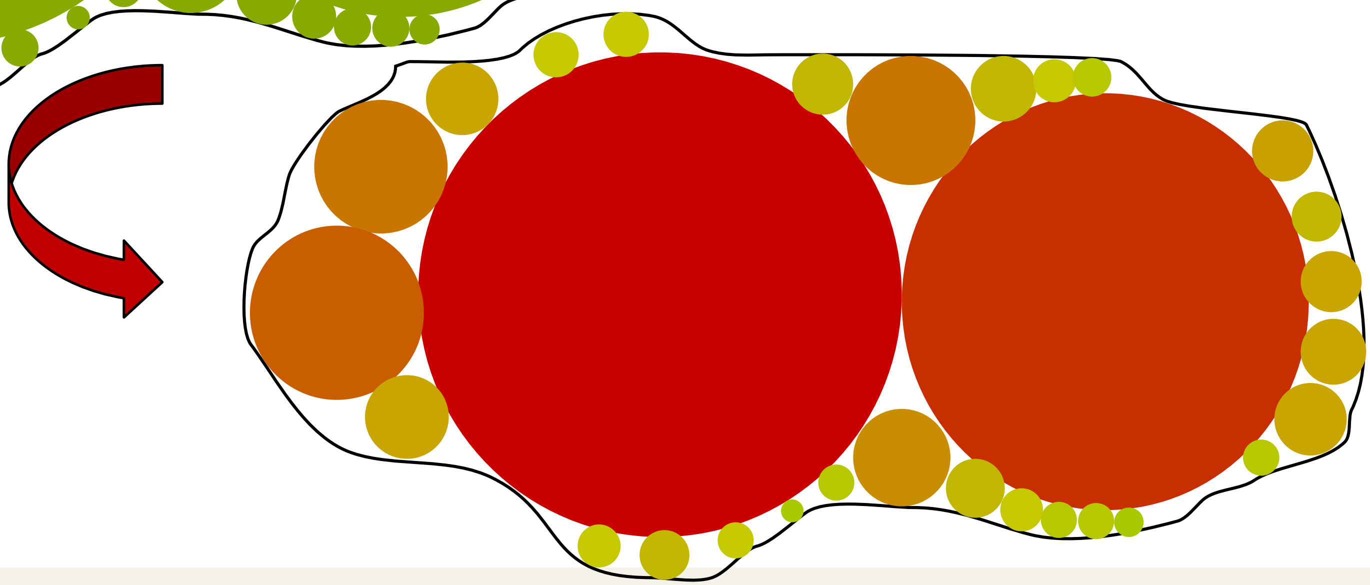
Volume Proportional Method

Objective is to distribute total mass among the spheres such that relative difference is as low as possible to the polyhedral model gravitational field

