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Assignment on Advanced Computer Graphics - Sheet 3

Due Date 19. 05. 2015 11:59pm
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Exercise 1 (Raycasting Fan Shot, *5 Credits*)

Imagine a ray casting "fan shot", defined by a start vector (with starting point S and direction d) and angle α:

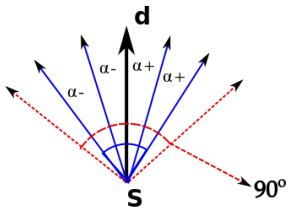

Figure 1: Fan shot view on one axis

The fan shot generates an equally distributed field of rays (see Figure 2), starting from the start vector, for the x and z axis (these are two vectors perpendicular to direction d) (applying the above illustrated scheme), whereas d shall be the pointing in direction y:

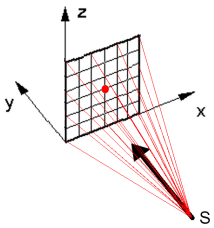

Figure 2: Complete fan shot

The fan shot shall be limited for each axis by a total of 90 degree as show in figure 1.

1. Give a pseudo code, which describes the generation of the fan shot rays
2. Implement your algorithm in C++. You can use the math template classes from the previous framework
3. Test your implementation with the following test cases and document your results:
   - Start vector with S = (0, 0, 0), d = (0, 0, 1), $\alpha = 45$
   - Start vector with S = (0, 0, 0), d = (0, 0, 1), $\alpha = 30$

   Please note: Your code should consider the general case (i.e. for any direction d and S) not only for d = (0,0,1) and S = (0, 0, 0).

Exercise 2 (BVHs for Ray Tracing, *5 Credits*)

You implemented a simple raytracer with a recursive ray trace function, a camera, a phong lightning and intersection test. The scene objects are managed in a list and you test every ray against every object in the list.

Your tasks:

1. To accelerate the ray intersection test, we want to use a hierarchical data structure called bounding volume hierarchy (BVH). Propose a (good) algorithm (in your own words or pseudo code) which creates a BVH for ray tracing applications.
2. Consider the following two-dimensional scene:
   - Triangle A = (-3, 3) B = (-1, 3) C = (-3, 1)
   - Rectangle A = (-2, 3) B = (-1, 3) C = (-2, 1) D = (-1, 1)
   - Circle r = 1 C = (2, 2)
   - Triangle A = (0, 2) B = (0, 0) C = (2, 0)
   - Triangle A = (1, 0) B = (1, -1) C = (0, -1)
   - Triangle A = (0, -2) B = (0, -3) C = (-3, -3)
   - Quad A = (2, -2) B = (3, -2) C = (2, -3) D = (3, -3)
3. Make a suitable two-dimensional sketch of the scene with bounding boxes
4. Build a BVH with your proposed algorithm (by hand - do not implement your algorithm) and give the resulting bounding volume tree. Argue why your bounding volume tree (respectively your proposed algorithm) is a good solution. (Remark: A good proposed algorithm has more advantages than a bad one)

Exercise 3 (On gradients/normals of implicit surfaces, *2 Credits*)

1. Find the normal $\mathbf{n}(\mathbf{x})$ for the implicit surface given by the function $f(\mathbf{x}) = x^2 + y^2 - z^2$ (hyperboloid).
2. Prove that
$$\frac{\partial}{\partial \mathbf{x}} \|\mathbf{x} - \mathbf{x}_i\| = \frac{(\mathbf{x} - \mathbf{x}_i)}{\|\mathbf{x} - \mathbf{x}_i\|}$$
where $\mathbf{x}_i$ is a constant point in space. It is sufficient to calculate just one partial derivative, e.g., $\frac{\partial}{\partial x}\|\mathbf{x} - \mathbf{x}_i\|$ Tip: try to write $\|\cdot\|$ in expanded form, then apply the chain rule.