Comparing Methods for Gravitational Computation: Studying the Effect of Inhomogeneities

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Session TP13 - Planetary Dynamics: Shape, Gravity, Orbit, Tides, and Rotation from Observations and Models

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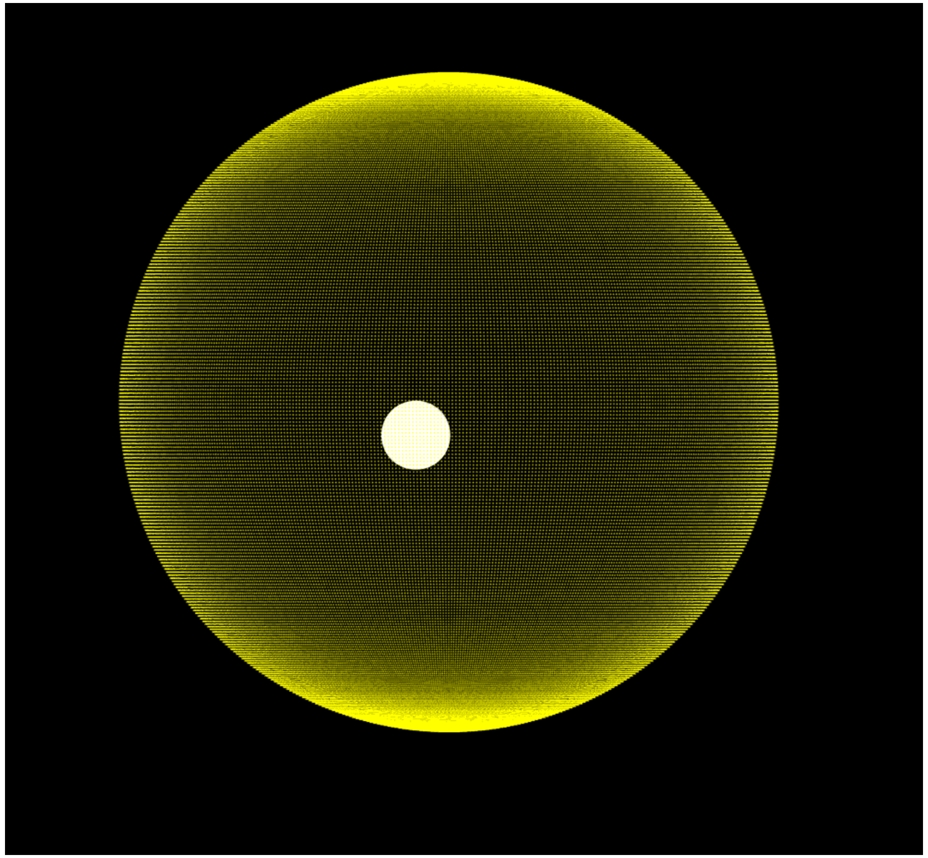