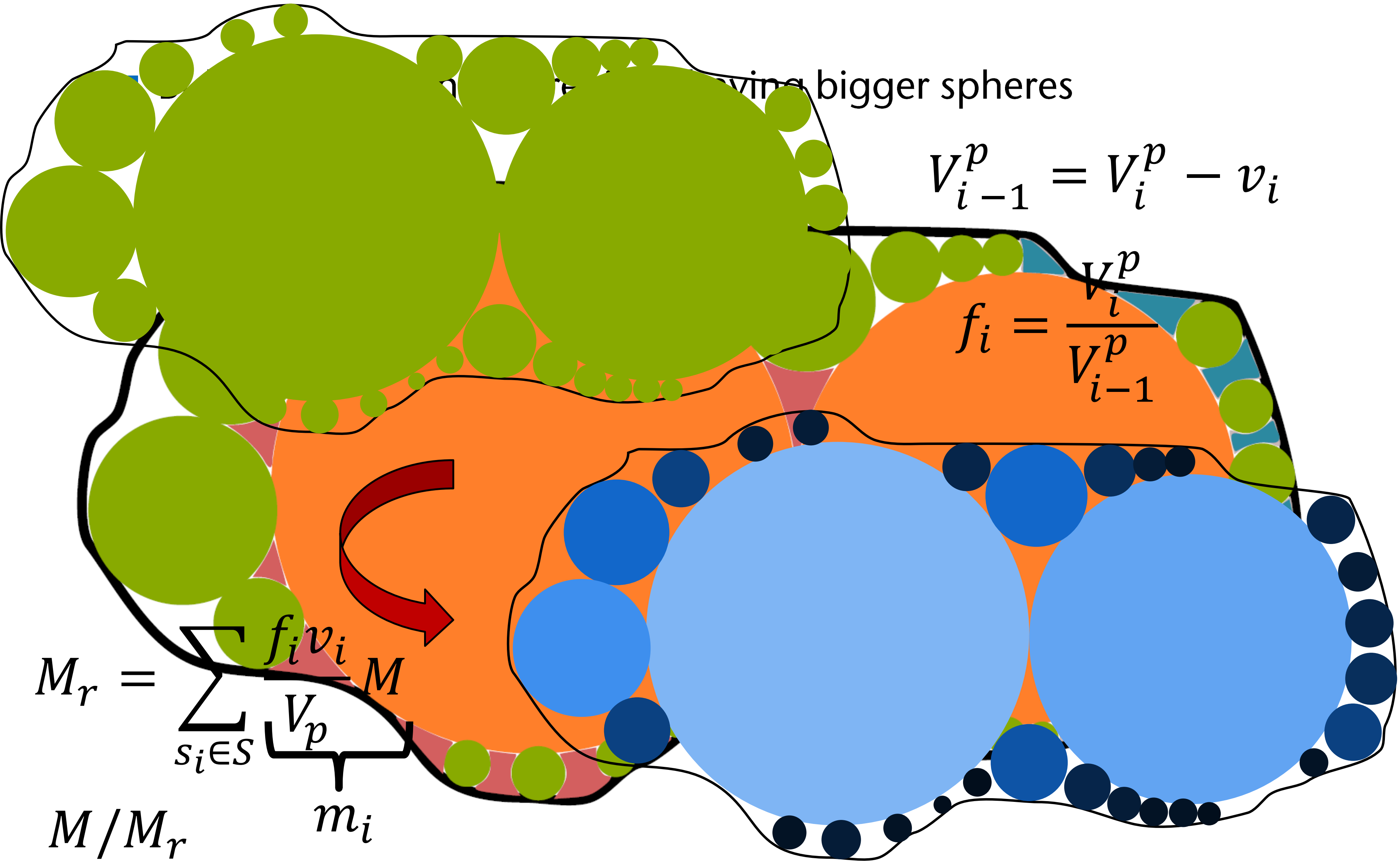


$$V_S = \sum_{s_i \in S} v_i$$

$$m_i = M \frac{v_i}{V_S}$$



Bulk Density Uniformity Method



$$V_{i-1}^{p} = V_i^{p} - v_i$$

$$f_i = \frac{V_i^{p}}{V_{i-1}^{p}}$$

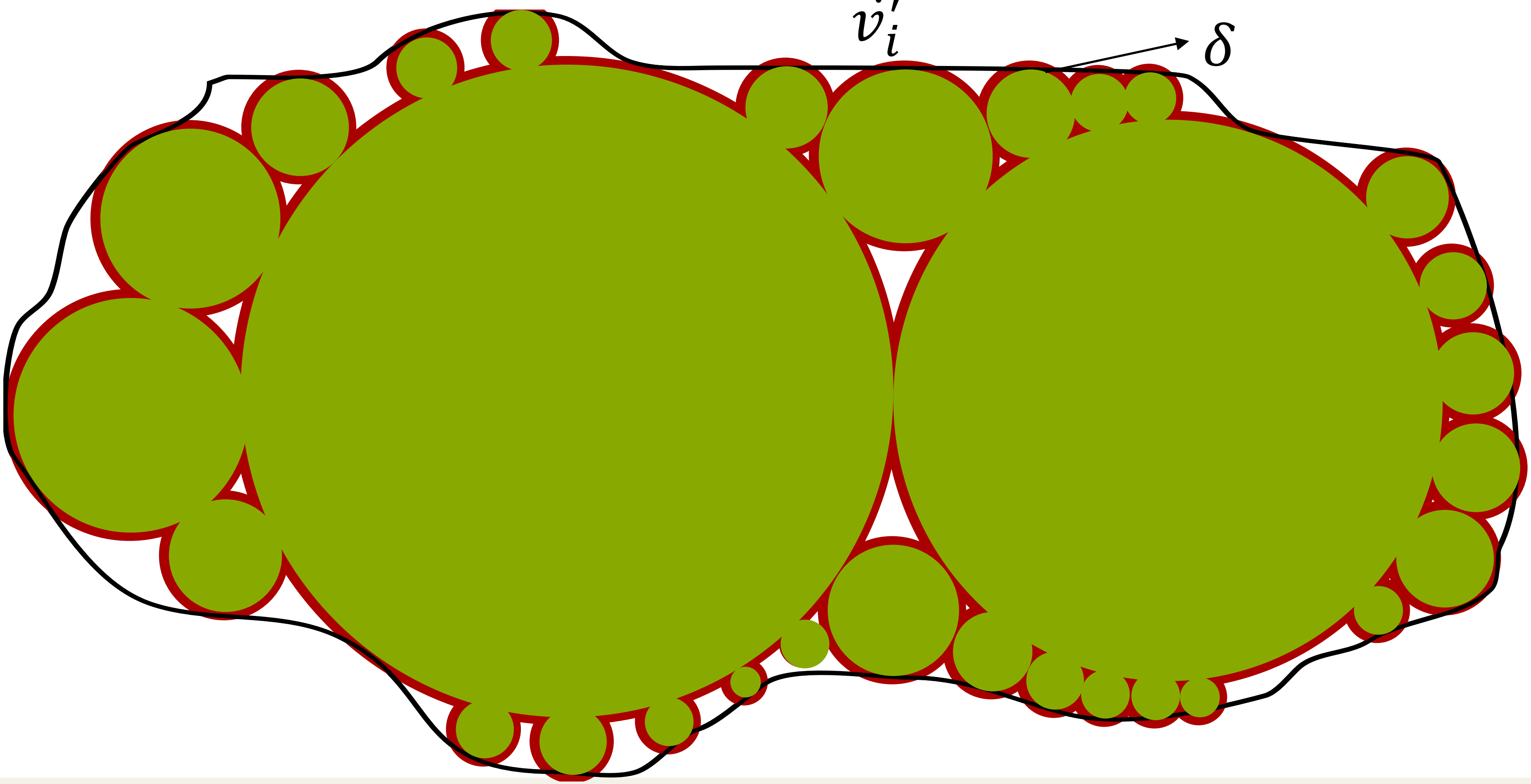
$$M_r = \sum_{s_i \in S} \underbrace{\frac{f_i v_i}{V_p} M}_{m_i}$$