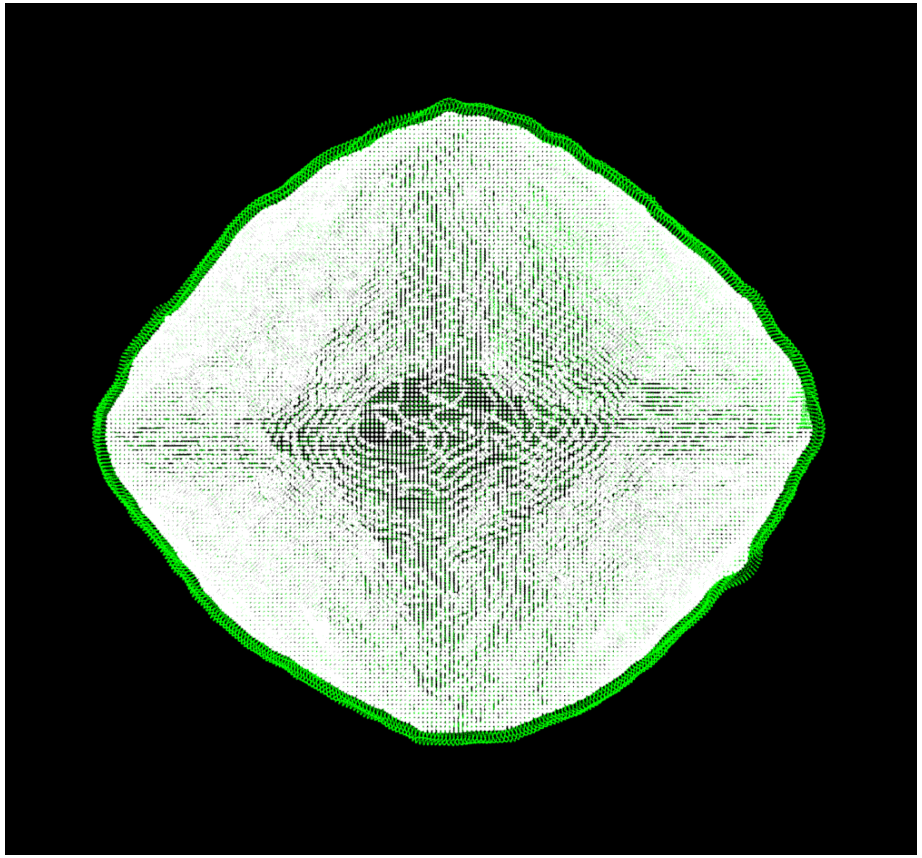

Credit Shape: OSIRIS-Rex team

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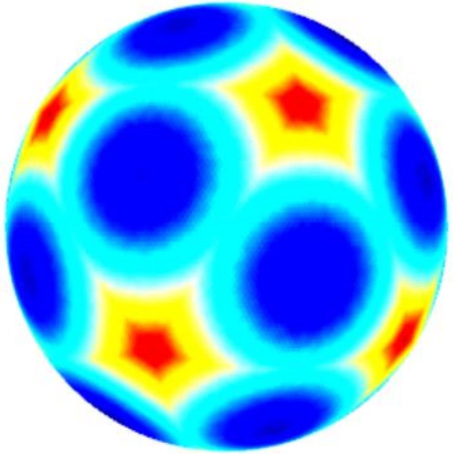
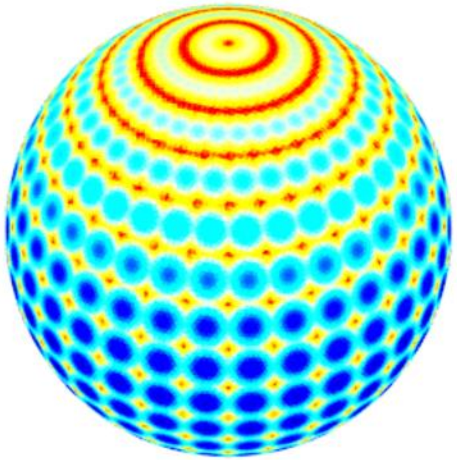
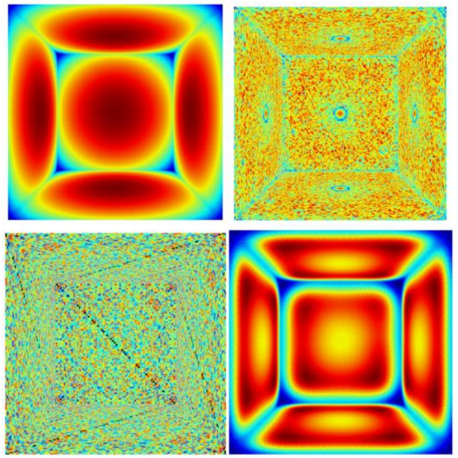
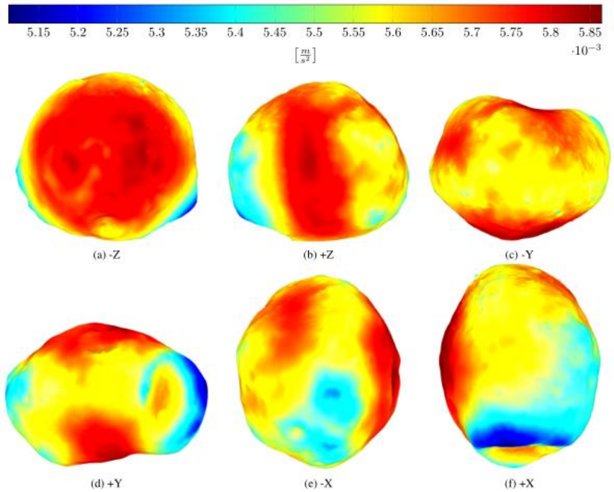
Motivation

- Various methods to model the gravitational field exist with different advantages and disadvantages
- Triggered by the development of the GRASS surface gravimeter for the ESA Hera mission (See talk by *B. Ritter* in MITM7 on Thursday), an international collaboration was formed to *compare* three different gravitation computation methods
- Here, focus was laid small, irregular Solar System bodies, with measurements on the surface (largest error, surface gravimeter)
- Thus far, only **homogeneous** case considered, **inhomogeneous** case presented today (+ongoing)







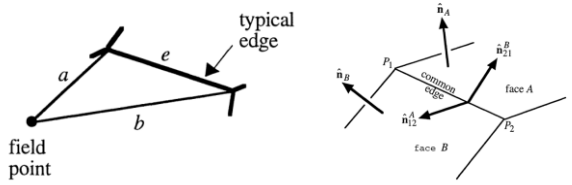


Credit: Meißenhelter, Hermann, et al (2022) IEEE Aerospace Conference (AERO). IEEE.

Comparing 3 Methods

Polyhedral Method (PM)

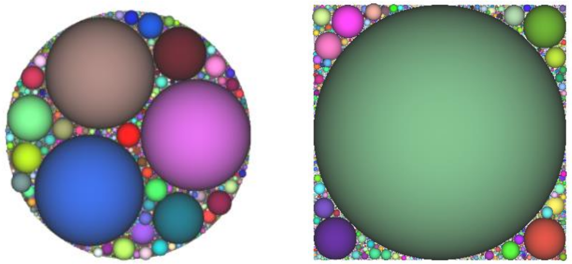
- Closed form analytical solution for gravitation of polyhedral by Werner & Scheeres (1996)
- Original form demands constant density ρ
- Found most precise in homogeneous case (e.g. cube), but expensive



Credit: *Werner & Scheeres (1997)*

Mascons: Sphere Packing (MSP)

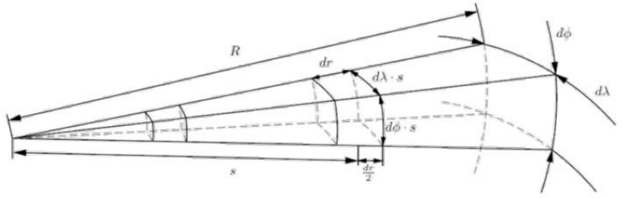
- Non-uniform sphere packing
  - Space-filling for an infinite number of spheres
  - A natural choice between accuracy and performance
- Fast computation with easy parallelization (Srinivas et. al., 2017)



Credit: *Meißenhelter et al. (2022)*

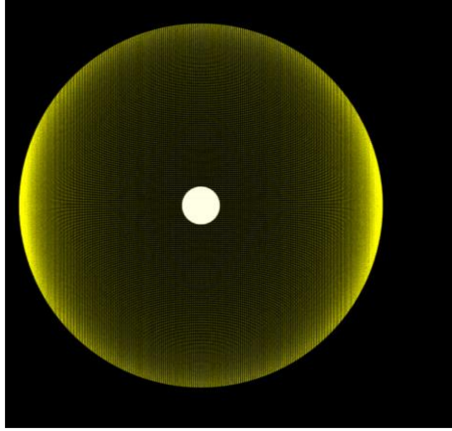
Mascons: Spherical coordinates (MASC)

- Divides the shape into subvolumes of adjustable size using spherical coordinates.
- Assigns a specific density to each partial volume.
- Sums over all the mass elements (parallel implementation) to calculate the gravity coefficients and acceleration at specific points