$$M / M_r$$

Delta Radius Increase Method

$$V_p = \sum_{i=1}^{n} \underbrace{\frac{4\pi(r_i' + \delta)^3}{3}}_{v_i'}$$

$$m_i = \frac{M}{V_s} v_i'$$

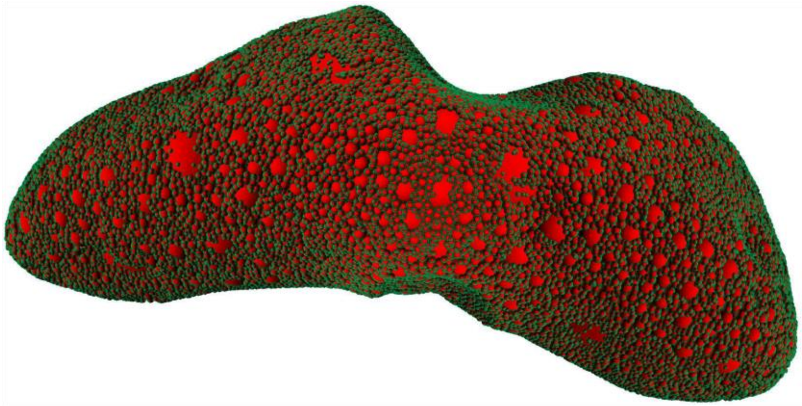


Delta Percentage Volume Increase Method

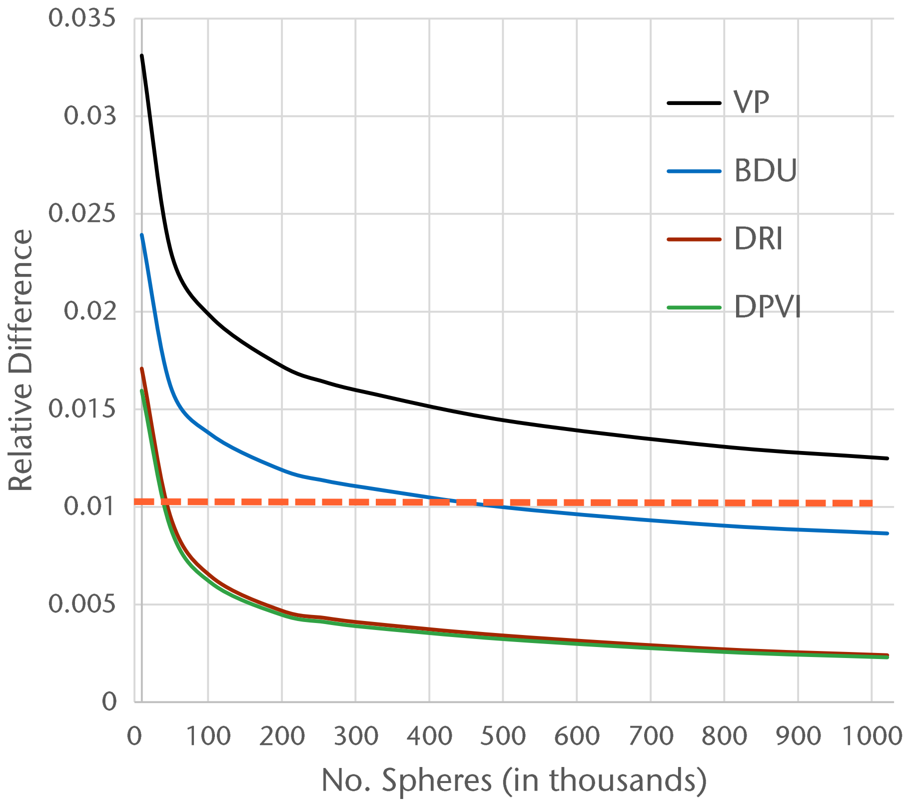
- Similar to the DRI method
- Increase volume of spheres by fraction inversely proportional to their current sphere radii