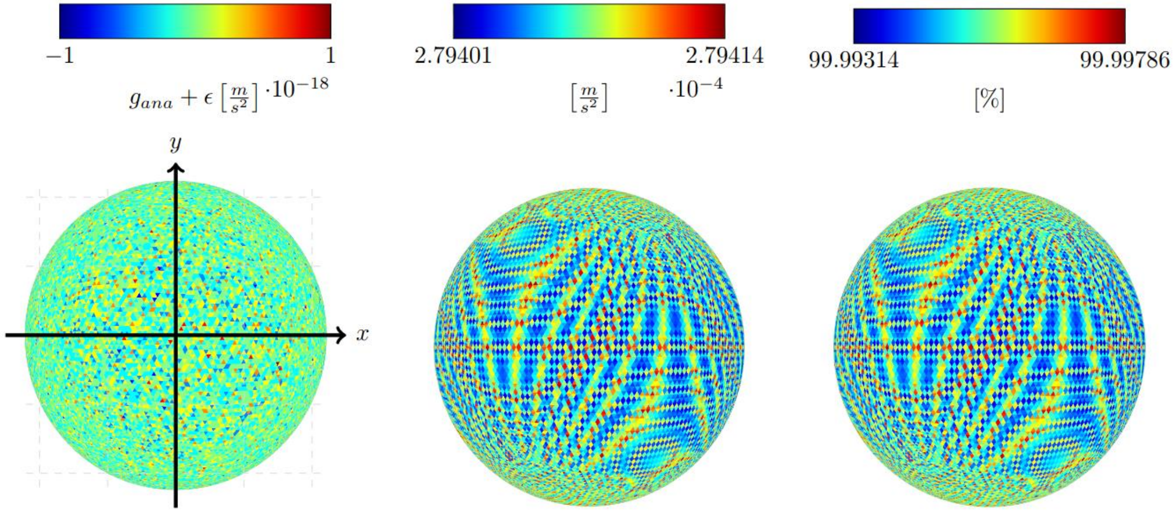
Credit: *Pätzold and Andert, et al., 2016*

Experiment 1: Sphere with central Core

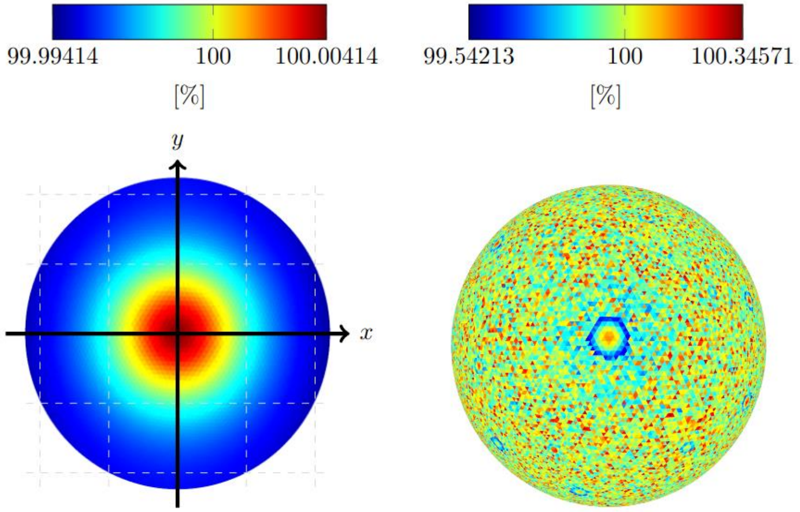


- Approximate perfect spheres with two UV-sphere (328,328 facets), core at center. r=1,000 m and a density of 1.0g/cm³. Inner sphere r=100 m and a density of 0.5 g/cm³
- PM error always <100 %, as UV-sphere lies inside
- Mascons overshoot locally >100 %
- MASC vs. MSP shows completely different behaviour, MSP has largest spread

Case	Method	Min. [%]	Mean [%]	Max. [%]	σ [%]
I	PM	99.993143	99.995461	99.997864	0.000942
	MSP 800k	99.542132	99.996047	100.345711	0.117449
	MASC 64.8k	99.994138	99.995459	100.004137	0.001201



(a) Analytic solution. (b) Solution with polyhedral method. (c) Relative error between solutions.



(a) MASC with core at center. (b) MSP with core at center.

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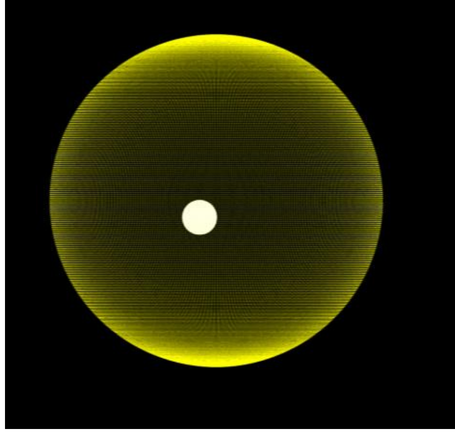
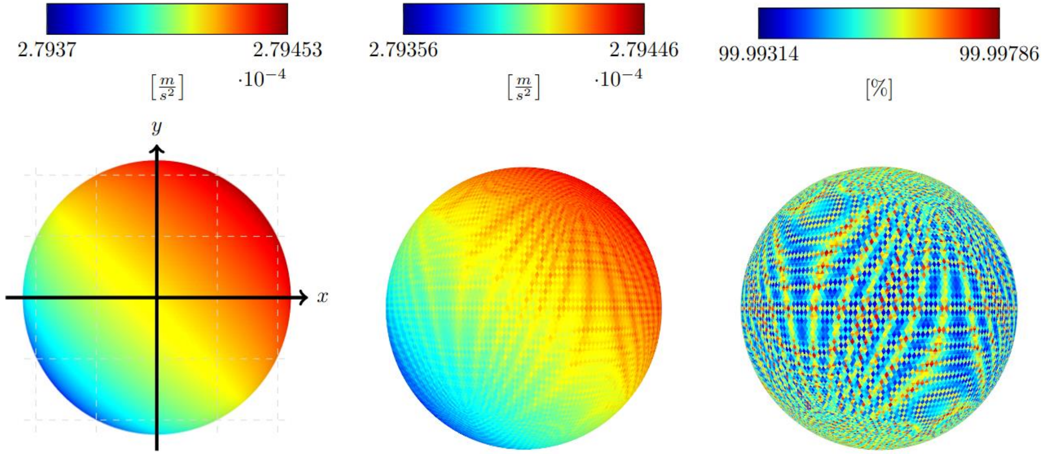
Experiment 2: Sphere with off-centre Core



- Approximate perfect spheres with two UV-sphere (328,328 facets), core off-center (X−r | Y−r) r=1,000 m and a density of 1.0g/cm³. Inner sphere r=100 m and a density of 0.5 g/cm³
- PM error always <100 %, as UV-sphere lies inside, PM (and MASC) indifferent of core position!
- Spread for MSP smaller (mean comparable, but worse agreement here)
- Mascons overshoot locally (>100 %)

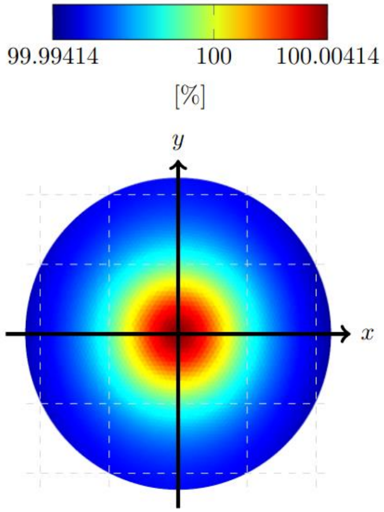
Case	Method	Min. [%]	Mean [%]	Max. [%]	σ [%]
II	PM	99.993143	99.995461	99.997864	0.000942
	MSP 800k	99.590773	99.995998	100.307859	0.111008
	MASC 64.8k	99.994137	99.995459	100.004158	0.001201



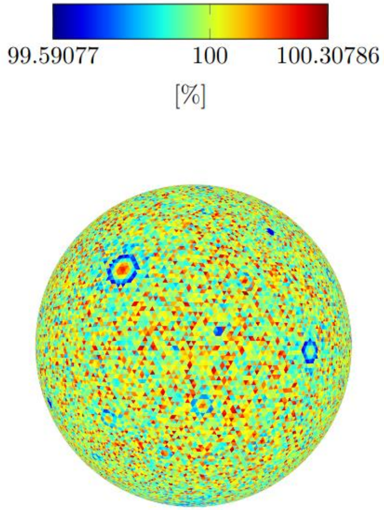
(a) Analytic solution.

(b) Solution with polyhedral method.

(c) Relative error between solutions.



MASC with core



(c) MSP with moved core.