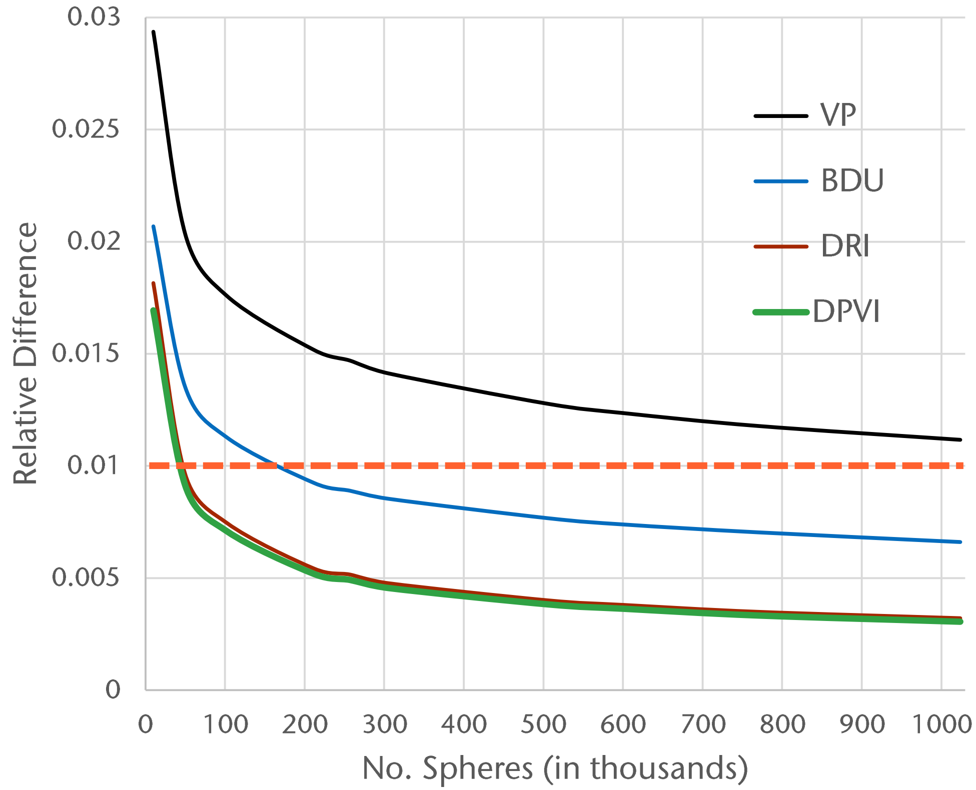
$$x = \frac{V_p - \sum_{i=0}^{n-1} v_i}{\sum_{i=0}^{n-1} v_i / r_i}$$

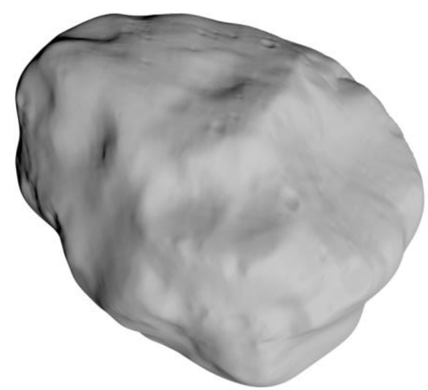
$$\frac{4\pi(r_i + \Delta r_i)^3}{3} = v_i + v_i x / r_i$$