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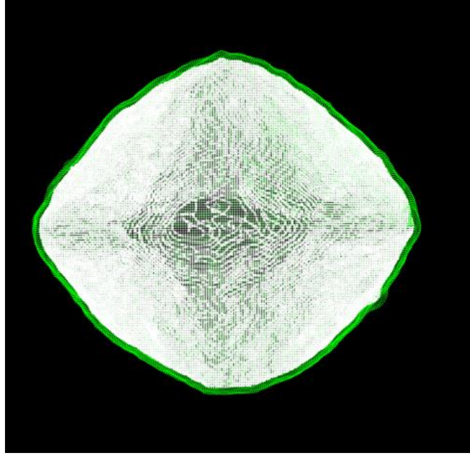
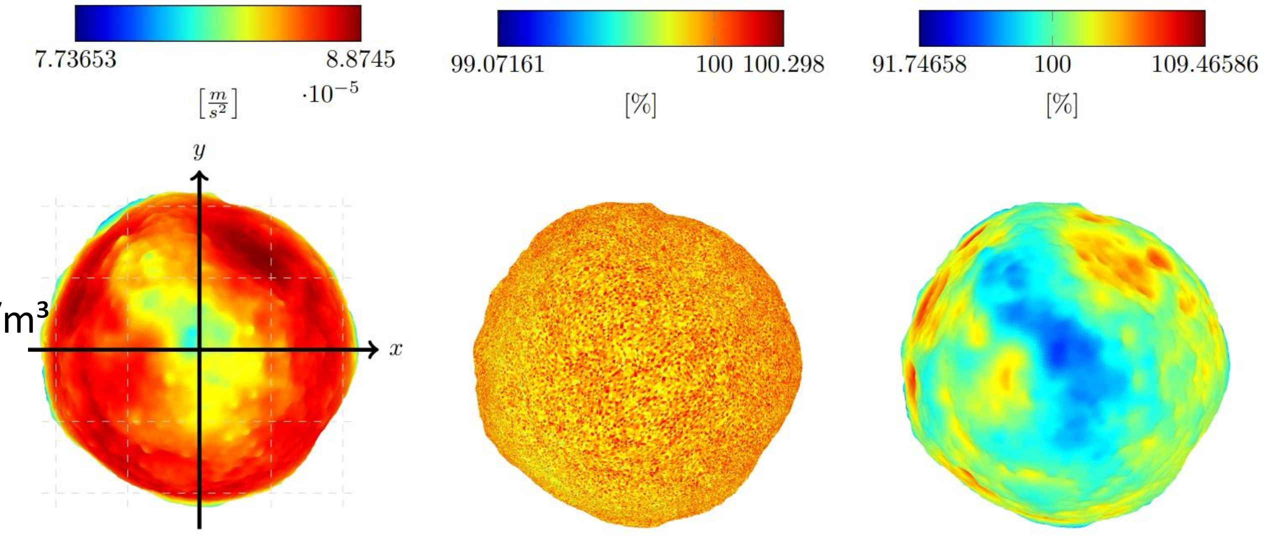
Ongoing and Future Work: Experiment 3



- Regolith Layer on Bennu as inhomogeneity (10 m surface with some smoothing at core shape.
- Make inhomogeneities increasingly complex.
- Kept total mass of Bennu constant
- Density: Starting from total mass 7.8*10^10 kg, density for homogeneous 1266 kg/m³
  Introduced density contrast for regolith -250 kg/m³ (assumed)
  Readapted core density to 1301 kg/m³ to keep **total mass** of Bennu **constant.**



(a) PM, gravitational acceleration. (b) MSP, relative error. (c) MASC, relative error.

Method	Min. [%]	Mean [%]	Max. [%]	σ [%]
MSP 800k	99.071611	99.962896	100.297938	0.102459
MASC 64.8k	91.746583	100.651412	109.46586	2.289015

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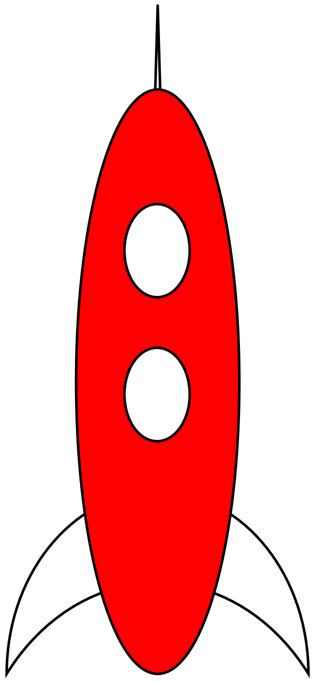
Thank you!