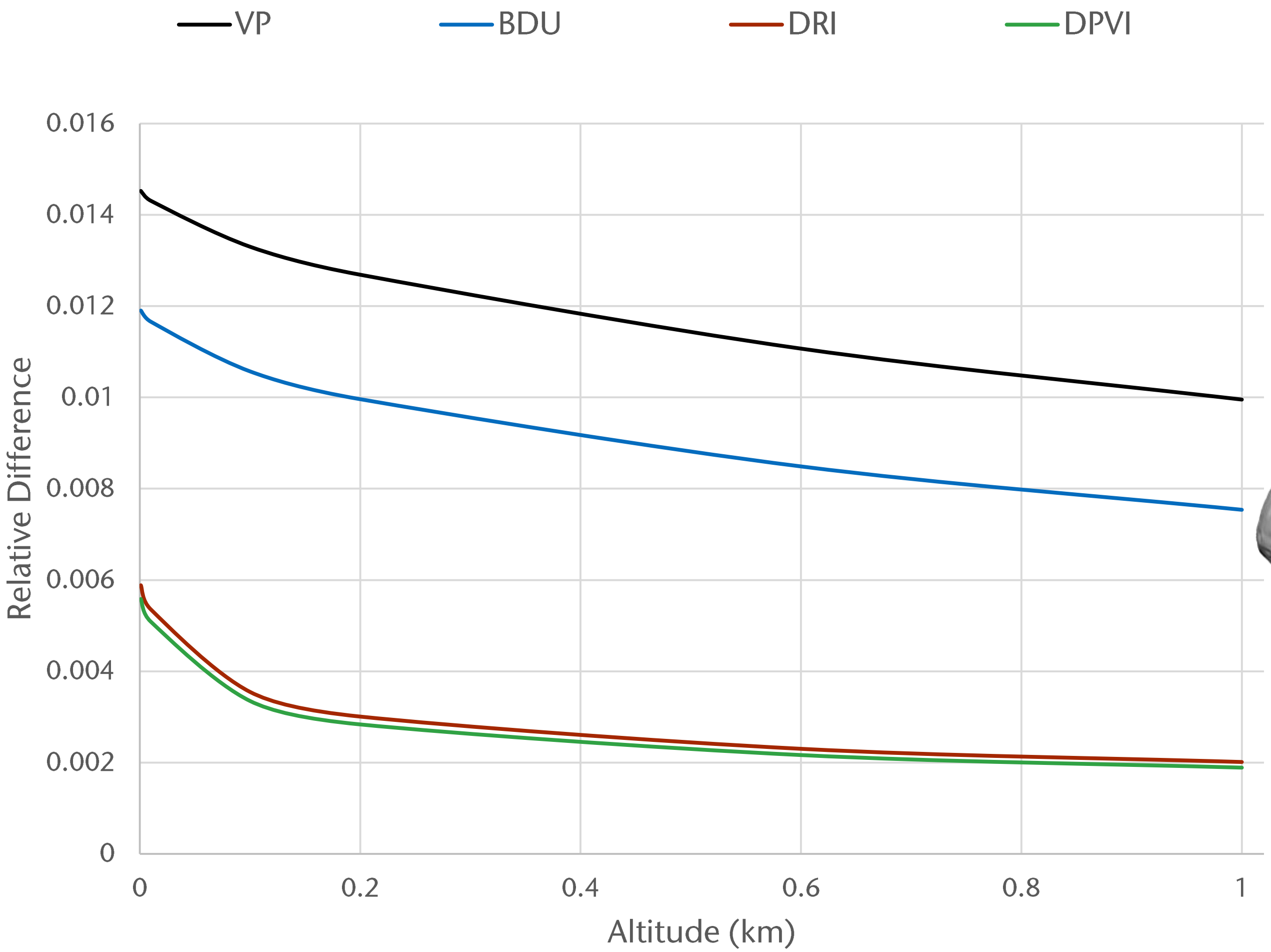
Results: Accuracy (1m Above Surface)

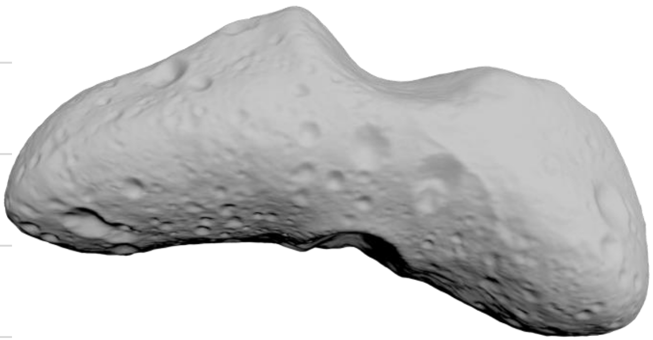






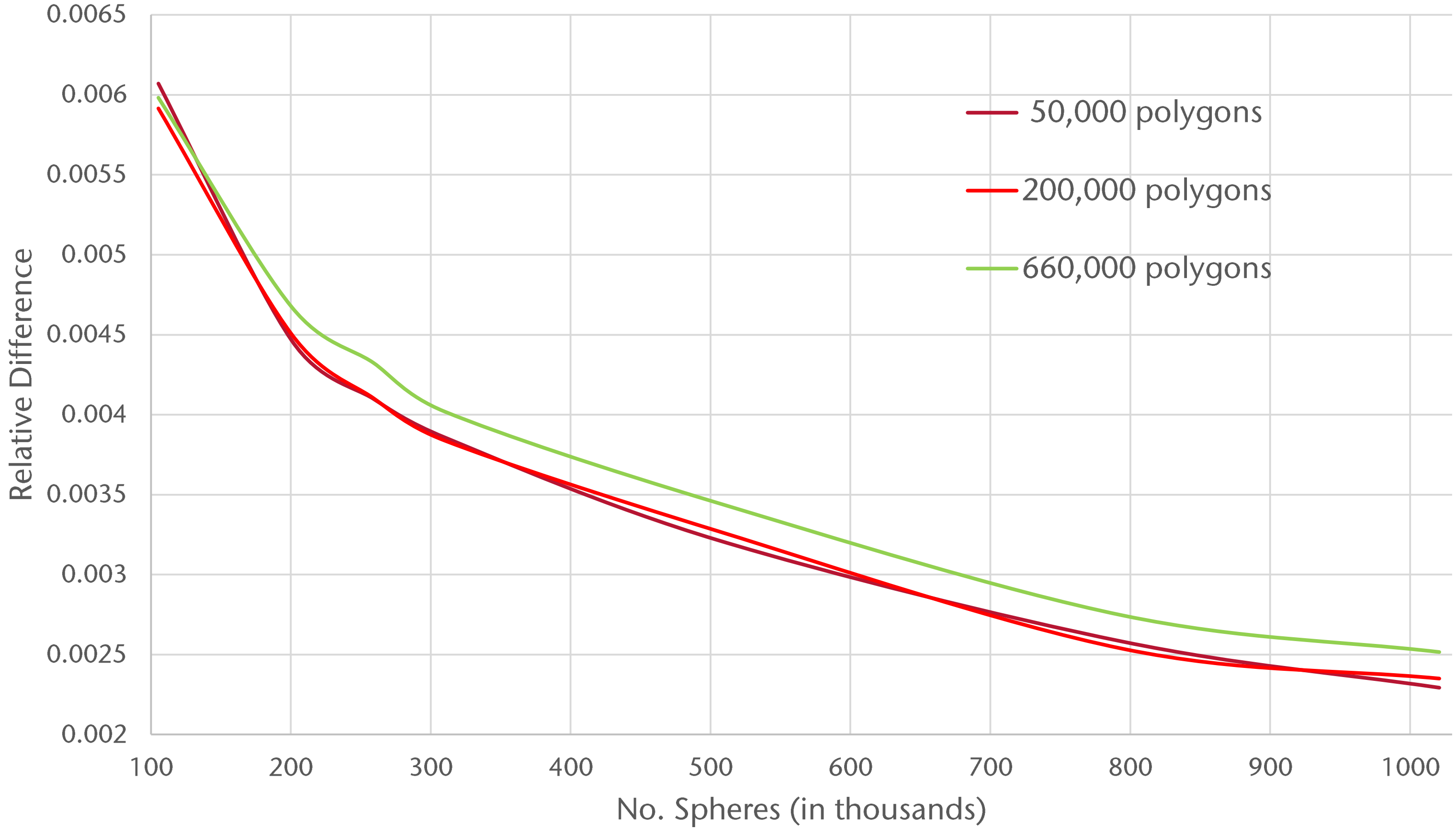
Results: Accuracy (Different Altitudes)