More information on homogeneous density gravitation computation comparison:

Meißenhelter, H., Noeker, M., Andert, T., Weller, R., Haser, B., Karatekin, Ö., Ritter, B., Hofacker, M., Machado, L. & Zachmann, G. (2022, March).
Efficient and Accurate Methods for Computing the Gravitational Field of Irregular-Shaped Bodies.
In 2022 IEEE Aerospace Conference (AERO) (pp. 1-17). IEEE.



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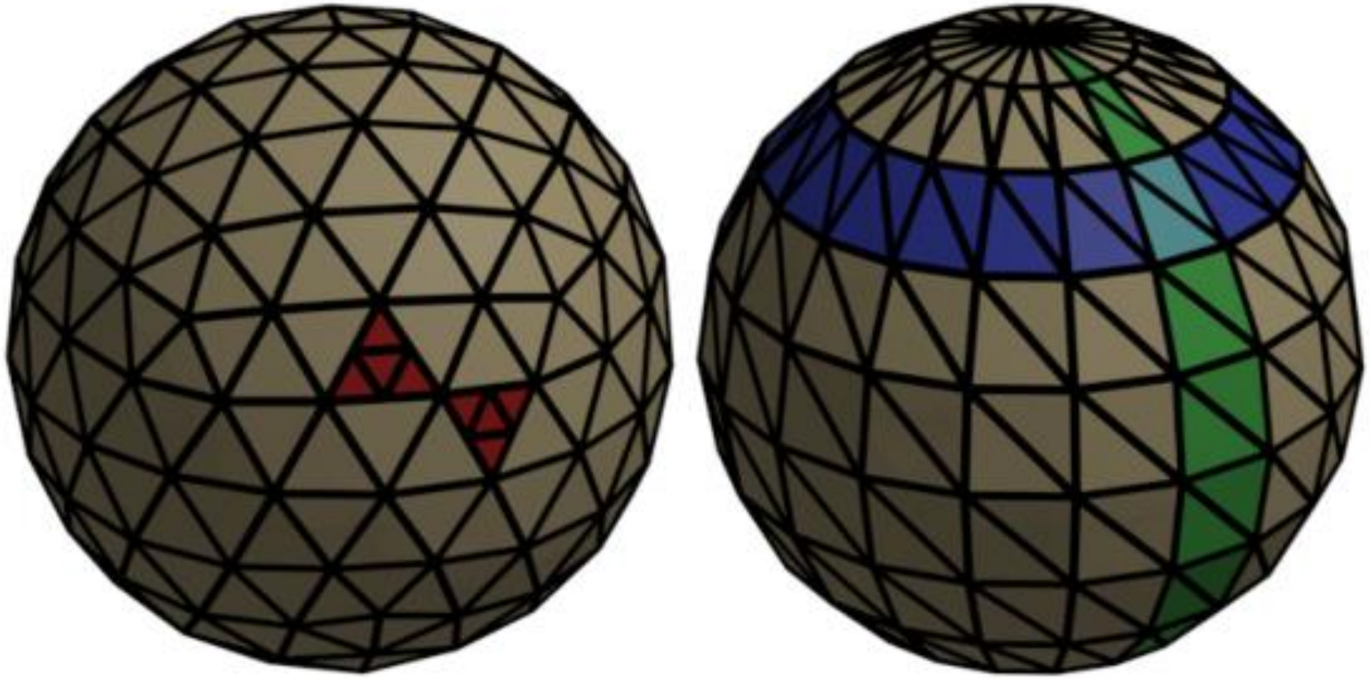

(a) (b)

Figure 6.1: (a) Icosphere (b) and UV-sphere. The number of facets for the icosphere grows by a factor 4 per subdivision, thus the precise number of facets cannot be chosen arbitrarily. On the contrary, the UV-sphere subdivision is controlled by spherical coordinates, and thus the latitudinal and longitudinal subdivision has a larger adaptability. From Meißenhelter *et al.* (2022)