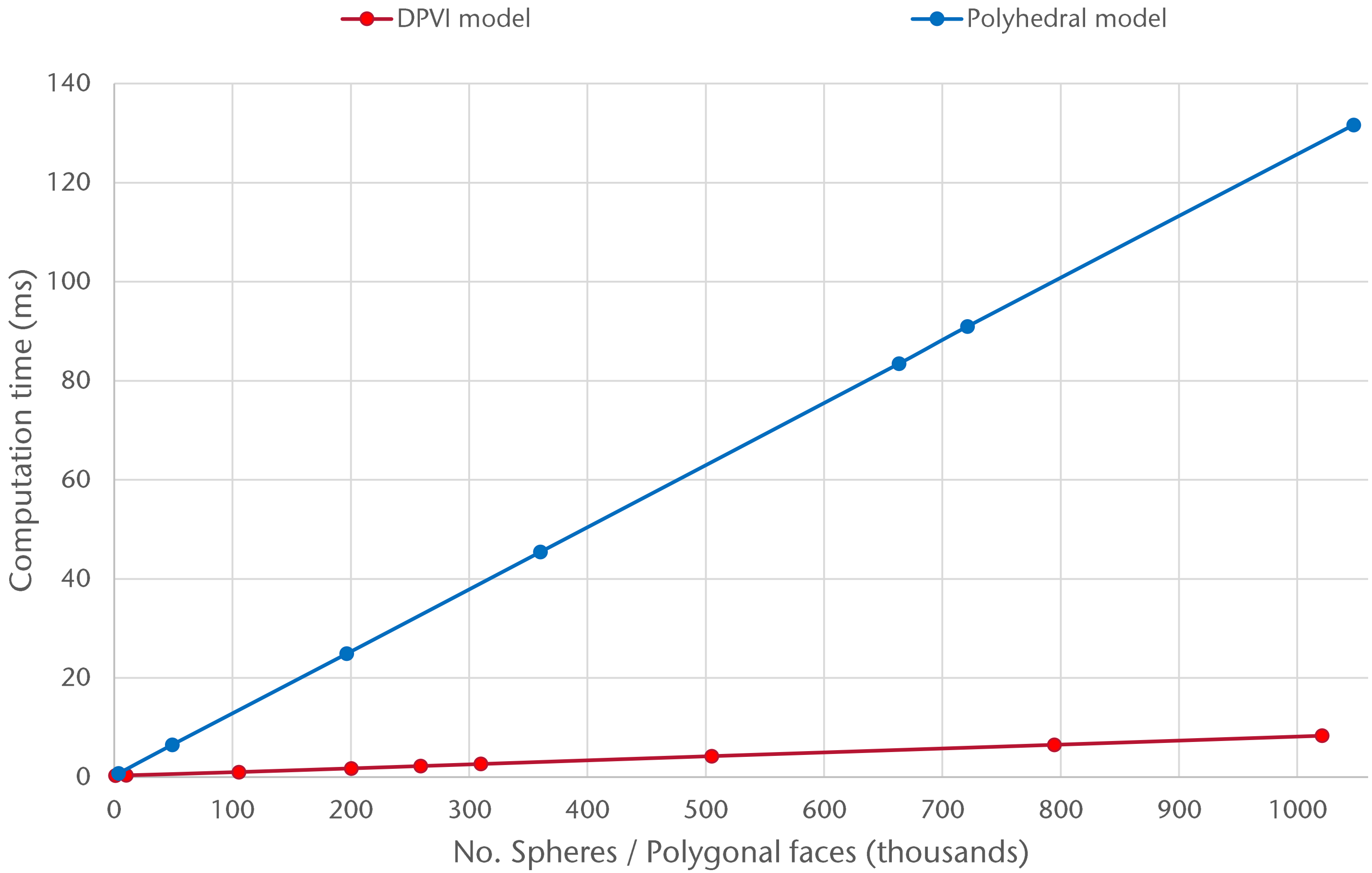
Accuracy at different altitudes from Eros surface (300k spheres)

Results: Accuracy (Different Resolution Shape Models)



Accuracy of DPVI method w.r.t different resolution shape models of Itokawa

Results: Performance Comparison



Time taken to compute acceleration at a given point

Conclusion

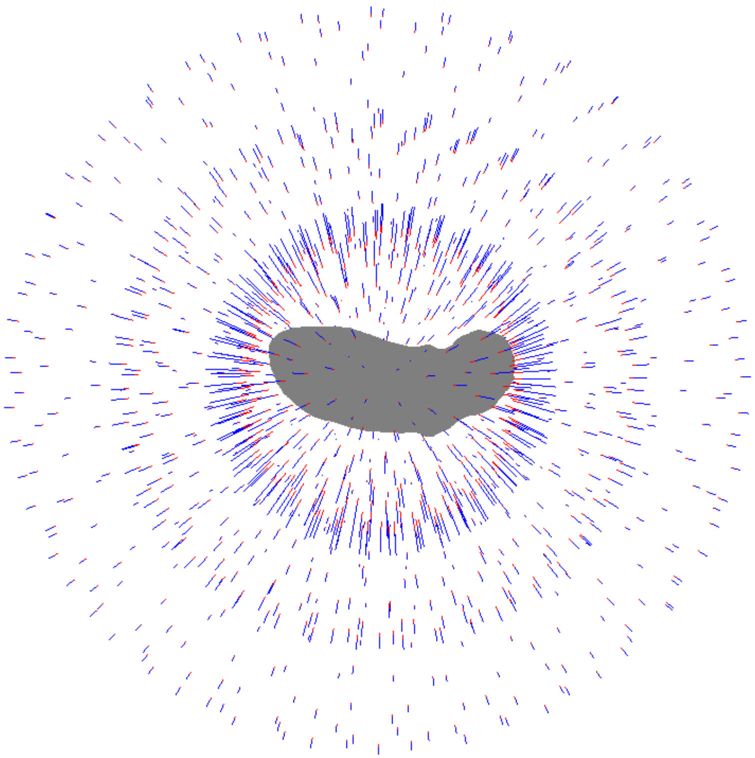
- A novel massively parallel algorithm to compute the gravitational field for arbitrary shaped small bodies like asteroids
- Four methods to distribute the asteroid's total mass on to spherical mascons were presented
- Out performs the traditional polygon-based method by a factor of ~300 while generating a gravitational field with a relative difference of < 0.3%
- Three times higher accuracy with the same number of mascons in comparison to other mascon models in the literature
- Perfectly suited for physically-based simulation of space missions in virtual test beds

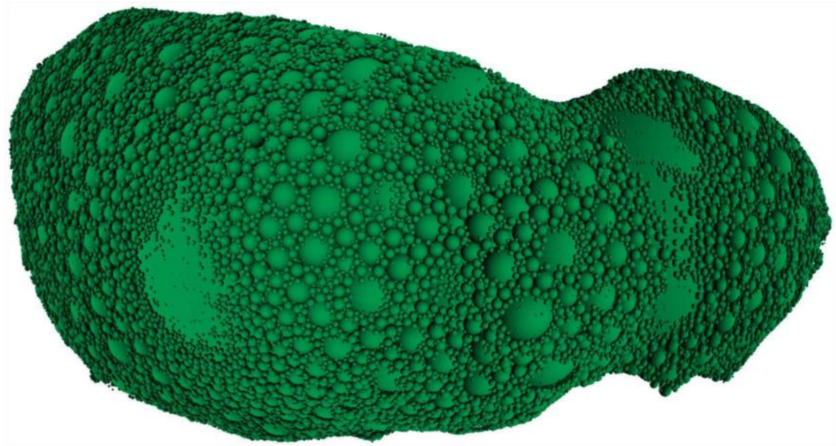
Bremen

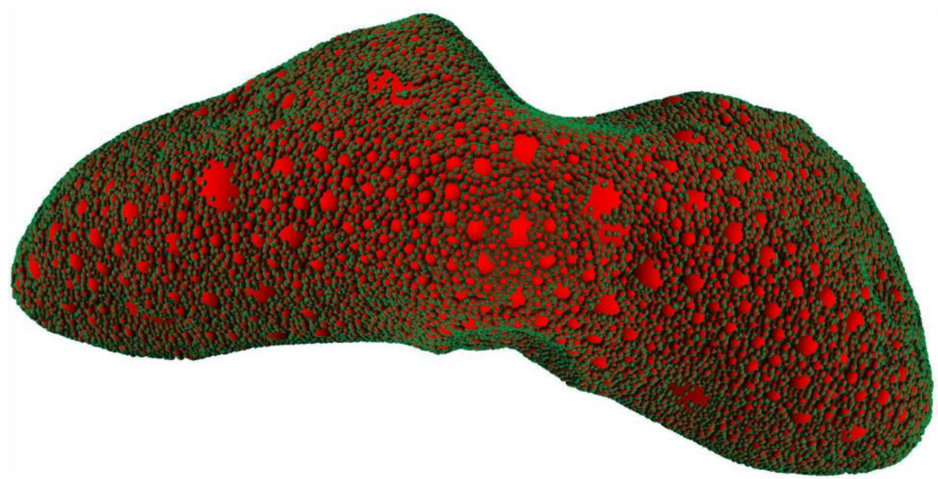
Future Work

- Investigate the sphere arrangement other than greedy sphere packing of Protosphere algorithm
  - Optimization criteria to influence the arrangement of the spheres in the packing
- Solving of inverse problem i.e. to compute a mass distribution for a measured gravitational field of a small body
  - Sphere packing approach in combination with an appropriate mathematical optimization scheme

Thank you!
Any Questions?







BMWi grant 50NA1318 & DFG grant SFB 1320/R03